# AIR PHOTOGRAPHY APPLIED TO SURVEYING

BY

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WITH DIAGRAMS AND ILLUSTRATIONS

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# FOREWORD

It is always a pleasure to welcome a book by a man who is a real authority on the subject upon which he has written. This book is one of that kind. As Senior Lecturer, for some years, in the Department of Engineering at University College, in the University of London, Dr. Hart gained wide knowledge of Civil Engineering and Surveying. To this, as a result of special study both in this country and abroad, he has added a deep insight into the science of aerial surveying and photogrammetry. Being thus an authority on both the ground and the aerial aspects of surveying he is, I think, eminently qualified to write this book.

Perhaps I may be allowed to take a sort of paternal interest in it, for its author tells me that it was a remark made by me some years ago that caused him to realize the great importance of the subject and determined him to give it his serious attention.

Air photography, like most other inventions, sends its roots deep into the past, it is almost co-eval with photography itself. The first serious application of the new art had, however, to await the war period of 1914–18. It was then that it was born, the years previous having been its period of gestation in the womb of time. Upon its immense importance and utility both to the Navy and the Army during that war, crude as were the materials and methods that had to be used, I will not enlarge. Everyone knows about them. In spite of this, when the war ended it nearly suffered sudden death, and it was only by the persevering efforts of a few brave pioneers that its development was assured. The survey of the Delta of the Irrawaddi River for the Government of Burma, carried out by one of these pioneer firms, early showed what valuable results could be obtained. In this area, owing to the difficulties of the terrain, a terrestrial survey was nearly impossible, or at any rate would have taken many years to complete.

Another outstanding example was that of a railway in West Africa which, as a result of a ground reconnaissance, was constructed for 120 miles, when it encountered a serious obstacle. A subsequent air survey showed a better route about fifty miles to the east and one by which running costs would be so reduced that it was decided to scrap, at a loss of approximately £1,500,000, the portion already made.

Examples such as these were sufficient to prove the value of air surveys for those parts of the world which had not already been mapped, and countries like Canada and America made full use of them. But in countries like our own where accurate and large scale surveys of the whole land exist, their immense utility has not yet been fully realized. "Considering the map as a representation of detail on the Earth's surface," says a recent American authority, "the aerial camera records these details in a manner quite impossible by any other method." Its outstanding factors are speed and the recording of detail.

We are, I believe, entering on an era when the old *laisser-faire*, happy-go-lucky tactics of our forefathers, will disappear, and when we shall have to adjust most of our operations to plans and programmes that have been carefully devised and revised beforehand. For this, like a General who has to co-ordinate in his plan of campaign not only military, but many other facts to achieve his objective, we shall require reliable and continuously up-to-date information of the whole domain in which we are to act. For the supply of much of this we shall, in the future, have to make greater use of the aerial surveyor.

I am reminded of echo-sounding, upon the development of which we in this country have done such excellent work. It used to take years of painstaking toil to procure a reliable chart of the ocean bed. But the Echo-sounder gives an instantaneous picture, exact in almost every ripple of the surface below, and I need not emphasize how immensely useful it has been to Navigators and Civil Engineers. In the same way the picture flashed to the aerial camera gives details that escape the eye and sense of the most accurate surveyor. While I do not think that we shall ever be able to dispense completely with the old methods, I do definitely consider that we shall find more and more how essential the aerial survey is to supplement and give details that are discovered later on the ground.

In the past books on this subject in Great Britain have mostly been written by men connected with commercial organizations, for specialists in mapping, photography or instrument making. Dr. Hart's book begins a new line. It is, I believe, the first to be written from the standpoint of the prospective user, and I personally welcome it as an excellent and most opportune addition to our literature of Air Surveying.

ALEXANDER GIBB.

## PREFACE TO NEW IMPRESSION

THE first edition was completed in August 1939 and presented an up-to-date review in respect of new developments and experiments as well as of methods in general use. Details of the employment and development of Air Survey during the recent war are now generally available and the present position may be reviewed.

Operational air photography often necessitated the sacrifice of technical to tactical requirements, and much effort was expended in securing survey photographs and in using them for making and revising maps. Established methods were employed and developed, while new approaches were tried and applied and the general effect of war-time mapping has been to establish and improve methods which are economical in man power. Many thousands of square miles of new tactical maps in all parts of the world were produced from air photographs of areas to which we were denied access at the time. In areas where the existing map framework was reliable air photographs were used for revising hundreds of thousands of square miles.

An important change in the scientific approach to survey has resulted from experimental work on the applications of radar to surveying. Pioneer research work was carried out in this country as a result of the necessity of producing tactical maps in virtually unmapped areas of S.E. Asia, then in enemy occupation. This work was initiated by the Directorate of Military Survey, War Office, and the author was actively concerned in these experiments. A suitable technique has been developed for the production of reasonably accurate maps up to a scale of 1/25,000 from air photographs fixed in position from remote radar controlling stations up to 200–250 miles away, and with no direct access to the ground in the area mapped. With such control air photographs can be taken accurately in the required positions in relation to the earth's surface, thereby economizing greatly in navigation. Moreover, it has been found possible to utilize the method of calibration of radar range measurement devised during the early experiments to the measurement of long lines up to 500 miles or so by taking a series of range readings from several flights across the base line, the accuracy being of the order of 1 part in 20,000. Such accuracy is significant for basic geodetic survey, particularly in bridging wide gaps across the sea.

The experimental work on radar controlled survey has been described by the author in two papers at the Royal Society Empire Scientific Conference in 1946. Although there is still much research to be done and results of its application in practice are awaited, without doubt radar control of one form or another will become an accepted method in the future.

During the recent war, mapping from the standpoint of the Allies was divided up between the British and American Survey Services, and in general both followed similar methods. Much of the plan work was based on the radial-line method, but the slotted template modification described in the first edition has proved remarkably effective and enables plans to be produced in about a third of the time and with less than half the ground control required by graphical triangulation as in the Arundel method.

The Multiplex method of projection has been widely used and is standard equipment in both the British and U.S. Survey Services. It is of great value for topographical mapping, and particularly so when used in conjunction with the radial-line method. The most effective application is for bridging height control over a number of overlaps by providing additional control heights and plotting control contours. The remaining contours are interpolated by simple stereoscopic examination as described in the first edition.

Many special problems were solved by photogrammetric methods during the war. For example, the gradients of many assault beaches to depths down to 20 feet below low water level were determined with sufficient accuracy by determination of wave velocities from accurately timed and overlapping air photographs. Also, airfield sites in enemy areas were selected from suitable air photographs and the approximate field work determined.

The war has finally established the vital necessity of adequate air photography to survey standards at a very early stage of economic development of a region, not only to enable the required maps to be prepared without undue delay, but so that photographs are available for interpretation by all the scientists and others concerned in such developments.

It has been shown also that successful surveying from air photographs depends upon full appreciation of the scientific requirements of air photography and of methods of mapping from them. Close technical liaison is necessary between surveyors and air photographers, in order that accurate navigation and altimetry can be achieved; and that photographic materials are selected and processed for minimum distortion of the perspective of the photographic image.

The more successful the photographers are in taking photographs

exactly where they are required and producing them with minimum distortion, the easier will be the task of the surveyor in producing maps of the desired accuracy.

As a result of developments in photographic materials; of precise stereoscopic measuring instruments and of the application of radar measurements, it is probable that it may soon be economically and technically sound to make engineering surveys of quite large scales from photographs taken by a gyro-stabilized camera fitted in a helicopter, the position in space being recorded within a yard or two by radar methods. By such a method a large proportion of the dense ground control now required for large-scale surveys would be eliminated, but here again much research is required.

In the first edition reference was made to the lack of academic recognition of these new developments in the British Empire as distinct from other countries. There is now a very live appreciation of the use of air survey in economic development throughout the Empire. For example the University of London has now established a Chair of Surveying and Photogrammetry tenable in the Faculty of Engineering at University College, where the establishment of a laboratory fully equipped with all types of modern apparatus is in progress for the purposes of undergraduate teaching and specialized research.

UNIVERSITY COLLEGE C. A. HART
LONDON
*March*, 1947

# PREFACE

THE employment of air photographs has now progressed considerably beyond the experimental stage in its applications to surveying, and increasing use is being made of them for mapping purposes and for designing engineering and other economic projects.

It is not always easy to convince those responsible for the financial side of large schemes that preparation of a reliable preliminary survey and accurate maps during the early stages of a large undertaking may save many times their cost. Formerly, on account of the time required, it was not generally possible to wait until the survey was completed and the map plotted before economic development such as the building of routes could be commenced. Air photography has furnished a reliable and rapid method of providing the necessary information for such purposes. In this way the area may be examined as a whole. It may be considered that the initial outlay is high, but this is offset by the variety of uses which may be found for the air photographs, and for which the respective Departments should pay. The value of time gained and the greater reliability of location are also factors which must be weighed when considering the suitability of the method.

It is important to realize at the outset that air photography provides another method of surveying which is additional to and *not* a substitute for existing methods.

Improvements in the accuracy of reproduction of photographic images, in the navigation of aircraft, in the determination in space of the position of exposure, and in the design of photographic and plotting apparatus have considerably reduced the costs and widened the scope of air survey during the last few years. This has been particularly so in the field of large-scale planimetry and of contouring which are of special interest to those concerned with the economic use of land.

Although the technique of air survey is now such that the resulting map will have, under suitable conditions, an accuracy comparable with that of a survey made entirely on the ground, survey control on the ground will always be required except for approximate reconnaissance surveys. For large-scale plans actual measurements to some points of detail are also required. Air photographs may be used for the preparation of the

lay-out or preliminary survey of an engineering scheme, but the final setting out must always be done on the ground.

Air survey technique has been chiefly developed by instrument makers and by military surveyors, whose policies have not always coincided. As a result, it is often difficult for those able to make technical use of air photographs to decide upon their suitability in particular instances. The Continental instrument-makers are primarily concerned with the manufacture and sale of their photogrammetric instruments and their recommendations as to method naturally involve use of their instruments. These makers do not, naturally, sponsor with enthusiasm any method whereby the employment of their instrument is not essential. The military surveyors, however (and particularly in Britain), have developed graphical methods of plotting and contouring. Recently, as a result of increasing accuracy in the navigation of aircraft by automatic pilot, the scope of these simple methods has been considerably enlarged.

There have been indications lately that a common level is being approached by using the more elaborate instruments to provide the control (as in the case of the theodolite in ground surveys), while simpler and cheaper instruments are used for contouring and filling-in detail in the same way as the plane-table on the ground. This procedure obeys the well-known axiom of surveying, namely of working from the whole to the part, and it removes one of the strongest arguments against the use of the elaborate instrument. This has been said to provide a "bottle-neck" in the plotting system because everything has to go through it when it is used for plotting the detail. The new procedure is to use the large machine to establish the absolute orientation of each pair of photographs and the positions of control points, both in plan and height. This setting can then be reproduced in a much simpler instrument, in which contours and detail are plotted. By employing several of these smaller instruments to one control instrument, the mapping proceeds with expedition.

Much more attention has been paid to the scientific study of photogrammetry elsewhere than in the British Empire. There is an International Society of Photogrammetry of which there are affiliated bodies in most countries except this one. In the United States, for instance, there is the American Society of Photogrammetry which issues a standard specification for photogrammetric work. In Continental countries and the United States, there are special University Departments in which photogrammetry is taught. These Departments are equipped with plotting instruments, which are in some cases available not only for training purposes and research, but also for plotting actual surveys. The outstanding example is perhaps the Department of Professor Schermerhorn of the Technical High

School at Delft in Holland. Among other instruments the Department is equipped with plotters, both of the elaborate and simple types, and in addition to teaching and research, much plotting work is undertaken.

Other Departments which may be mentioned as examples are those of Professor Hugershoff at Stuttgart, Germany; of Professor Earl Church at Syracuse University, U.S.A.; the Technical High School at Zurich in Switzerland, and of the University of Liége in Belgium.

In some cases a specialized training in surveying, including photogrammetry is provided, but generally the Department gives elementary training to Civil Engineering students, while there is opportunity, as for example, at Syracuse University, to concentrate more on the subject by selecting it as a special subject in the Civil Engineering Course.

The author, no doubt in common with other lecturers in surveying at the British Universities, has been able to give some elementary training in photogrammetry as part of the advanced course in Surveying and Geodesy, but there seems to be a clear need for a special institute or department in Great Britain, where training and research in the subject can be undertaken.

In the preparation of this book an attempt has been made to present the point of view of the prospective user rather than as has generally been the case that of the official surveyor, instrument maker or commercial operator. For this reason, the historical section is followed by a discussion of the engineering, economic and scientific applications of air survey, and this part of the book may be read separately from the rest. Then follows a description of the principles of photography, navigation, perspective and stereoscopy. These should be understood before the various methods of plotting, plan and contours which follow, are studied.

Actual examples of air surveys mentioned in the text are used in order to give an idea of the scope of the applications of air photography to surveying but are not intended in any way to be a complete list.

With regard to photogrammetric instruments no attempt has been made to describe more than a selection of them; there is a number of other instruments available, notably in France and Italy.

In the sections on interpretation and economic applications, photographs have been freely reproduced as illustrations, but it is necessary to understand that these have been selected to show particular effects. In practice, experience in interpretation and in stereoscopic observation (with which the former should always be coupled) is necessary before success can be expected. Since, however, the possible user of air survey is not likely to have available a library of photographs for his practice, it is to be hoped that the inclusion of these illustrations will make the possibilities of the applications clearer to him.

Those who wish to study thoroughly the scientific basis of photogrammetry cannot do better than to read Hotine's book *Surveying from Air Photographs*. Major Hotine has probably done more than any other man to put air survey on a sound and rational basis. This book, and others by Hotine, have been consulted freely by the author. There is a reference to these and to other books in the bibliography.

A nucleus for the book has been provided by a Chapter on air survey written by the author in a book, *Principles of Road Engineering*, of which the joint author was Professor H. John Collins, M.C.

The author is indebted to the Institution of Municipal and County Engineers for allowing use to be made of material and illustrations from a series of articles on air survey written by him and which appeared in their Journal between February and May 1937.

In the preparation of this book many publications have been consulted and much information has been gathered from persons concerned with various aspects of the subject. Notes used in lectures given at University College, London, have also been consulted. The author has done his best to credit investigations and information to the original source, and he takes this opportunity of apologizing for any unintentional omissions or misdirections. As far as possible all references and acknowledgments have been made at the appropriate place in the text.

In addition to the Bibliography, a separate list is attached of those whom the author desires to thank for the assistance they have given him in various ways.

In conclusion the Author's grateful thanks are due to Sir Alexander Gibb, not only for the Foreword which he has kindly written, but also because it was he who first gave him the incentive to make a study of the subject.

C. A. HART.

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CHAPTER I

# INTRODUCTION—HISTORICAL—OUTLINE OF METHODS

## INTRODUCTION

ALTHOUGH surveying from ground photographs had been employed to some extent before 1914, it was not till after the War that, with the increased use of aircraft, surveying from air photographs developed into a practical method for general use, and all subsequent apparatus has been influenced by the requirements of the air photograph.

Progress has been somewhat retarded, partly due to conservatism and partly due to the over-enthusiasm of those who anticipated the almost complete elimination of ground measurements in the preparation of surveys. The surveyor, cautious and thorough by training, needed time to investigate the speed, accuracy and cost of the new method before he could accept air survey and define its range of application.

War-time air surveys of military expediency lacked refinement of technique and suitable apparatus, so that maps and plans did not reach the standard required for non-military surveys.

In recent years methods of taking air photographs and of plotting from them have, for a variety of purposes, been developed in many directions. It has been difficult to follow progress in current literature, because many publications are not unbiased.

Early experimental surveys made it clear that air survey could never be entirely independent of ground control except perhaps in cases of military necessity or for rough reconnaissances. It is now admitted that air survey, if used, may be employed in preference to, or in conjunction with, earlier topographical methods, the choice being influenced by the ruling conditions of accuracy, speed and cost.

In these days of rapid local development, particularly in this country, surveying from air photographs is an expedient constantly in the minds of engineers, town planners and industrialists. Owing to war-time depletion of Ordnance surveyors, and subsequent economy, large-scale ordnance plans, namely six inches to one mile and 1/2,500, were found, in many cases, to be twenty or more years out of date. Many local authorities were forced,

as a result of post-War development and legislation, to undertake expensive local surveys, and air photographs have been extensively used for this purpose.

The Ordnance Survey Department, after a long period of experimental work, has concluded that air survey should play an important part in the revision of large-scale plans.

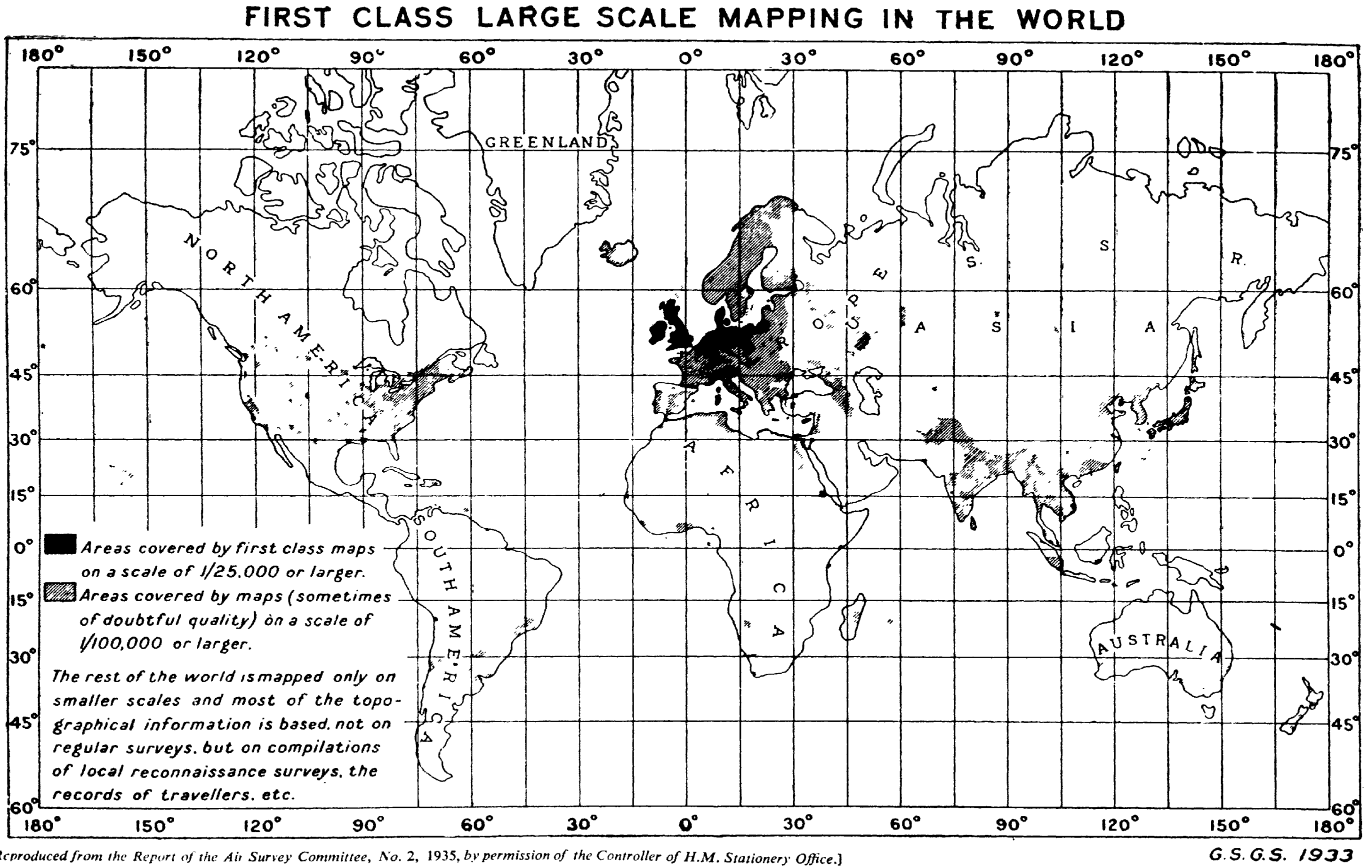
Many air surveys undertaken in Great Britain are required for revision purposes only, on account of the high degree of mapping progress of the national survey. Those who live in Britain find it difficult to realize that many parts of the world are not mapped accurately, and where maps are published they are usually on medium or small scales only.

The progress of first-class large-scale mapping in the world up to 1933, is shown in Fig. 1. It is only in Britain that complete, or almost complete, publication of sheets at scales as large as the six inch and twenty-five inch of the Ordnance Survey has been achieved so that the plan of any required area can be purchased on demand. It may be observed that it is obviously impossible for a diagram at the scale of this one, to indicate all those scattered areas of small extent where large-scale surveys have been made.

Most of the continental large-scale surveys, where complete, are available only in manuscript form. In the 1935 report of the Air Survey Committee [6] it is pointed out that the existence there of close networks of triangulation stations and much levelling data, together with the lack of published large-scale plans, provides a considerable market for precise large-scale surveys by air photography. As a result of this need for large-scale contoured plans, Continental instrument makers, such as the German firm Zeiss of Jena, and the Swiss firm Wild of Heerbrugg, have developed highly specialized methods of plotting which are dependent on elaborate and expensive stereoscopic plotting machines.

Photographic surveying, or photogrammetry as it is often called, has perhaps its greatest value in those countries where development is as yet in its early stages. Nevertheless, there are many instances in which it can prove of value even in countries which are mapped in great detail.

Photographic surveys can be plotted from ground photographs taken by the engineer, who can carry through the complete operation. It is now, however, more common, except in the case of very large scales in mountainous country, for the survey to be plotted from air photographs. In such cases the photographs must be taken by a specialist organization. Air photographers and surveyors will undertake to photograph on any desired scale and produce the completed plan; or they will take photographs in co-operation with the engineer who can provide the ground




Reproduced from the Report of the Air Survey Committee, No. 2, 1935, by permission of the Controller of H.M. Stationery Office.]

FIG. 1.

control necessary. The engineer may later carry out the plotting from the photographs.

It is obviously impossible for every engineer himself to acquire elaborate plotting machines and he must decide whether he will rely upon the specialist for the completed plan, or use a plotting method which will enable him to keep both a practical interest and control in the plotting. His knowledge of the district will be of value in this work, particularly in the provision of ground control and in interpretation.

Government Departments, such as the Ministry of Transport, which has lately been giving attention to air survey, might benefit by setting up a section to deal with air survey. Such a Department could well afford some elaboration in the choice of special plotting apparatus and this might also be available for selected work of local authorities.

The London, Midland and Scottish Railway has established its own air survey department, chiefly employing graphical processes on rectified prints.

In the less-developed parts of the world air survey offers unlimited scope. Satisfactory development of a "new" country must depend upon a clear knowledge of the topography, and economic and agricultural possibilities. Nothing is more effective than a reliable topographical map in the early stages, and nothing can provide this more rapidly than air survey. The great advantage of air photographic methods is *speed.* Engineering surveys by ordinary methods are often of no value except for the particular project in hand, but those produced from air photographs will also make available photographs which can be used for a variety of purposes—geological, topographical, prospecting, forestry, etc. After they have been taken, the photographs may be filed until they are wanted for the plotting operation, which may be spread over a period of time.

These photographs will be of value in engineering reconnaissance surveys. By locating routes from the air a wider range of country can be covered in the time available than by ground methods, and this gives more alternatives from which to select the final route.

It is now established beyond doubt that in countries which are largely undeveloped a satisfactory balancing of speed and costs can be achieved by a combination of ground and aerial work. It is not usually an economic proposition to make air surveys of small areas, unless photography for a number of small surveys is grouped.

The influence of climate is important, and in this country the number of suitable days for air survey photography does not average more than about thirty a year. A high cloud layer is essential. A small cloud and its

shadow will not only blot out detail but also, almost invariably it seems, cover a part of the photograph vital to plotting. It may be weeks before a suitable half-day occurs and the photographic personnel must be ready for it. Given a clear day photographs can be taken to keep plotting going for weeks.

It is intended here that the viewpoint of the user, such as the engineer, should be taken, rather than that of the specialist map maker. The user, with draughtsmen on his staff who can plot consistently to at least ·01 inch, should be able to produce reliable plans from properly taken air photographs. Owing to the rapid development of and improvements in methods, details of photographic survey technique are not widely known among engineers and on account of the specialized nature of some stages of the process it has been difficult to form unbiased opinions. In this country research has been largely in the hands of the Air Survey Committee of the War Office, which is naturally more interested in military mapping problems; and of the Ordnance Survey, which is concerned chiefly with revision. In Canada a special method has been evolved for preparing small-scale plans of flat country. In the United States, departments such as the Geological Survey and the Soil Conservation Service have mapped thousands of square miles and made valuable contributions to the subject. From Germany, particularly, many elaborate instruments and dissertations have emanated, the manufacturers seemingly having a "bottomless" store of funds for research in the scientific field of photogrammetry.

There are a number of competent British firms which are ready to undertake the photography of areas for preparation of plans on any scale. The Air Survey Committee in its recent report (1935)[6] states that the British Companies "have carried out several contracts in India, Africa, Iraq, South America and other places . . . but, unfortunately, the cartographic technique employed on the majority of these contracts has been of the simplest kind. Their work has proved, on the whole, to be of little value to the Committee in throwing light on the strictly cartographic problems which it had to study."

The British firms were formed on a commercial basis, and research on special problems and methods has been limited owing to lack of funds. In its final report (1938) the Departmental Committee on the Ordnance Survey [72] (known as the Davidson Committee) has recommended that an official co-ordinating body should be formed to consider the general problem and control research.

Recently a definite advance has been made in the organization of British air survey. The firm of H. Hemming and Partners Ltd. has been able to combine the interests of their company with those of the Aircraft Operating

Company Ltd., the Aircraft Operating Company of Africa (Pty.) Ltd., and Aerofilms Ltd. The Aircraft Operating Company has now assumed control of the group and these four companies, together with the Air Survey Company Ltd., are officially recognized by the Government.

In view of the recommendations contained in the Report of the Davidson Committee, the Aircraft Operating Company are establishing an Air Unit in the British Isles. The nucleus of this consists of two De Havilland 89 Rapide Aircraft specially modified for air photography, and fitted with Smith's type automatic pilots. Also precision cameras are being supplied by the Williamson Manufacturing Company Ltd., to satisfy the specification laid down by the Ordnance Survey.

In order that all classes of survey may be dealt with, apart from those requiring rectification of photographs and simple graphical methods of plotting, not only have two Cambridge Stereocomparators been installed, but also a much more elaborate instrument in the form of a Wild Autograph, Model A5.

The Aircraft Operating Company of Africa apart from extensive work in Africa, also undertakes special work for the Government of South Africa, and aircraft and personnel incorporated in the South African Defence Force under the title of the Transvaal Air Survey and Photographic Squadron. It is anticipated that the British Air Unit will be considerably enlarged in the near future, and during the winter months the aircraft will be available to augment the Unit in South Africa and to carry out work in other parts of the world.

The Air Survey Company while carrying out surveys in any part of the world, has particular interests in India, where an associate company has mapped considerable areas.

The future looks much brighter for air survey and it is to be hoped that this nucleus of an Empire Air Survey Organization will enable some co-ordination to be achieved in Empire Survey policy.

## Historical Development

### *General Development of Photographic Surveying.*

Surveying from photographs is by no means a new method. Soon after the introduction of photography just over a hundred years ago it was realized that photographic images were formed according to the laws of perspective. If a photograph is taken from each of two stations at a known distance apart, any point which appears on both photographs can be located in space. Surveying cameras and plotting methods were developed by various people during the nineteenth century, but the preparation of

plans was so laborious and photographic technique so crude that little impetus was given to it.

At this stage surveying from air photographs was no more than an idea, although the possibility of surveys from balloon photographs had been considered and experiments made.

Photographic surveying, as it is known to-day, has developed from the crude methods evolved during the Great War. As in so many other instances, the exigencies of war forced improvements for military needs and these improvements have since proved adaptable to the needs of peace. At first, photographs were taken by hand-camera and it was found that a series of exposures taken at, say, daily intervals of an area where the enemy was active, would show any changes clearly, and "bushes" which grew overnight were at once suspect. Camouflage had to be good to escape the searching eye of the camera; dummy trenches, effective on the ground, threw the wrong shadows in an aerial view: tracks made by a body of troops were easily discerned, and the later stages of the war were notable for the ingenuity with which attempts were made to deceive the air camera. Actually the air photograph has made camouflage very difficult as, for instance, in the overhead cover of an artillery battery which is easily detected by stereoscopic examination of a pair of photographs.

It was realized that maps could be made from vertical air photographs, and if a sufficient number of photographs were taken from about the same height with the aircraft approximately horizontal, they could be joined together to form a "mosaic" or photographic map. Mosaics are in great demand for economic purposes, but when presented in the form of photographic maps, suffer from the disadvantages that they can never be quite true to scale and that too much detail is shown. These mosaics were much used for trench maps.

It was found that when certain conditions of flying were observed and vertical photographs taken with about sixty per cent longitudinal overlap, line maps could be produced very rapidly, the accuracy being largely dependent upon the quality and amount of ground control provided by the ground surveyor.

During the War much progress was also made by Germany, but on different lines and with a tendency to rely upon elaborate stereoscopic viewing apparatus and automatic plotting machines, rather than upon the simpler semi-graphical methods used by the Allies.

Subsequent advances in many countries resulted in divergences in methods and technique according to requirements and inclination. On the Continent ground photogrammetry by stereoscopic methods had been developed at the same time as air photogrammetry. In Britain the practice

has generally been to simplify plotting processes as much as possible, although it is agreed that contouring with minimum ground control requires somewhat more elaborate apparatus.

*Ground Photographic Surveying.*

It was in this field only that practical results were obtainable until the "heavier-than-air" craft became established. In 1851 Laussedat of the French Army commenced experimental work and in 1861 produced a survey of a village near Versailles. Following Brunner's photo-theodolite of 1859, others improved the camera, theodolite and measuring devices. Laussedat's work, was primarily intended for military purposes, and much of the subsequent research work in other countries was undertaken by the military authorities. In time, this led to instruments of precision such as the Zeiss Field Photo-theodolite in Germany in 1906 and the Bridges-Lee Photo-theodolite in England at about the same time, or a little earlier.

The year 1888 saw the introduction of photographic surveying in Canada by Captain Deville, Surveyor-General to the Dominion Lands. Surveys were made from ground photographs in the inaccessible districts of the Rocky Mountains. Similar operations were commenced on the Alaskan Border shortly afterwards, and were later continued by the United States Geological Survey.

European developments on a practical scale began in Switzerland in 1890, when ground photogrammetry was used to prepare plans for the Jungfrau Railway. Much of this route was through inaccessible, mountainous country which would have been extremely difficult to survey by any other method. The real object of these early surveys was to plot inaccessible areas, but the method was not considered suitable for general use, though it could be profitably employed in mountainous country and where it was necessary to complete the ground work quickly for climatic or other reasons. In all cases the surveying camera was mounted on a tripod.

From then onwards until the Great War a number of photographic surveys, some by stereoscopic methods, were made chiefly by Canadian, German, Swiss and Russian surveyors, and frequently for railway survey in difficult country. At the present time ground photographic surveying is not much used except for large-scale surveys in very inaccessible country, such as the mountainous regions in Canada and the Swiss Alps. Vagaries of climate have been against any extensive use of ground photographic methods in Britain.

*Measuring and Plotting Apparatus for Ground Photographic Surveys.*

Much of the plotting apparatus employed for air surveying has been developed from apparatus originally designed for use with ground photographs. This has been particularly so on the Continent, and in some cases lack of true appreciation of the essential problems of air survey made these machines somewhat of a makeshift.

In 1902, Fourcade, a South African Forestry officer, described and later in 1902, Pulfrich of the German firm, Zeiss, described and produced a "Stereocomparator." This is an apparatus in which photographs exposed horizontally and in parallel direction can be examined and measured stereoscopically, arrangements being made for extension of base between the pair and for magnification. The stereoscopic measurement was on the basis of "difference of parallax" which is a measure of relative displacements of points appearing in two photographs.

About 1907, Major F. V. Thompson, R.E., of the British War Office, added a mechanism known as a "parallax drum" which enabled distances to be read without computation and he also added a simple lever for plotting. [55] The machine was not entirely automatic as certain readings and settings had to be made. This stereo-plotter was used experimentally during the survey of Fiji in 1908–10.

The first machine on these lines which gave automatic plotting was the Von-Orel Stereoautograph in 1908. Hotine [55] considers it very satisfactory for ground work but less so when adapted to air photographs. Sander [46] points out that "the stereocomparator with a copying system of such a form that in bringing the image of the (floating) mark into touch with the image of the ground, the necessary motions of the slides of the stereocomparator call for motions of the slides of the copying system" which keep the plan and height in correct position.

In 1913, Kammerer [46] put forward one of the first proposals to plot from plates inclined to one another at any angle. This solution did not employ stereoscopy, as is usually the case.

*Surveying from Air Photographs.*

The first survey from air photographs was plotted by Laussedat in France in 1858, the photographs being taken from captive balloons or kites. Later, in 1881, Woodbury in England tried plotting from panoramic balloon photographs. In 1893, Adams in the United States patented the principle of photographic intersection from balloon photographs, which has been the basis of a number of varieties of the "radial-line" method.

Even in those days the importance of stabilization of the aircraft, and

of knowing its exact position relative to the ground, led to such experiments as those of Stolze who, in 1881, not only used a ground mark two hundred metres square on level ground to establish tilt and height, but also made a proposal for gyrostatic stabilization. Schniffer, in 1892, used wires hanging down to establish the plumb-point of the photograph.[46]

Mention has been made of the small amount of progress in air survey until the beginning of the War, but it was soon extensively employed by both sides as being the only method available. In France, verticals were chiefly used, extensive ground control being available from the existing surveys. Some two thousand square miles of broken country were surveyed in great detail in Palestine on a minimum of ground control. "The assumption," states Captain H. Hamshaw Thomas, "was made that 'planes could be flown level for a certain distance at a uniform height and with wings level."[62] This assumption, vitally important for simple plotting from verticals, did not enable precise results to be obtained at this stage. The impossibility of measuring height of aircraft or tilt accurately necessitated extensive ground control for good results.

The two types of air photograph from which surveys are plotted are illustrated in Fig. 2. The distortion of an oblique photograph is obvious and is similar to the effect obtained by the amateur photographer who takes a photograph of a person in a recumbent position with his feet pointing towards the camera.

For large and medium scales, surveys are usually plotted from vertical photographs, while for very small scales the oblique is sometimes found to be preferable. Oblique photographs are generally "high obliques," i.e., those in which the horizon appears near one edge of the photograph. "Low obliques," where the horizon is not seen, are used for pictorial purposes rather than for surveying. As an alternative to the oblique, in some cases, the multi-lens camera, which takes a group of photographs at one exposure, has been developed.

In the autumn of 1920 experiments on flying for survey were undertaken by Professor B. Melvill Jones and Mr. J. C. Griffiths of Cambridge University. A grant was made by the Department of Scientific and Industrial Research and assistance was given by the Royal Air Force.[7] The results were most valuable in pointing out the directions in which improvements could be made, because air survey was still very much "under suspicion" at this time, and the conclusions reached expressed clearly many of the limitations then existing. The present scope of air surveys shows the great advances made since 1925. Some of the conclusions reached at that time were as follows:

"It is not, for instance, always possible to obtain by it the high degree

of accuracy which almost automatically is realized in the majority of ground surveys. This is particularly the case in those types of aerial surveying which lend themselves to rapid and economical working; that is to say, in just those cases where the economic factor is likely to be strongly in favour of the aerial method. . . . Stress laid upon geometric accuracy will generally, but not always, react against the aerial method."

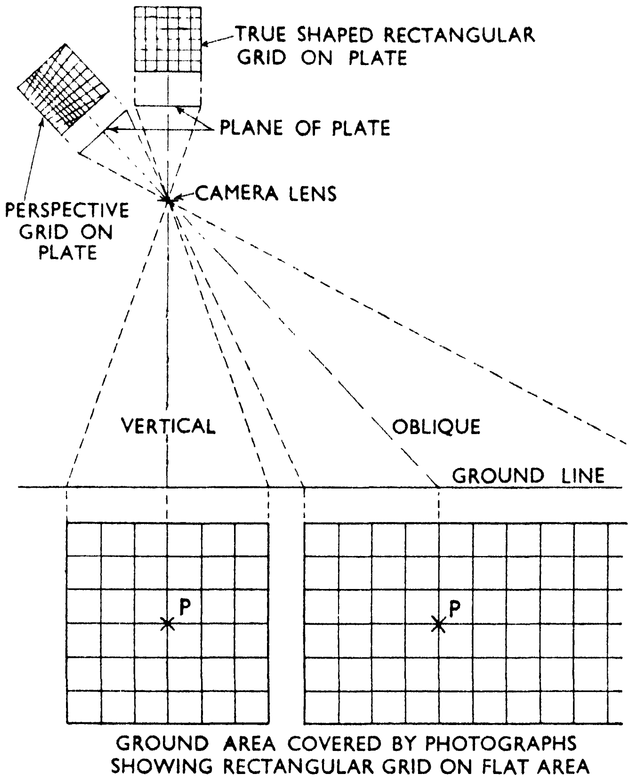



FIG. 2.

Most of these objections have now been eliminated by improvements in photographic apparatus and materials and in plotting.

The experiments showed that only specialist pilots can obtain the best results from vertical photographs. It was found that a first-class survey pilot can, in general, keep the tilt of the aircraft to less than 2°, the length of straight strip being about ten miles, ground control being at this spacing. Maps of indifferent quality could be obtained from such photographs, provided that the differences of local height did not exceed ten per cent of the

flying height. Although recording apparatus has improved, it is only by use of automatic control that these *flying* conditions can be improved.

In the case of levels, however, the position was different. "It is not possible to determine the absolute height of any point with useful accuracy." Much subsequent research work has been directed towards improvement of contouring and determination of levels.

Experiments were also made in high oblique photography. This was found to be "suitable to any kind of country, however hilly, and is capable under favourable conditions of giving some indication of absolute height." It was intended for small-scale mapping, the suggested method being long, straight, parallel strips up to ninety miles (the photographs at each point being two obliques and one vertical), with ground control fifty miles apart. Errors in distance were not greater than 1½ per cent, while ground heights could be determined to within about one hundred feet. High obliques have been used extensively for small-scale planimetry and particularly in Canada, which has been well to the fore in developing air survey, and as early as 1923 the Canadian Air Board reported that much progress had been made.

Meanwhile in 1921 the Air Survey Committee was instituted by the War Office. It had as its main objects the development of technique and apparatus for the production of (i) a line map at a scale of six inches to one mile, and (ii) a contoured map at three inches to one mile. These were to be comparable in accuracy with maps prepared from ground surveys.

Jones and Griffiths [62] observe, "While there seems to be very little scope for aerial survey in England, it may be a means of facilitating the progress of civilization in many lands overseas."

Colonel Sir Charles Close remarked in 1924 that the position was that air survey was indispensable in war, but its uses were limited in peace time; it had possibilities of immediate application to surveys of deltas, estuaries and creeks (e.g. to Irrawaddy and Nile deltas) and also for making surveys of native towns. He concluded, "The position may change, but at present it must be considered as an auxiliary method only, but one of particular importance in an area difficult of access to the surveyor." Subsequent improvements and requirements have shown the adaptability of the air photograph for all types of survey, including large-scale surveys in Britain.

The devastated areas in France were re-surveyed by the Compagnie Aerienne Française, using identifiable points of the original detail to establish the scale of the vertical photographs.

An indication of the commercial application of air survey for new mapping was given in 1924 from vertical photographs by the forestry

survey of the Irrawaddy Delta. In this survey one thousand square miles were photographed from a height of just over 9,000 feet, at a scale of 3·4 inches to one mile. Of this, waterways and unclassed forests formed three hundred and fifty square miles, and owing to the swampy land it would have been very difficult and tedious to complete the work by ground methods.

*Experiments by the Ordnance Survey in the use of Air Photographs.*

Experimental surveys in England in 1925–6 and 1928–9, tested the applications of a simple method of plotting from vertical photographs. From this has been evolved the Arundel Method, very largely due to the work of Major M. Hotine, R.E. From the military point of view it was found that plans at six inches to one mile and contours at three inches to one mile could be produced in country where the variations of height are not greater than ten per cent of the flying height. Ground control points were required every five miles or so, and spot heights at about six per stereoscopic overlap. While the preparation of a plan at this scale was satisfactory, accuracy of contouring was not so certain. The military desideratum of contoured plans for artillery action on enemy areas could not be satisfied where the ground control was non-existent. Efforts were made to produce reliable contours, but the recent tendency has been to depend more and more upon instruments of precision to bridge over non-controlled areas.

At this time imperfections of flying with regard to tilt and indeterminate height made the application of this simple form of air survey for large scales somewhat doubtful. The Ordnance Survey was actively interested, particularly with regard to revision of 1/2,500 plans which had dropped sadly behind owing to depleted personnel and funds.

The first experiments were made in 1925, and many difficulties were met. The first Report of the Ordnance Survey on Air Survey (1927) [70] makes very interesting reading in view of present conditions. It is stated:

"Delays and wasted flights were caused by defects in the sights, films and other equipment, and were in no way due to the contractors,* though the extra expense caused thereby had unfortunately to be borne by them. The first films were almost ruined by static markings. . . ."

In comparing times taken for the ground revision for 1/2,500 plans and that required when the photographs were available, the Report goes on to say: "The above figures show that by the use of aerial photographs the field work was reduced by about forty-nine per cent. It might even have been further reduced if the revisers had been able to get away from

* The Aircraft Operating Company Ltd.

their old habit of visiting all the ground. It was only after each man had gone over several Field Traces that he was convinced that the photographs had given information of all the existing improvements."

It was found as a result of this experiment that the air method cost forty-five per cent more than the ground method. The area covered was near Eastbourne and included a general variety of country and detail.

It was concluded in this report that aerial photographic methods already had advantages over ground methods for towns in flat country and areas such as estuaries or tidal flats and it was decided to initiate a series of experimental revisions from air photographs.

A second report in 1930 states [71]:

"The experiment shows that the use of air photographs as a reconnaissance is uneconomical in open downland and in suburban areas of rapid growth. It is thought that this will be true also of all other classes of property, except closely built town areas. . . . The use of air photographs as a form of reconnaissance will tend to increase the cost of revision work, compared with methods now in use. . . . In order to maintain our present standard of revision, the whole of the ground must be visited by the reviser."

The report of the Davidson Committee on the Ordnance Survey issued in October 1938, shows that conditions have changed somewhat since 1930. While it is still true that in sparsely developed areas it is cheaper to continue with ground methods, it is pointed out that in areas where there has been much development it may be both economical and useful to employ air photography to expedite revision of the 1/2,500 plan. It is also stated that in the proposed plan series, all plotted on a standard national projection, air photographs should be of value in the consequent "overhaul" of plans.

*Development of Air Survey in North America.*

McKinley, speaking principally of conditions in the United States in 1929 says [67]:

"In the single decade since the World War, aerial photography has progressed from crude rule-of-thumb work, to that of a science. Cameras and instruments are improved to a degree which at last makes accurate work possible. . . . Engineers are gradually accepting aerial photography as a sound supplementary mapping method. It seems inevitable that the aerial survey must, in time, become indispensable for mapping. In mapping inaccessible territory, the aerial survey has already assumed priority as shown by the success of the Hamilton-Rice Expedition in mapping hitherto inaccessible regions in tropical South America. . . . Considering the map as a representation of detail on the Earth's surface, the aerial

camera records these details in a manner impossible to obtain by any other method. . . . Air survey has justified its existence as a surveying method by its two outstanding factors—namely speed, and recording of detail."

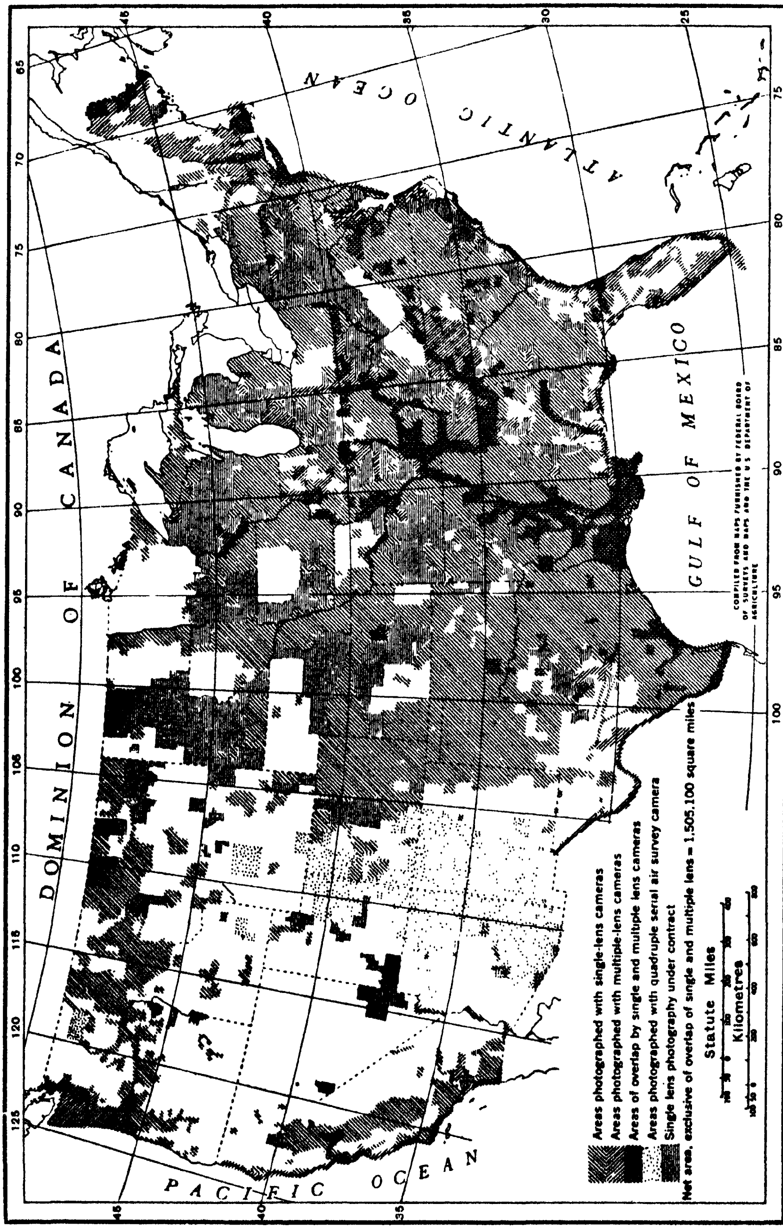
Surveying from aerial photographs was taken up seriously in the United States after the War. The United States Geological Survey is responsible for the mapping of large areas at medium scales. In 1920, using the tri-lens camera [77] the systematic air survey of Santa Domingo and Haiti was commenced. In 1920 a quadrilateral about fifteen miles square was mapped from single-lens photographs, contours being fixed by ground topographers. It was estimated that there was a saving of \$6·77 per square mile. In 1921 the Topographical Branch of the United States Geological Survey formed a Section of Photographic Mapping.

It must be remembered that the problem was quite different from that in Great Britain, because large areas were not adequately mapped topographically. In 1924 after careful investigation stereo-photography was taken up and stereoscopic plotting machinery installed. Although the procedure was quite satisfactory for planimetry, the use of air photographs for contour mapping did not become assured until 1933 when the first Zeiss Multiplex Aeroprojector was installed.

The area covered by air photography in the United States up to June 1938 is shown in Fig. 3. This diagram is self-explanatory. The greater part of this photography has been done for the various agencies of the United States Government, including the Geological Survey; the Coast and Geodetic Survey; the Tennessee Valley Authority; the Corps of Engineers, U.S.A.; the Brazos River Conservation Authority; and the Soil Conservation Service, the Forest Service, and Agricultural Adjustment Administration of the Department of Agriculture. [1]

The area given in the diagram is the total area covered and does not include re-photography of some areas where photographs are no longer suitable. Thus the Department of Agriculture in U.S.A. photographed or arranged for the photography of 1,582,052 square miles between 1926 and June 1938.

In Canada, while vertical photography has been used extensively, oblique photography was, and is, used widely in the Laurentian Shield country where there is little relief and much coastline. In this small-scale work, flying control is not so vital and the camera was hand-held at first but now more rigid mountings are used, ensuring that the horizon shows in the photograph. The Canadian air photographic library has increased from 22 photographs in 1922 to 780,000 in 1938.




[Courtesy of United States Department of Agriculture.

FIG. 3.

*Instruments and Apparatus for Air Photography.*

From the end of the Great War period onwards, efforts were being made to provide instruments which would stabilize the aircraft and camera and enable determination of the position in space at exposure to be arrived at with a minimum of ground control.

McKinley considered that $3^\circ$ was a "slight" tilt and expressed the opinion that a good photographic pilot should be able to fly strips twenty-five to thirty-five miles long with an altitude variation not greater than a hundred feet.

Many of the early efforts were with gyroscopic forms of control, either of camera or aircraft. Very little success was met with for a number of years, though instruments for indicating turning and tilt were developed. The ordinary altimeter, which records on the photograph, is very approximate, and the statoscope (or differential aneroid) enables heights to be recorded fairly accurately with reference to a particular flying height. Aneroid tables and details of a standard aneroid and corrections to be applied have been published by the War Office. [93]

For a number of years it has been possible to project accurately through lenses of limited field, but photographic surveying inaccuracies arose chiefly because of displacements of image between exposure and final printing, due largely to distortions of film and paper. Recent improvements in the production of film bases and papers with very little distortion, have made it generally possible to fix the position of the image within a plottable error. It is now also possible to produce an accurate photograph over a much wider angle of lens than formerly, the distortions being quite small in such lenses as the new Ross Wide-angle, and the Zeiss Topogon.

Some of the original experiments made by the Royal Air Force using the "Three-axes Automatic Control," show that "stabilized flight is now no figure of the imagination as it was in 1923; level flying has become a *mechanical* possibility, to the immense advantage of air survey." [6]

It was stated in 1935 that Royal Air Force policy did not provide for routine air survey photography.—"It is considered that routine work should generally be carried out by Civilian Firms. . . . It is the intention of the Air Staff, to keep available a trained personnel from which a survey unit can be formed at reasonable notice." [90] In Canada, however, much flying for Air Survey photography has been carried out by the Royal Canadian Air Force. The Departmental Committee on the Ordnance Survey [72] has recommended the formation as soon as possible of a special Survey Unit capable of satisfying the requirements of the

Ordnance Survey, and the Aircraft Operating Company as mentioned on page 5 is going ahead with this project.

The satisfactory development of the automatic pilot has completely changed the position. Flying "is so superior to that of the piloted aircraft for survey purposes, that it would seem most undesirable nowadays to attempt any air survey photography without its assistance." [6]

Major D. R. Crone, R.E., has shown that flying straight and level in taking obliques has enabled a reliable levelling method to be devised. This method he has used with success, particularly in the Himalayas.

The present position is that the aircraft can be set on a course and fly straight with a tilt of the order of $\frac{1}{4}°$. It has been a tendency of the automatic pilot to fly with a slight bias probably due to some unbalanced gyroscopic force, and experiments on a method of navigation depending on tuning in to a broadcast wireless station have been carried out by Captain Charles Lloyd who claims to have reduced this effect. The successful introduction of the automatic pilot has widened the field of survey very considerably.

Instruments such as the Aldis Camera Sight and the Course and Distance Calculator have made it much easier for the pilot to fly the area and ensure that it is all covered with the requisite overlaps.

## Air Survey Methods and Plotting

Before tracing the development of air survey technique it is desirable to separate the various methods by which maps and plans can be produced. In each case attempts have been made to satisfy particular requirements of accuracy and scale for definite types of country.

The conclusions reached by Jones and Griffiths [62] in 1925 were that "possible methods of aerial surveying can be sharply divided into two groups, one of which will give accurate results, but is not very economical, whilst the other may be very economical, but is as yet incapable of giving very high accuracy. The methods of the first group which have been intensively studied in Germany, will be of value in connection with surveys in which the factors of accuracy and detail are of great importance, as compared with the factor of economy."

In the 1935 report of the Air Survey Committee it is stated that "almost inevitably . . . photographic methods of surveying imply precise measuring apparatus and/or specially designed optical methods of enlargement. Consequently, the development of aerial surveying consists largely in the invention and design of apparatus which will measure the much reduced photographic images of the landscape in the most convenient, the most rapid, and the most accurate manner."

In almost all cases, even if the plotting is not based on a stereoscopic method, stereoscopic interpretation of the photographs is employed.

The main methods of air survey may be grouped as follows:

(i) Graphical or semi-graphical methods of plotting from vertical photographs for medium and fairly large scales.

(ii) Very accurate methods on large scales, contours and plan being produced by elaborate machines.

(iii) Mechanical methods of plotting plan and contour for medium scales with minimum ground control.

(iv) Small-scale surveys by oblique photography, from photographs taken with a multi-lens camera, or with an ultra wide-angle lens.

*Simple Methods of Plotting from Vertical Photographs.*

Plotting machines are very expensive and can be employed only by large organizations. Even then it is considered by some that such instruments may prove a "bottle neck" in the work and the War Office have therefore directed much of their attention towards plotting by simple methods from photographs taken as vertically as possible in strips, with about sixty per cent longitudinal overlap between photographs to ensure stereoscopic vision over the entire area.

It was shown that from a point on the ground vertically below the lens, known as the ground plumb point, the angle between any two lines was equal to that subtended at the point vertically above the lens, known as the photo plumb point. Great difficulties were experienced, due to unknown tilts of the aircraft at exposure which made the plumb point indeterminate unless extensive ground control was available; and to distortions caused by varying heights of the points. Instruments such as the McLeod Tilt Finder were designed to cope with this problem, but were not used extensively except in those large-scale maps where the photographs were rectified with respect to ground control before plotting. Efforts were made to improve flying conditions so that a graphical routine having known limitations could be evolved.

McKinley [67] describes this "radial line" method used in the United States. Although this technique is the basis of the Arundel Method, much of the credit for the latter must go to Hotine, who was at the time Research Officer to the Air Survey Committee. He worked out a definite procedure for graphical plotting, aided by stereoscopic examination. At first tilt distortions limited application to areas where the height did not vary by more than ten per cent of the flying height, a ground control point being required every five to ten miles for plan and five or six points per overlap for levels. Recent improvements of

photographic materials and the use of the automatic pilot have increased the scope of this method. Contouring by simple methods still presents many difficulties, and without extensive ground control, or ground levelling, it seems almost inevitable that a plotting machine will be required.

A modified Arundel Method has been adopted by the Ordnance Survey for the 1/2,500 revision and overhaul, which does not include levels. The photographs are corrected for tilt by rectification with respect to at least four known points per overlap, while the Thompson Comparator has been designed to eliminate some stages of the graphical plotting. An earlier instrument still sometimes used for revision is the Barr and Stroud Epidiascope where the photograph is projected onto a screen in coincidence with identifiable points on the old plan, so that the amendments can be traced direct.

In this, as in all large-scale surveys, ground work and measurement is necessary to fill in the data which cannot possibly be supplied by the air photograph.

Even when using simple methods, vertical photography becomes uneconomic if the scale is much less than 1/30,000 unless one of the new ultra wide-angle lenses is employed. Limitations of flying height and focal length, together with the time taken in plotting and reduction, encourage the use of multi-lens or oblique photographs.

*Accurate Large-scale Surveys.*

Most of the work in this field has been done on the Continent. Here in many cases an adequate ground control exists and there are few large-scale plans published. The photographs are taken in stereoscopic pairs, the most commonly used method of plotting being that of stereoscopic fusion.

Although fairly large-scale plans can be produced by simple methods, any attempt to produce accurate levels necessitates the employment of special instruments. The early machines used in this work were modifications of those used formerly for plotting from ground survey photographs.

Apart from special instruments for rectifying photographs in relation to ground control, few special developments took place until the appearance of the Hugershoff Autocartograph in 1920. This machine had appeared somewhat earlier in a cruder form, while a further improved model appeared in 1925. It was rather cumbersome and elaborate and consisted of three principal parts. (*a*) The stereoscopic observing system, (*b*) the surveying system, formed by two theodolites coupled by levers for computing, (*c*) a plotting system. The procedure was to set a pair of photographs by stereoscopic observation into the correct relationship as

at exposure, with reference to the horizontal plane and four ground control points. The setting of the stereoscopic floating mark to the ground at various points operated the plotting mechanism and automatically produced a contoured plan.

In 1923 [5] it was estimated that for one pair of photographs two days were required for computation and preliminary work, and one day for plotting on the machine. The reference plane was the horizontal. Hotine [55] points out that in using the apparatus a tedious computation is required to effect a resection in space in order to fix the three-dimensional co-ordinates of the two air stations forming the end of the line.

In 1924, Barr and Stroud produced a photogrammetric plotter, which was later abandoned in favour of an instrument employing the Fourcade principle.

Meanwhile, the Zeiss Stereoplanigraph and the Wild Autograph were being developed on similar lines, while the Hugershoff Autocartograph was superseded by the Hugershoff-Heyde Aerocartograph.

In 1926 [38] Fourcade introduced his "correspondence" theory, and thence evolved the Fourcade Stereogoniometer (described later) in which, for the first time, the setting of the photographs was related to the air-base. This omission in earlier instruments caused great trouble because the air-base is rarely horizontal, the aircraft being unable to fly along a horizontal plane. Fourcade showed that if five points can be accurately identified on each photograph of a pair, the photographs of the pair can be set in their correct relationship to each other without reference to ground control. One great advantage of setting with relation to the air-base is that orientation is greatly accelerated. A plotter for use with the Fourcade instrument has been devised by Captain E. H. Thompson, R.E., lately Research Officer to the Air Survey Committee and now Air Survey Officer to the Ordnance Survey. In 1932 Professor Hugershoff became associated with the firm of Zeiss and the Hugershoff-Heyde Aerocartograph has been abandoned in favour of the Stereoplanigraph which has been much improved in design. The Wild Autograph and other instruments have also been much improved in design and facility of operation.*

In France, air survey has been extensively employed, first for re-mapping devastated areas and subsequently for town planning purposes. The method, which is similar to that which has been used extensively in this country, is to take photographs as nearly vertical as possible and rectify photographically with respect to ground control.

*Mechanical Methods for Smaller Scales.*

Elaborate plotting machines of the Fourcade, Zeiss and Wild types work

* The Poivilliers-S.O.M. Stereotopograph is similar in principle and achievement to the Wild and Zeiss Instruments.

to a luxurious standard for medium scales. It therefore becomes desirable to simplify the plotting and reduce ground control, consistent with the required standard of accuracy in planimetry and contours.

In 1915, Gasser proposed a system for projecting a pair of stereoscopic air photographs apparently partly based upon a method suggested earlier by Scheimpflug. A recent adaptation has led to the Zeiss Multiplex Aeroprojector, in which photographs are projected alternately in red and blue and then observed through spectacles with eye-pieces of complementary colours. This gives a plastic effect to the landscape, and by using a plotting table of variable height and carrying a floating mark, the plan and contours may be plotted. This is, in application, the principle of the anaglyph, and the method is being given much attention at the present time. The introduction of this plotter is an important step forward. It is being extensively used in the U.S.A. and elsewhere. Wild have also recently produced a plotter (Autograph A6) of about the same standard. Both these instruments are now being used in conjunction with the larger ones. Recently, the Canadian Radial Stereoplotter [13] has provided a means of eliminating some of the graphical work in the Arundel Method.

The great difficulty in all simple methods is the production of reasonably accurate levels with a minimum of ground control. This is particularly important from a military point of view. Winterbotham [58] remarked in 1929, that in easy country a height control of four points per overlap would cost almost as much as a complete one-inch map. He emphasized the importance of the strip method and remarks that "the Fourcade instrument seeks to free the surveyor from many ground control requirements. The larger instruments are being increasingly used for horizontal and vertical control of large surveys, with minimum ground control, the detail and contours being plotted with cheaper instruments or by simple methods.

*Small-scale Surveys by Oblique Photography or by Multi-lens Photographs.*

The possibility of oblique photography interested those who wished to produce small-scale maps rapidly with a minimum of ground work.

Gordon [45] proposed a method of plotting from an oblique photograph. This method assumed that the point where the optical axis meets the plate and the tilt of the photograph are accurately known. This, however, is not the case and the difficulty of such determination has resulted in the method not being adopted.

It is chiefly in Canada that the oblique method has been developed and utilized, although experiments on a "navigational control" method of

combined oblique and vertical photography were carried out in England by Jones and Griffiths prior to 1925.

The Canadian method is to fly a strip along which obliques are taken to the front and to the sides. Later a perspective grid suitable to the photograph is placed over it and plotting effected on the plan by proportion on to the squares of known size. Narraway [67] states that given a suitable control of high accuracy a detailed topographical map can be plotted, on scales of the order of one inch to the mile, to an accuracy greater than the likely plotting error. He remarks that in one hundred and twenty-five thousand miles of Canada surveyed (1929) any point should be within 1/20 inch of its true geographical position. "This," he concludes, "is better than could be expected of a similar ground survey." It may be mentioned again that this method is only applicable to areas of slight relief.

More recent experience is given by Burns [11] and by Burns and Field [12] who describe a simple plotter for high oblique photographs. "It was obvious that the machines and methods first developed in Continental countries had no economic application in the construction of small- and medium-scale maps."

Difficulties of obtaining accurate wide-angle camera lenses, and the inherent advantages of the vertical photograph for plotting purposes, led to the development of the multi-lens camera. This has taken several forms, but consists generally of a group of lenses in fixed relationship, so that one vertical and a number of obliques are taken at each exposure. If the obliques are rectified, the final effect will be as taken by a single lens of very wide angle.

The tri-lens camera in the United States, the nine-lens camera of Zeiss and the seven-lens camera of Barr and Stroud, have opened up this field. The multi-lens seems to have possibilities for small scales, although recent improvements in wide-angle lenses have made single-lens photography available for much smaller scales.

CHAPTER II

# INTERPRETATION OF AIR PHOTOGRAPHS: MOSAICS AND PHOTOGRAPHIC MAPS: SCOPE AND COST OF AIR SURVEYS

## INTERPRETATION OF AIR PHOTOGRAPHS

THE identification of detail from air photographs and the interpretation of information given on them, requires experience for accuracy. Even then, considerable care and common sense must be exercised, or erroneous conclusions may be reached.

Information given by an air photograph and the ease with which it can be read accurately depend on a number of factors such as the angle of photograph, season, weather, scale, development and printing. While it is possible to give certain examples here as a guide to the correct identification, it should be realized that a few hours' practice in reading air photographs and in subsequent checking on the ground will indicate much more clearly what may be expected.

Oblique photographs, although not much used for large-scale mapping, are easier to interpret than vertical photographs because they present the usual view from the top of a hill or building. Oblique and vertical views of the Houses of Parliament, London, given in Fig. 4, show quite clearly how the oblique may be more easily interpreted by the inexperienced observer. He has a conventional mind-picture of objects as they appear from his "worm's-eye" view; while the skilled interpreter will have adapted himself to expect the "bird's-eye" view in a vertical.

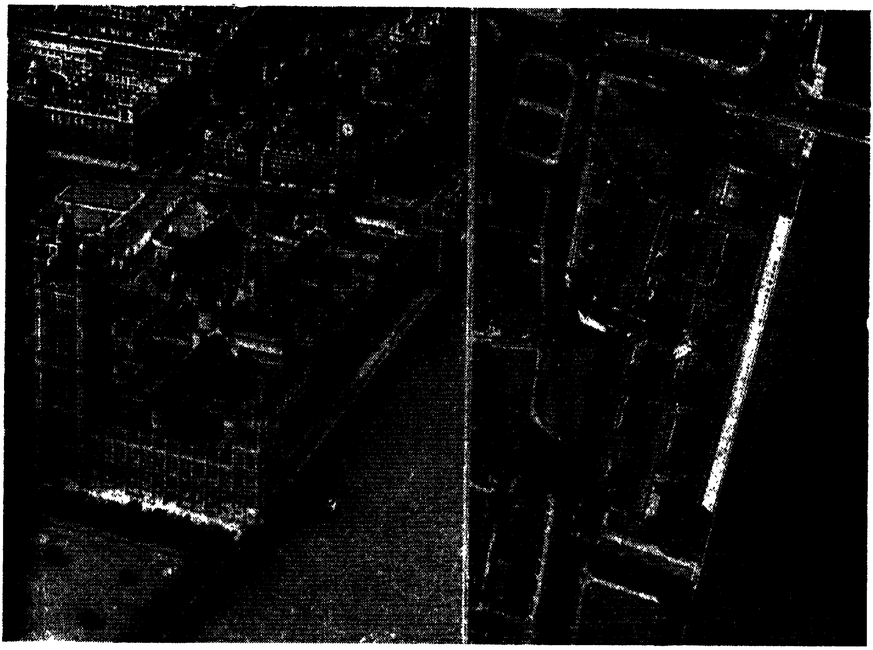
Oblique photography has been used extensively because it presents an accurate detailed view which can be appreciated easily by such persons as lawyers and members of non-technical committees.

The value of the pictorial photograph to those who are developing estates is emphasized by Collins: [22]

"A well-placed oblique of a housing estate may often (depending, of course, upon the estate) be worth columns of advertisement. While depicting the boundaries of the property with unquestionable fidelity, the map fails to distinguish between a disreputable backyard and a garden of roses; or between a field of mangold wurzels and verdant pasture. A prospective purchaser is often more concerned with the

general aspect and salubrious nature of the district, than with several decimal places of the exact acreage."

Two excellent examples of the useful employment of obliques are given by Wills [96]. Brewery companies with foresight will purchase possible sites for hotels in undeveloped areas or along new arterial roads. When development has passed the early stages, application is made to the



[*Courtesy of Aerofilms, Ltd., London and Wembley.*

FIG. 4—OBLIQUE AND VERTICAL VIEWS OF THE HOUSES OF PARLIAMENT, SHOWING HOW THE OBLIQUE IS EASIER TO INTERPRET.

Licensing Magistrates for a licence and evidence must be produced that such an application is justified, together with plans of the area. Air photographic views taken for this purpose show the roads and buildings clearly over an area of approximately a square mile and are often studied instead of the plan.

The other instance given is that of a Parliamentary inquiry. The Monmouthshire County Council in opposing the Cardiff City Extension Bill 1937, had aerial photographs taken of two or three parishes which Cardiff desired to incorporate. These photographs showed the disputed

areas so clearly that it was amicably agreed by the parties to use them so that the opposing Counsel could demonstrate their points.

The air photographer, when commissioned by persons such as Parliamentary agents, has to choose the angle and direction of photography with some care in order that he may best illustrate the important details. For legal purposes photographs cannot be joined together and they are usually mounted consecutively on linen.

In the examination of air photographs the value of stereoscopic examination cannot be overestimated. Man is able to see in the "solid" because each eye records a slightly different view of the same object in space, which enables the brain to interpret shape and size. There are, however, other factors, such as the known relative size of well-known objects, by which it is possible to judge size and shape without making much use of this stereoscopic power.

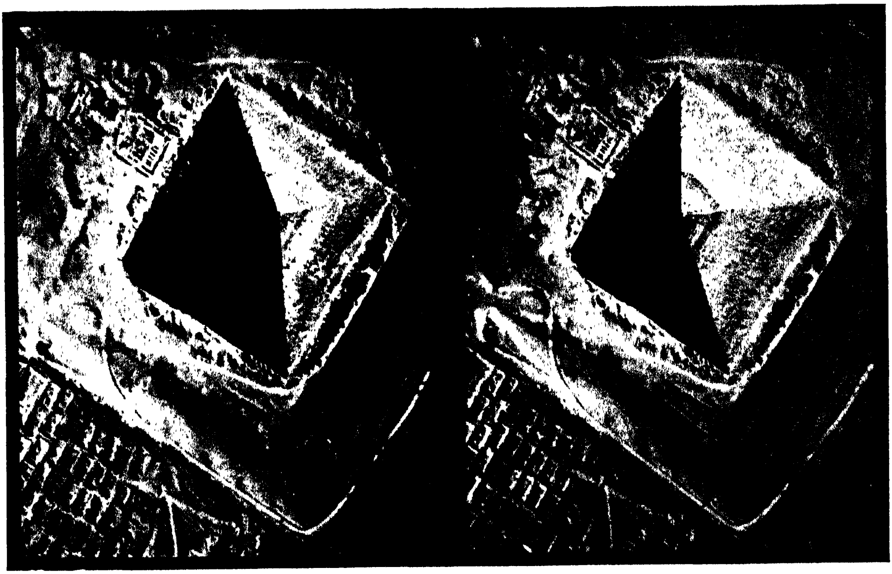
Salt [85] mentions that it is only carnivorous animals which have stereoscopic vision; the eyes of herbivorous animals being situated so that they view the world as a flat panorama. Beasts of prey have a rather larger eye-base than human beings which gives them a sharper appreciation of relative distances. When the ground is observed from an aeroplane at a height of several thousands of feet most of the relief disappears. Therefore, instead of using the view which an observer would obtain from the aeroplane, two overlapping photographs taken from different camera stations, which may be a mile apart, are viewed in a stereoscope. The effect is a view which would be seen by a giant having one eye at each end of the air base between the two photographs.

Such a pair of photographs is shown in Fig. 5. It will be noticed that in each photograph the apex of the pyramid is in a different position relative to its base. This is due to the displacement of the image resulting from its height. The length of the shadow of an object thrown by the sun on to the ground gives some idea of this if it be imagined that the sun is the camera station.

To examine a single photograph for detail is like going about with a shade over one eye. Objects which cannot be picked out in a single photograph often stand out very clearly in a stereoscopic view. For example, by stereoscopy, it is easy to differentiate a cutting from an embankment, or to separate a tree from its shadow. Again it is easy to make wrong assumptions from a single photograph. The writer, on one occasion, was out fixing points for ground control, and working from a single photograph found what looked like a very suitable road junction in a small group of new houses. On arrival at the site it was discovered that the supposed group of houses was a stack-yard and the road a very rough and

muddy farm track. This was clearly shown by subsequent stereoscopic examination of a pair of photographs. Slight irregularities made it clear that the objects were stacks and not cottages. Hotine has remarked very truly that the study of a pair of aerial photographs under a stereoscope gives the most detailed and comprehensive view of the earth's surface yet obtained by man.

While the oblique photograph gives a picture which can be appreciated by all and is thus of great value in evidence or as an indication of progress



[*Courtesy of the Wild Surveying Instrument Co., Heerbrugg, Switzerland.*

FIG. 5—STEREO-PAIR OF PYRAMIDS.

at a given date, it does not show as much detail as the vertical photograph due to some of the background being hidden by objects in the foreground. On the other hand the abundance of detail on a vertical makes it more difficult to read. Maps, which have the same viewpoint as a vertical, indicate the various objects by their conventional signs and are therefore more easily interpreted.

There are a number of factors which are likely to give to the same object widely varying appearances under different conditions.

In the first instance the scale must be considered because the basis of interpretation will differ according to the size of the image in the photograph. As will be shown later, the scale of a vertical photograph is the

ratio of the focal length of the camera to the flying height at exposure: thus if the exposure is made at a height of 15,000 feet with a lens of 7 inch focal length, the scale will be about 1/25,000 or 2½ inches to 1 mile; and if from 7,500, with the same focal length, it will be at 1/12,500 or 5 inches to the mile. A cottage 24 feet square will be 0·12 inches square if the scale is 1/2,500 but only 0·012 if it is 1/25,000.

On a large scale, therefore, it is possible to pick out quite small objects such as fences, walls and details along roads while on small scales the interpretation must be more general. A railway can be differentiated from a road because of the greater lengths of straights, long uniform curves, the absence of ribbon development and the presence of stations and sidings. In such cases it is the topography rather than detail which is being studied.

Shadows play an important part in the study of air photographs, and in order to avoid a pseudoscopic effect in which everything is seen inside-out, the shadows should run towards the observer and away from the source of light. Trees, in particular, are apt, in the single view, to mingle with their shadows, but in the stereoscope the separation is at first quite startling. For example at a scale of 1/5,000 a foot-bridge at a railway station will stand out very clearly from its shadow. Where there are shadows there is some danger of obscured detail and accordingly the films are developed very carefully to retain as much information as possible. An object which cannot be picked out in the photograph can frequently be identified by its shadow. In the vertical view of the Houses of Parliament (Fig. 4) the shadow of the well-known tower can be seen. Fence lines, or electric power pylon lines, can often be followed by their shadows. Air photographs taken when the shadows are long will often assist interpretation in such cases. A number of important archæological discoveries have been made owing to the flat rays of light accentuating slight irregularities in the ground in the same way as the headlights of a car show up corrugations of a road. Ancient roads, camps, and irrigation lines, may show up quite clearly and can be checked by subsequent examination although they have not been noticed on the ground.

The air observer has not the assistance of colour which is so valuable on the ground, because the landscape tends to appear monochromatic from a height and similarly the photograph is a monochromatic representation of the landscape. Detail can thus be identified only by its tone, and this is one of the reasons why an object may appear indistinguishable from its shadow in the single view. The tone is determined by the quantity and quality of light which imprints the image on the negative.

The modern panchromatic film is sensitive to light of long wave-length, and thus differentiates between red, orange and yellow, particu-

larly if filters of the last named colours are used to retard the blue light. Photographs taken through such filters have been found to be easier of interpretation, and Hemming [51] records the use of special filters in the survey of the Northern Territory, Australia, in order to differentiate more clearly between various types of vegetation, soils and geological formations. The tone values recorded on an infra-red film are completely different from those shown on panchromatic stock, and a different technique of interpretation is required. The use of any filter necessitates an increase in exposure and, at present, the exposures demanded by an infra-red filter are too long for the method to be used successfully in air survey."

The quantity of light reaching the film is dependent upon the reflecting power of the object. On a vertical photograph the obliquity of the rays results in a very different light value from that which is registered on a photograph taken from a ground station. Atmospheric conditions also greatly influence the tones. A field of corn will appear light on a windy day, but if the air is calm the photograph will show the ground at the bottom of the stalks, and as this has a lower reflecting power, the field will then appear dark. Water when calm may appear anything from white to black even on adjacent photographs. The amount of light reflected depends upon the direction of light, and upon the depth of water, which influences the amount of light reaching the camera. This effect of depth on tone has been found very useful in hydrographic surveying. One way of ascertaining if the area observed is water is to notice in the stereoscope if it is a smooth level surface. When the surface of water is disturbed it will appear grey. Dark objects, such as wet rocks, will often appear light in tone if the light is reflected into the camera. Variations of tone of ripening crops in apparently uniform fields has been found to indicate the presence of remains under the soil, and it has been shown that this difference is due to the varying surface compression of the soil. Where the soil is more compressed, as, for instance, over an old foundation, the crop receives less moisture and ripens earlier than over the rest of the area, causing a difference of tone.

These effects also have considerable military value because that part of a meadow which has been pressed down by the passage of men and materials will show up unmistakably, and many a gun position was discovered in the War by photographing the track of the ammunition line made the previous night. Incorrect shadows thrown by dummy trenches were also easily seen in the stereoscope.

It is important when examining a photograph to know at what time of the day and season it was taken. Survey photographers when working on large scales sometimes try to take the pictures early in the year before

the leaves cover the trees and hide the detail beneath them. Often, however, the points of economic importance are more easily appreciated from the photographs when the trees are not bare. Photographs of the same area taken in winter and summer will appear very different, and it is remarkable that in a photograph it is often the least significant detail which shows up the most clearly. If, at the time of photography, fields are being ploughed or harrowed or hay or corn is being cut, the line of demarcation will be much bolder than anything else. In the case of the crop being cut, that which is left standing will be dark, while the remainder will appear light.

Robbins has done much pioneer work in the economic use and interpretation of air photographs. His ecological work, that is in relating the topographical features to geology, forestry, agriculture, etc., has been of great value in advancing the science. He makes the following observations: [82] "The actual aids that the interpreter uses in his work are multitudinous, as they include every possible aspect of the effect of all natural forces. First and foremost is the general impression, and I can best illustrate the ecologist's attitude of mind and powers from that of our own photographers. The latter, on looking at a print will note subconsciously all its good points and its deficiencies, and will frequently discard a print for some fault that the ordinary layman would never notice. There may be some slight darkness or lightness or lack of contrast that is not visible except to the specially trained eye and it is such slight differences that really occur on the ground and can be seen on the photograph that aid the ecologist in making his conclusions."

Economic interpretation of air photographs, particularly in connection with geological and prospecting surveys has been comprehensively dealt with by Gill. [43]

## Mosaics and Photographic Maps

So much detail is shown by the air photograph that some have advocated the substitution of photographic mosaics and maps for the ordinary line map. A mosaic is made by joining up a series of vertical photographs which have been taken of an area so as to give a "bird's-eye" view of the area photographed. It is not possible to fix the scale of the photographs with precision owing to variations of height and tilts, so that when the photographs are joined up they will give only an approximate representation of a desired scale.

Since the scale of a photograph is fixed by the ratio of focal length of camera lens to the height above the ground, the effect of height will be

that each contour will be on a different scale. Consequently no form of photographic representation can be to a definite and uniform scale unless the ground is quite flat.

In reconnaissance surveys a mosaic is produced by joining up the photographs with approximate orientation, which ensures that all the area has been covered. This enables preliminary examination of the area to be made from the mosaic in conjunction with separate prints which have been studied stereoscopically. By this means the topography, geology and vegetation can be studied in connection with engineering projects, and useful information gathered regarding such details as swampy areas, limiting flood lines and geological fault lines.

The commercial air survey firms have considerably developed the art of the mosaic. In order to produce a photographic map or controlled mosaic which shall be reasonably near to a desired scale, the photographs are rectified, in printing, for scale and tilt distortions by reference to ground control points. In order to eliminate the effects of height distortions as far as possible the prints are rectified to scale at the mean height of the area. Photographic maps have occasionally been made with the aid of stereoscopic apparatus with scale correction for each contour. This method is somewhat cumbrous and the additional accuracy is not warranted when the limitations of use of this type of map are considered.

An example of a photographic map and the corresponding line plot is given in Figs. 6 and 7. In this case the original scale of the mosaic was 1/2,500, each photograph being rectified and enlarged from one taken at a scale of 1/5,000. This illustration is reproduced at a scale of about 1/25,000 from the photographic map which was itself reduced to six inches to one mile (1/10,560) from the mosaic at 1/2,500. The final scale of this illustration is therefore ten times smaller than that of the original mosaic, or two and a half times smaller than the photographic map. The line plot was also drawn to a scale of six inches to one mile and is reproduced here at about 1/25,000. In this particular instance the plans are being used by the owner and his estate agent in connection with future developments, and for recording information such as crop rotation, position of drains and services.

Great success has been achieved by the companies in the art of rectification to a required mean scale, and where there are no great variations of height, the accuracy obtainable by scaling from the photograph is often remarkable. Professor S. D. Adshead and Mr. R. A. Hudson during the re-planning of Brighton have used rectified enlargements at a scale of 1/2,500 for map revision. In a letter dated 13 May 1936 to the contractors for the air photography, Aerofilms Ltd., they remarked:

"The rectified enlargements to the 1/2,500 scale have proved extra-

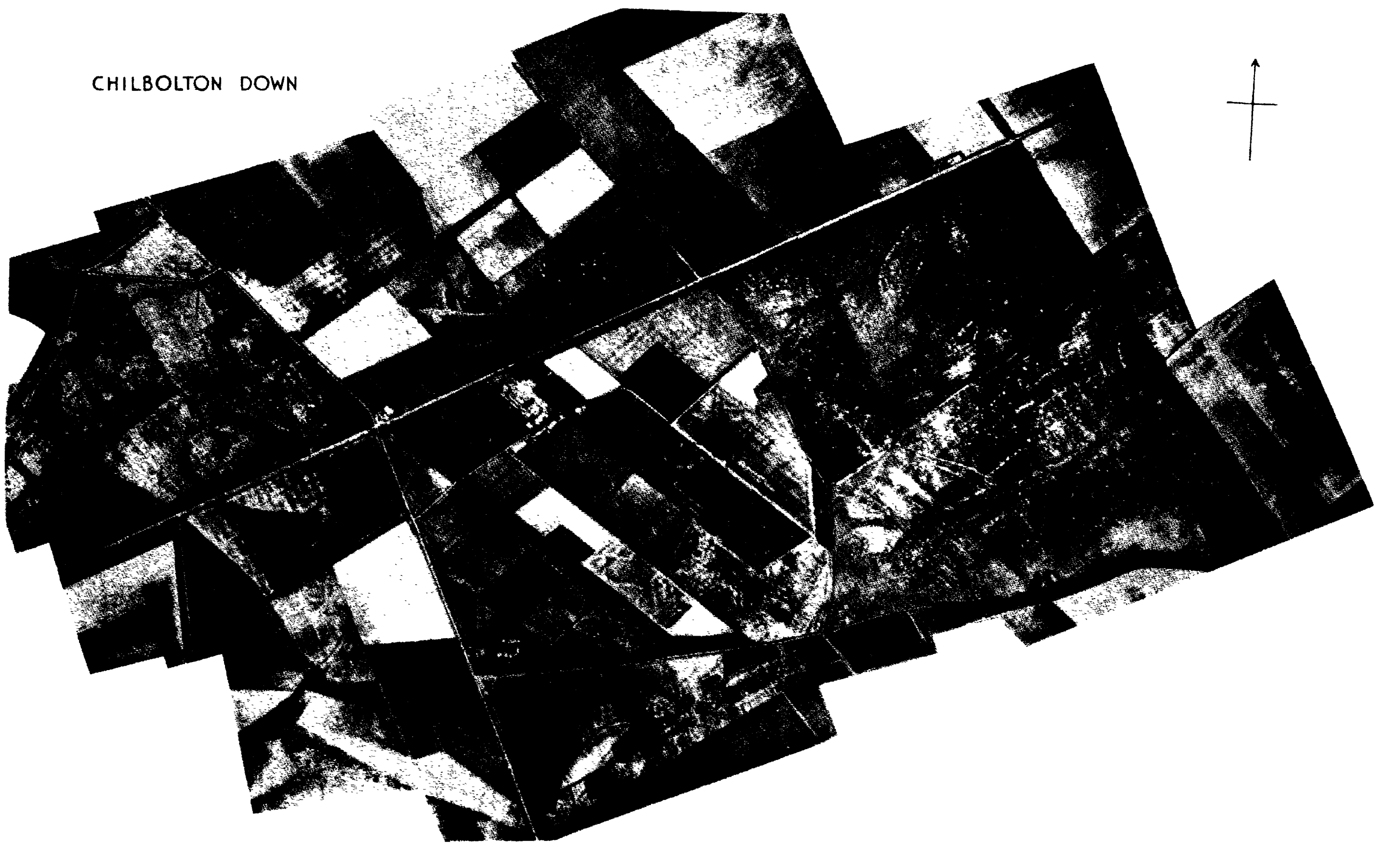



FIG. 6.

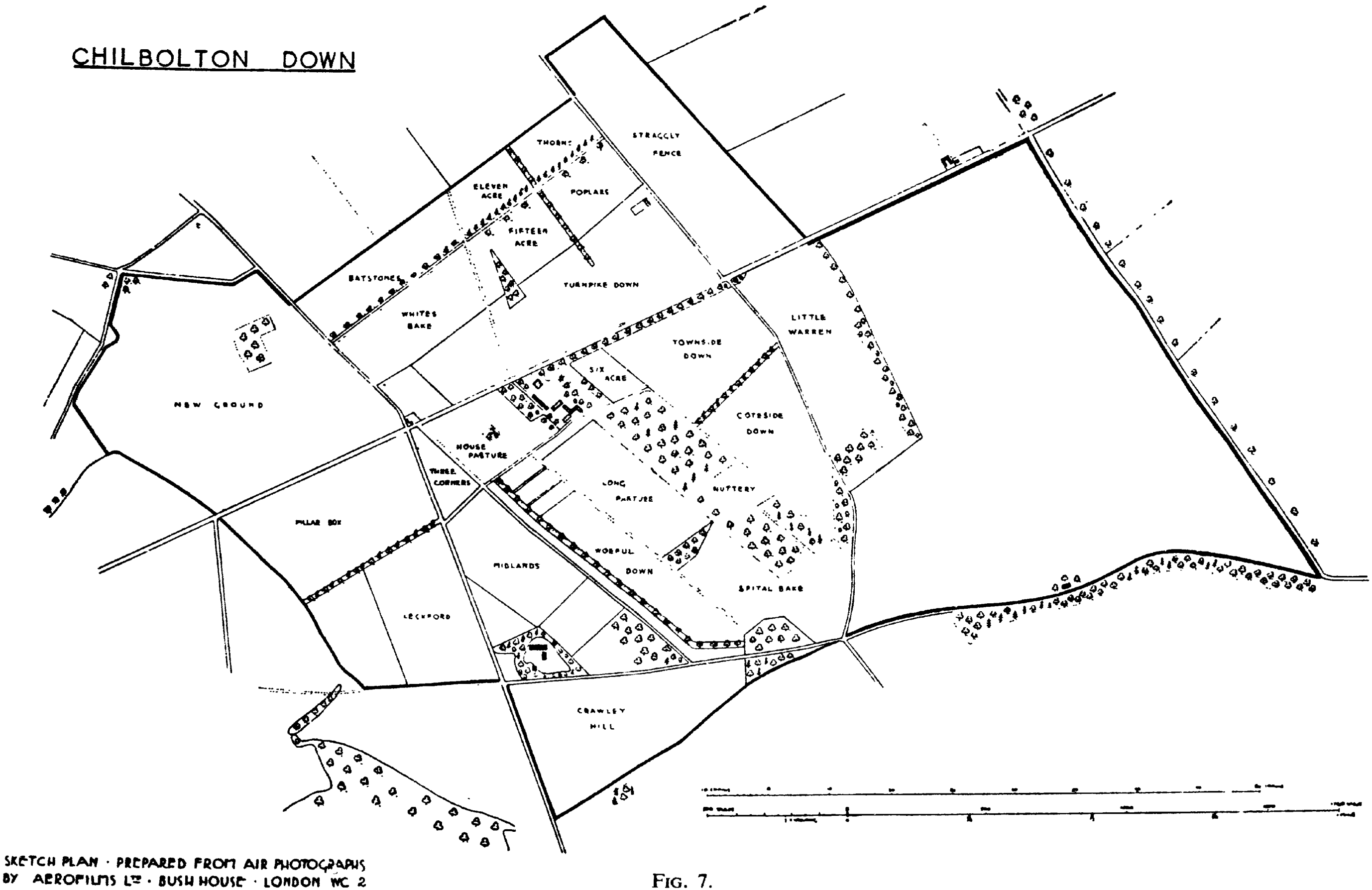

CHILBOLTON DOWN
STRAGGLY FENCE
THORNS
ELEVEN ACRE
POPLARS
FIFTEEN ACRE
BATSTONES
WHITES BAKE
TURNPIKE DOWN
LITTLE WARREN
TOWNSIDE DOWN
SIX ACRE
COTSIDE DOWN
NEW GROUND
HOUSE PASTURE
THREE CORNERS
LONG PASTURE
NUTTERY
PILLAR BOX
WORPUL DOWN
MIDLANDS
SPITAL BAKE
LECKFORD
CRAWLEY HILL
SKETCH PLAN · PREPARED FROM AIR PHOTOGRAPHS
BY AEROFILMS L^TD · BUSH HOUSE · LONDON WC 2

Fig. 7.

ordinarily accurate for the purpose of Ordnance Map revision. . . . A case arose in which it was necessary to check measurements from the photographs with measurements on the ground, and the difference between the measurements was negligible on this scale: for example—street measured on the ground was found to be 1,201 feet in length and on the photograph was 1,200 feet."

Separate photographs used in conjunction with a mosaic or rectified photographic map, and a corresponding line plot will give very complete information about an area for all sorts of purposes. The wealth of detail shown on a mosaic is invaluable for all kinds of engineering and economic activity but it is very difficult to "see the wood for the trees" even when the mosaic is not reduced in size as in the example shown. The substitution of unfamiliar objects for the usual conventional signs leads to confusion among the uninitiated, especially if stereoscopic examination of prints is not employed. Although the study of a mosaic and separate prints will in the case of a route location leave no doubt as to the best route, this excess of detail is so great that it is only on a line map containing nothing but essential information that the proposed route can be conveniently laid out. Again, it is only on small scales that the air photograph can provide all the topographical information. The ground measurements which are certain to be required for large scales are better plotted on a line map together with the detail plotted from air photographs. It has therefore become commonplace to produce a mosaic as well as a line map for surveys on medium and large scales. There seems to be no limit to the use of the photograph.

In Canada at least twenty uses have been found for photographs other than those for which they were taken.

Robbins [82] points out that the mosaic is of great use in the case of companies operating far away from, say, their London office. The possession of such mosaics and prints by both offices has often assisted rapid decisions to be made with little discussion.

## The Scope of Air Surveys

Before discussing in some detail the various engineering and economic applications of air photographic surveys it may be desirable to indicate the scales at which they can be produced and their suitability for various purposes.

*Large Scales—up to* 1/5,000.

Ground photographic surveying has been applied to scales as large

as 1/200, but air survey does not appear to have been applied successfully to survey or revision on a scale larger than 1/500. The ground photographic method has been found very suitable for inaccessible sites as, for instance, the Boulder Dam site in the United States.

In general, however, this range of scales may be considered as being 1/1,000 and 1/5,000. Such scales are particularly applicable to engineering surveys, to plans of developed areas, and to plans for intensive mineral prospecting. Air surveys on such scales are now quite practicable although the technique required is different from that used for smaller scales. Obscured detail always presents difficulties on the larger scales and important boundaries may not show up. Points required may be obscured by shadow or at the edges of the photographs by images of other objects due to the oblique rays. To fix such points some direct measurements on the ground will be required in addition to the ground control for scale. Plans on such scales become available for most engineering surveys except that for final construction for which it is still necessary to measure out and peg on the ground. Accuracy of height determination on large scales has not been brought to that required for the construction survey. It is not likely ever to supersede ground levelling, because the measurements which must be made on the photographs are extremely small and this makes it difficult to detect changes of level of two or three feet; very small inaccuracies in measurement are likely to introduce errors of this order. It is claimed that recent improvements in stereoscopic measuring apparatus, particularly abroad, have made it possible to eliminate much ground levelling so that such quantities as volumes of water and earthwork can be determined at least accurately enough for the preliminary stages of a project.

An excellent example of pioneer large-scale survey from air photographs was that made some years ago at Rio de Janeiro by the Aircraft Operating Company. Skeleton plans were drawn of the district from a ground survey and vertical photographs used for filling in detail at scales varying from 1/20,000 in the outlying areas, to 1/1,000 for some of the central districts. The work of producing line and photographic plans by means of ground survey assisted and accelerated by air survey was completed in about three years. By ground methods alone, ten years was considered the shortest possible time in which such a survey could have been completed.

The London, Midland and Scottish Railway Company are now using air survey methods to produce plans on a scale of two chains to one inch (1/1,584).

Surveys have also been made for road improvements and for restric-

tion of ribbon development on scales as large as 1/1,000. Large-scale cadastral plans are also prepared from air photographs in some countries where conditions are suitable.

Much progress is now being made in the production of large-scale contoured plans by means of stereo-plotting machines.

*Medium Scales*—1/5,000 to 1/30,000.

This range of scales includes large-scale topographical maps, and reconnaissance and preliminary surveys for engineering works. In addition there are many economic applications. Until recently the smallest scale on which vertical photographs could satisfactorily be taken in practice was about 1/30,000, and it was common when taking photographs for smaller scales than this to revert to oblique or multi-lens photography, owing to the high cost of taking vertical photographs and then reducing the scale. Modern wide-angle lenses are, however, making smaller scale verticals possible. Robbins [82] stresses the necessity for careful consideration of the factors affecting the cost of production of an air survey. He points out that the cost of producing a map at 1/50,000 with prints at 1/5,000 will be more than double the cost of a map produced from 1/10,000 prints. The contour interval also has a considerable effect on cost.

It is possible on scales of from 1/20,000 to 1/30,000, given suitable ground control, to produce by simple methods contours of the same order of accuracy as those obtained by the older forms of ground topographical survey. On the larger scales, fair accuracy can be obtained by such methods, but to obtain results of real value more elaborate methods of plotting must be employed.

Research work by the War Office has been largely directed towards the perfection of methods and plotting from vertical photographs on the military scale of 1/25,000 which is about 2½ inches to 1 mile. The use of the Arundel Method lends itself to simple graphical methods of plotting and, with improved flying technique and photography, it is now being applied to the larger scales. The scope of this method is limited when ground heights vary considerably.

For surveys of large areas, particularly in Colonial districts, the opinion is gaining ground that excellent results are obtainable from the employment of stereo-plotting instruments, which enable the photographs to be set in their relative positions as at exposure. By this means aerial co-ordinates of certain control points may be found from a limited ground control of the order of a secondary triangulation, and detail and contour plotting effected either in the same machine or in a less elaborate auxiliary one.

When locating a route, the value of a plot from air photographs, used

together with the photographs, is considerable, especially in countries not very highly developed. By this means it is possible, in a very short space of time, to select the best route which, after months of ground reconnaissance, might be missed. In preliminary reconnaissances, which are concerned with relative differences of level rather than with absolute levels, the ground control can be very much reduced.

*Small Scales*—1/30,000 *to* 1/250,000.

These scales are rather the concern of the mapping surveyor than of the engineer. Owing to the high cost of vertical photography which must be much reduced in size when plotting on very small scales, a greater area is covered per photograph either by taking high oblique photographs or, as is now becoming common, by using a multi-lens camera. The latter type of photograph, when rectified, gives the effect of a vertical taken by a camera with an ultra-wide angle lens; much wider, in fact, than is possible with a single lens. Mapping from oblique photographs is being superseded by ultra wide-angle or multi-lens photographs, except where there is little likelihood of any maps being required on larger scales. In the United States, where the Zeiss Multiplex Aeroprojector is used extensively, limitations of accuracy in plotting make it desirable to plot at a scale two to five times that required for the map. Hence with ultra wide-angle photographs, small-scale mapping can be carried out with the usual routine.

Many thousands of square miles have nevertheless been satisfactorily mapped by the oblique method in Canada, Alaska and Northern Rhodesia, chiefly on scales of the order of 1/250,000 or ¼ inch to 1 mile. The methods employed are more particularly applicable to areas where outline is the chief consideration with only approximate determination of heights. In India the methods employed have led to the successful determination of topographical heights from obliques.

*Map Revision.*

The air photograph is proving invaluable for revision on a variety of scales. The Ordnance Survey has experimented with air survey for some years and is proposing to employ it extensively for the revision of the 1/2,500 series of plans. Also it is expected that most of the tertiary triangulation will be eliminated by means of observations on air photographs with the Thompson Comparator. Air photography is being used for revision by many Local Authorities and, in many instances, abroad. Special methods, or adaptation of methods, and plotting apparatus are used according to the circumstances.

## COST OF AIR SURVEYS *

One may conclude that in most cases it is practicable to make a survey with the aid of air photographs. Whether it is an economic proposition in the circumstances must depend upon a number of factors. It is, however, necessary to investigate the question of costs relative to ground methods and to compare the quality and quantity of the information which can be obtained by the two methods. In considering costs the great advantage of the speed of air surveys over ground methods should be related to the "invisible assets" of time saved and the fact that much additional information is available from the photographs. On the other hand it is necessary that the technical standard of air photographs should be as high as possible or the results of a survey may be disappointing.

A large number of factors influence the cost of air surveys and the following list is based on that given by Hemming, [50] who is a pioneer of its commercial application. This list indicates the information which the commercial operator must know before he can give a reliable estimate.

(i) Size and position of the area.
(ii) General description of the topography, supported by any available maps, sketches or reports.
(iii) If possible, the purpose for which the survey is to be made.
(iv) Whether obliques, or verticals, or both, are required.
(v) If they are to be rectified.
(vi) If mosaics are required, and if they are to be rectified.
(vii) If maps are required.
(viii) The required scales for (iv) (vi) and (vii).
(ix) Particulars of any existing ground control.
(x) If the contractors are required to provide the ground control.
(xi) The standard of accuracy required.
(xii) If contours are required and at what vertical interval.
(xiii) The facilities existing for the operation of aircraft and the housing of the expedition and if they are to be provided by the contractors.
(xiv) Particulars of the meteorological conditions in the area. These are particularly important in relation to conditions of cloud and mist.

It will be seen from the above list that the factors which influence cost and method are in two groups: (*a*) field conditions and (*b*) operational factors. The latter are decided by the requirements of the survey, so that it is quite impossible to give hard-and-fast rules about the costs of air survey, and, for any particular case the various factors must be weighed

* 1939

carefully. The items given above are intended to apply to the general case of a country which is not, as yet, completely mapped. In this country some of the queries would obviously not arise. It must also be remembered that the time factor is often of more importance than the actual cost, particularly where economic developments on sound lines are dependent on the production of reliable maps at an early stage in a country's history.

If a country which is incompletely mapped has a modest survey programme costing a moderate annual sum which will be spread over some sixty years, that period will be occupied in the production of a complete topographical survey, and at the end much of the mapping will be years out of date, and useless. In such cases, Hemming [51] remarks that the survey of the whole country could be completed in three or four years with the aid of air photography and the total expenditure would be merely a fraction of that incurred by continuing the programme of ground survey.

Carpenter [17] stressing the necessity of accurate topographical maps for national defence of the United States remarks: "At the end of the World War we were spending 24 million dollars per day. The Board of Surveys and Maps estimates the cost of completing the mapping of this country as 117½ million dollars. The cost of this mapping at the above daily rate represents just five days of warfare. . . . It is significant that one per cent of our present annual expenditure for highway construction would, in twelve years, pay for the entire mapping programme of the United States."

In an actual case it was estimated that a saving of half a million dollars in the cost of construction of an oil pipe-line run from Pecos County, Texas to the Gulf Coast was made as the result of location from air photographs.

Unfortunately it so often happens that the administrator who can provide the necessary funds is not one who uses and appreciates maps: also bigger and better mapping programmes have not much political weight because the great advantages of a complete and accurate map are not generally realized.

Bearing in mind these varying factors, the following examples of costs and estimates may be taken as giving some guide, but in no way are they intended to give definite values.

An area of 63,000 square miles was surveyed by oblique photography at a scale of 1/250,000 by the Aircraft Operating Company in Northern Rhodesia some years ago at a cost of £1 per square mile. Of this about 15*s*. was for photography and 5*s*. for the plotting. This did not include

any contours. Vertical photography of a similar area would cost about 30*s*. per square mile and the completed plan with levels between £3 and £4 per square mile.

Hemming[50] estimates that 400,000 square miles in South America could be mapped from verticals in five years at a scale of 1/50,000 with 25-metre contours at £3·77 per square mile. He also states that it would be possible to make an economic survey of 200,000 square miles of bush country, such as that found in Northern Rhodesia, including ground control and interpretation at under £2 per square mile. This would include a reconnaissance map of the whole at 1/125,000; a drawn map at 1/50,000 of selected areas totalling 20,000 square miles, and mosaics of these areas at 1/25,000. The work could be completed in three years.

The total cost of photographing an area of 1,582,052 square miles by the United States Department of Agriculture between 1926 and 1 June 1938 (which includes re-photographing of certain areas) will be $6,049,724[1], i.e., approximately $4 per square mile.

The whole of the United States, covering an area of 3,000,000 square miles is being covered by air photography at a scale generally of 1/20,000. Scaled enlargements are made and mounted as mosaics.* After copying and enlarging so that 1 square inch of mosaic covers an area of 10 acres, each sheet covers about 9 square miles and is about 2 feet square. The cost to the Government of the maps is about $35 each. The maps are being used extensively in connection with the Soil Conservation Service and Forest Service as described in Chapter III.

In the Second Report of the Air Survey Committee, 1935[6], is given an analysis of the cost of photographing an area of 1,000,000 square miles with a Williamson "Eagle 4" Camera, with a focal length of 25 cms. at a flying height of 15,000 feet above the mean ground level. The basis of an estimate of cost is gone into very thoroughly, and it is assumed that there is a central air-survey organization established on a permanent basis and undertaking large-scale operations. It is emphasized that the figures given are intended only to give a general idea of the various operations and not for checking quotations for specific items.

The costs given are for initial air reconnaissance which includes oblique photography, followed later by vertical photographs over the whole area. The cost of ground control and preparation of the plans are not included. It is stressed that the different factors have varying importance in relation to the scale, but a table is given showing the approximate relationship between the costs per square mile of photography under the above conditions for varying areas.

* *Reader's Digest*, December 1938.

The necessity of continuity of work for such an air-survey organization is stressed, so that the experienced personnel required should have security of tenure.

| *Size of area, square miles.* | *Cost per square mile in shillings.* |
|---|---|
| 1,000,000 | 23 |
| 100,000 | 35 |
| 10,000 | 60 |
| 1,000 | 110 |
| 500 | 180 |

The Report also states that the figures quoted in the table above are not applicable to urban areas where the cost of photography may in certain circumstances reach £50 to £150 per square mile (1*s*. 8*d*. to 5*s*. per acre).

Two examples of present costs for revision of Ordnance Survey plans are as follows: (1) For a town of 60,000 population, covering an area of 6,000 acres, photography at 1/5,000 would cost some £375, including enlargement and rectification to a scale of 1/2,500. The preparation of the revised Ordnance Survey plans would cost £450, the respective costs being 1*s*. 3*d*. and 1*s*. 6*d*. per acre respectively.

(ii) Photography at 1/5,000 for 1/2,500 plans of a town about 25 square miles in area, and a population around 250,000, as, for example, Leicester, would cost about £20 per square mile and the production of up-to-date plans a further £100 per square mile.

The price will vary with the area and the amount of new building.

In the Ordnance Survey Report for 1937–8 [73] it is stated that in built-up areas, especially those newly developed, the saving in cost over ground methods is estimated at one to two shillings per acre or even more.

Contouring from air photographs will increase total cost by at least twenty-five per cent, the vertical spacing of the contours controlling the actual amount.

In Switzerland where the photographic method has been used for a number of years and is peculiarly adapted to the topography, the estimated saving by the use of this method instead of ground methods has been in the neighbourhood of 50 per cent. In Palestine, Wolff in advocating the employment of air survey [99] estimates that the saving in Palestine would

be of the order of 60 per cent for the property and taxation map alone. Other authorities consider this estimate somewhat excessive.

Finally it must be reiterated that no hard-and-fast details of costs can be stated, and even if they are given, the economic advantages of an early map coupled with economic interpretation of photographs are immeasurable.

CHAPTER III

# PRACTICAL APPLICATIONS OF AIR SURVEY: MAPPING: ENGINEERING: SCIENTIFIC: ECONOMIC

In considering the applications of air survey it is proposed here to deal first with the development of "new countries," and to consider, in the light of the general problem, the special problems which arise when the development is at a later stage.

## Mapping from Air Photographs

*Reliable Maps as the Basis of Development.*

It has been stated [22] that the importance of an accurate, comprehensive and detailed survey in the early stages of the development will enable a country to utilize its natural resources, and develop its civilized resources much earlier and with greater utility than if, as is so often the case, development precedes the survey itself.

It is recorded by a well-known authority that when settling in a part of Africa, then called the East African Protectorate, some thirty years ago he was allotted a small concession of land. After spending money on its development he was informed officially that, because there were no reliable maps of it, he did not possess any land. On pointing out to the authorities that they had given him this land he was informed that it was not known what it was or where it was. Other people had taken up land in the vicinity which had been recorded officially, and it consequently appeared that the pioneer possessed no land at all.

Another authority points out the difference between the old days when settlers went out and became self-supporting, and to-day when the settler must develop his land economically so that his products may be exported in order to provide him with the necessary funds to maintain a civilized standard of existence. It is evident that development must be planned on sound, economic lines. This particularly applies to railways, which should be located to provide efficient through routes, and to roads, which should be planned to connect with the various farms or other sources from which products can be obtained. Any process which advances this object more rapidly is of material value and it is in this connection that air survey can

be of the utmost service in enabling development to start on the right lines. Having selected, by means of aerial reconnaissance and photography the most profitable areas for development, the engineer can proceed to locate the essential lines of communication.

Economic location is ensured from the air survey, which has, in addition, a great number of economic uses. For this reason it is quite wrong basically to utilize air survey simply for the purpose of rapidly completing the topographical survey.

Bourne [7] late of the Imperial Forestry Institute, Oxford, emphasizes that the future prosperity of the British Empire depends to a great extent upon the development of the agricultural, forest, and mineral resources of the Dominions, India and the Colonies. The various official, scientific and commercial interests tend to approach the problem from different aspects, and he considers that the paramount importance of co-operation can only be ensured in a reasonable time and at a reasonable cost by air survey.

The first requirement is a proper mapping system, because land, the basis of all wealth, cannot be efficiently allotted unless its position, size and natural features are known.

Before the days of air survey, when speedy reconnaissance of an area was necessary, the result was often a map of little subsequent value. It is recalled by a pioneer that on one occasion in Northern Rhodesia he made a reconnaissance survey of some 4,500 miles in six weeks. Distances were measured with a "perambulator," consisting of the front forks and wheel of a bicycle, fitted with a cyclometer, while bearings were read by compass at distances of about one-third of a mile.

The result of many early surveys was a "plum-pudding" map, i.e one which had blank areas in various places. In the instance described these were avoided as far as possible by climbing hills and trees, taking compass bearings and observing areas with binoculars. When the readings were plotted, the detail was filled in by the information gathered by observation. In other cases, early surveys of this type would not fit, and, on more than one occasion, blank areas on the sheet had to be labelled: "These areas do not exist."

*Organization of Survey in "New" Countries.*

Initiative in mapping is usually taken by the military authorities, who realize from experience the paramount importance of first-class maps. The greatest difficulty of the surveyor is that in most cases the civil administration will not incur much expenditure without immediate return and since reliable maps form an investment which pays but a small divi-

dend at first, survey departments are frequently found struggling along with a minimum staff and microscopic funds.

Air survey enables a reliable topographical map to be produced in a few years instead of after two or three generations. Expense, spread over these few years is heavy, not on account of greater cost, but because the outlay is concentrated into such a short period. Benefits to agriculture and industry are immediate, but the exchequer benefits only indirectly. There seems no doubt that in such cases, the cost of mapping should in part, at least, be shouldered by those who benefit the most, and one solution is to raise a "survey loan" repayable over a number of years. The cost can be distributed over the various departments concerned, such as survey, public works, geological survey, forestry, agriculture.

Air survey in this way rapidly provides the necessary scientific and economic data to further best the cause of empire development.

General policy in the British Empire has been to encourage commercial enterprise in regard to air photography and although the Royal Air Force has co-operated in numerous experiments, for most of the work contracts are let to the private firms.

It does not seem practicable for any government department to set up its own special air photographic organization because the time of employment would be small.

An Imperial Survey Organization was first suggested a number of years ago by Hemming, who was supported by Salt[85] (at that time Research Officer to the Air Survey Committee) in 1933; and, later by others.

The recent amalgamation of British air survey firms has been mentioned in Chapter I, the ultimate aim being the formation of an Empire Survey Organization. Such an organization would produce complete plans and maps, but the government concerned could also set up a survey department for ground control and plotting, and this department could work in co-operation with the photographers.

Progress in Canada, in which photographical surveying was first practically applied, has been somewhat different.* The Topographical and Air Survey Bureau of the Department of National Defence has been largely responsible for the photography, although such other departments as the Bureau of Geology and Topography of the Department of Mines and Resources is also actively concerned. Photography has been carried out by the Royal Canadian Air Force as well as by private firms. A library of some 780,000 photographs is now available for various purposes

* The writer is indebted to Mr. A. M. Narraway, Consulting Aerial Surveys Engineer, Bureau of Geology and Topography, Ottawa, for most of this information.

and these are being extensively used. These photographs are available to the public as well as being used by the Topographical Survey, Geodetic Survey, Hydrographic Survey, Dominion Water Power and Reclamation Bureau, and the Forestry Department. Canada is in a rather special position, because the vast and inadequately mapped areas contain, particularly in the north-east, great mineral and other natural wealth. The vast nature of this problem has led to an organization which would hardly be justified elsewhere. On the basis that a photograph shows each tree, rock, exposure or stream, it is possible to go photograph by photograph and examine all these in detail from the Atlantic Ocean at Halifax to the prairies at Winnipeg; thence to and along the Arctic Coast—a distance of some 4,300 miles without a single breach.

It is generally assumed that the average photograph will have some five official uses and each user is concerned only with his own share of the cost.

With regard to the United States, much progress has been made in air photography as shown by the sketch map in Fig. 3, by which it can be seen that up to 1 June 1938 some one and a half million square miles had been photographed, while contracts have been let for an additional 435,000 square miles.[1] Also some 55,000 square miles have been photographed in Alaska. The photography has been mainly carried out by the United States Department of Agriculture, for itself and for other departments mentioned on page 15.

In the case of the United States Geological Survey, up to the end of 1937, 210,313 square miles were photographed in the United States, and 22,000 miles in Alaska. Of these areas 137,021 square miles had been compiled in planimetric maps in the United States and 16,315 square miles in Alaska. The total area contoured from air photographs was just under 2,000 square miles.[77]

The Soil Conservation Service of the Department of Agriculture has, since its inception in 1933,[63] mapped some 400,000 square miles by air survey.

About two-thirds of the area photographed by the Department of Agriculture has been primarily for the Agricultural Adjustment Administration for the determination of crop areas and not for mapping purposes.[1] The photographs are, however, available for other departments.

Although much progress has been made in the United States in the field of air photography and survey, there is, as is mentioned on page 51, some dissatisfaction there because of the lack of a real national survey programme.

*Pioneer Surveys.*

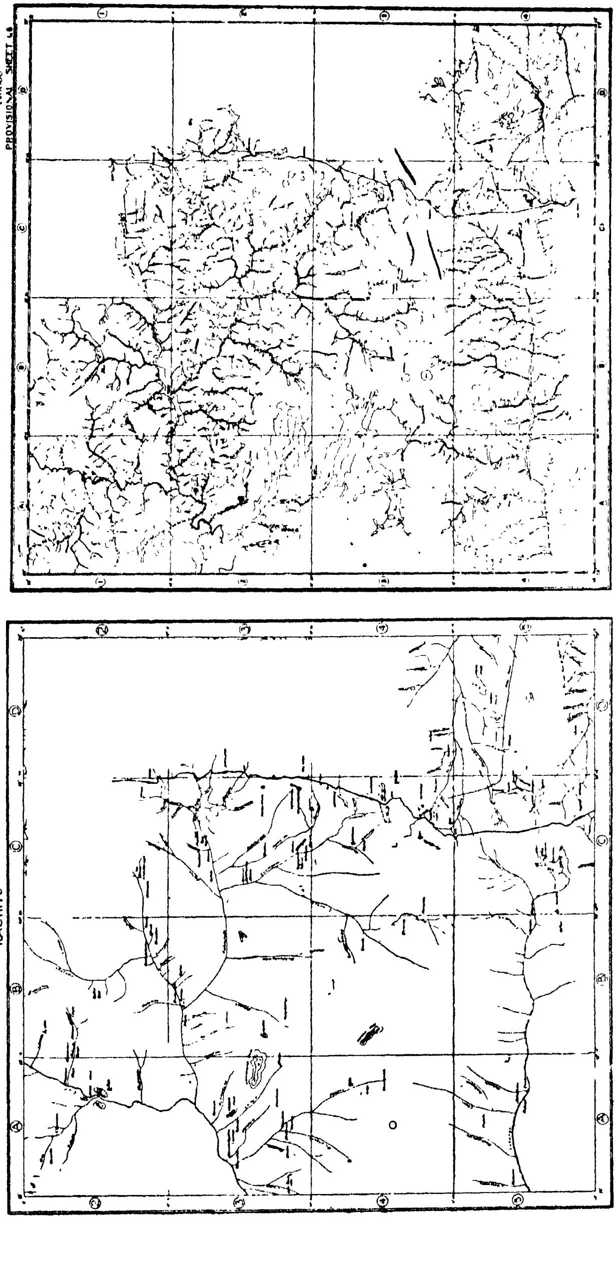
Surveys are required early in development for planning settlement and lines of communication. The latter usually follow either the shortest line or the line of least resistance and have the object of establishing or opening up a productive centre. The original alignment to a large extent influences the trend of settlement, and these areas may not be the best for the purpose so that the pioneer settler may have a harder time than necessary, the consequences being far reaching. Bourne [7] shows how this has happened in Northern Rhodesia for the simple reason that in the early days the railway was aligned along a watershed for economy. He stresses the necessity for a general air reconnaissance survey of the country and this he considers would repay the cost in a hundred ways. It is interesting to note that since that time (1928) a large area of Northern Rhodesia has been surveyed in this manner.

In Fig. 8 on the left is shown a very much reduced copy of a Government map of an area in Northern Rhodesia. This map was produced largely from "travellers' tales," sketches and rough surveys without the aid of air photographs. On the right is a reduced copy of the air reconnaissance survey of the same area produced from obliques. Even on this reduced scale it can be seen that the detail in the air survey line-plot is remarkably full when compared with that obtained from ground survey.

Good photographs often show up variations in surface detail which warrant further inspection on the ground, and observation from the air will indicate areas of swamp or desert where further investigation is unprofitable. The stereoscopic effect is invaluable and more can often be seen from the photograph than by observation, because of the exaggerated effect of an ordinary stereoscopic pair.

The location of a route on the ground in accordance with sound engineering and economic principles is lengthy and expensive and even after months of ground-work it may happen that the best route may not be chosen. Five per cent of the total sum allowed for survey and estimate costs for a road or railway is usually allocated to reconnaissance. Economy on this reconnaissance may show an apparent saving, but often actually results in heavy loss.

Hemming [50] states how economy in survey programmes often results in grievous waste at a future date. He quotes an example given by the late Sir Gordon Guggisberg who gave a striking illustration of the waste due to lack of survey, in the case of a railway in the Gold Coast where one and a half million pounds had been spent in straightening out the railway. He added "if you can get a photographic map first, you are going to save




[Courtesy of Aircraft Operating Co., Ltd.

FIG. 8—PART OF NORTHERN RHODESIA, SHOWING ON LEFT OLD GOVERNMENT MAP, AND ON RIGHT THE SAME MAP PREPARED FROM AIR PHOTOGRAPHS.

[Original scale 1/250,000; scale of reproduction 1/1,250,000 approximately.]

yourself literally hundreds of thousands of pounds in road and railway construction; but if you are going to conduct that survey by working through forests, you are going to take a very long time."

Many similar instances have occurred because the engineer could not wait for maps to be produced by the Government survey.

Air survey can also make this work much easier by indicating two or three possible routes, even if it cannot definitely establish the best route. These can then be surveyed in sufficient detail to enable estimates to be prepared.

The survey of the Imperial Airways route through Northern Rhodesia at a scale of 1/500,000 was carried out by the Aircraft Operating Company of Africa. This involved surveying some 16,000 square miles and re-plotting about 30,000 square miles of the survey previously made for the Government. This map shows hill features, forests, villages, roads and railways for air navigation purposes.

The characteristics of likely settlement areas can be explored from the air. It is essential to select areas where the soil has not eroded so much that it has become unsuitable for cultivation. Also the tsetse fly which causes such havoc in tropical Africa cannot be located direct because it does not, like the locust, feed on the vegetation.[55] Knowing that the fly infests certain types of vegetation, it is possible to tell from the air photograph where it *may* be found—just as pools of stagnant water show where the malarial mosquito is likely to be found. Elimination of such pests is essential for successful settlement and development, and having located the possible areas, remedial measures can be taken by the administration.

*Mapping and Administration.*

Mapping is primarily initiated by the administration, and it is first proposed to consider the application of air survey to mapping proper, before describing the special uses of the air survey.

(i) *Topographical Surveys.* Whatever method is employed for the detail, it is still necessary to provide a complete geodetic control, which must provide an adequate number of accurately fixed points. Hotine[55] is very emphatic in reply to those who consider that the precautions taken for accuracy in this control are unnecessary, and points out that the main object of the control is "no less practical than the prevention of gaps in subsequent detail surveys, leading to the same area being mapped twice over on different sheets or being omitted altogether. The introduction of air survey emphasizes rather than reduces the necessity for such control."

In Canada, the Topographical and Air Survey Bureau reported in

1933[15] that in ten years of air survey, 277,500 square miles had been surveyed by oblique, and 125,000 square miles by vertical photography:

"It has been possible to extend exact geographical knowledge of North Canada to a greater extent in the last ten years than in the preceding century of exploration and discovery, and to depict accurately the myriad lakes and water-courses of the great Laurentian Plateau for the prospector, lumberman, traveller and others. . . . Formerly the ground control in the lacustine region was established by traverses, but this is generally replaced by astro-radio fixings of suitable points, so that the delays and expense consequent on the ground movements on a traversing party are obviated, which is no small saving in Canada."

The state of mapping in the world has been shown in Fig. 1 and the amount of work to be done can be imagined. We in this country are very fortunate in being able to purchase on demand, not only topographical, but also medium-scale maps and plans at cadastral scales.

In other developed countries the mapping programme is not so far advanced. In a report to the United States Board of Surveys and Maps in 1937, Colonel Watkins[94] has stated: "The United States, Australia, and the cold regions near the North and South Poles are practically the only large areas of the World not covered by complete topographical maps including relief, drainage, vegetation, communications, boundaries and other cultural features. About fifty per cent of the U.S.A. is covered only by the new sectional aeronautical charts which are incomplete in topographical detail. The United States is the only industrial country not covered by accurate topographical maps. . . . About twenty-five per cent of the United States is covered by adequate topographical maps on the scale of 1/62,500 or 1/125,000. About twenty-five per cent additional is covered by topographical maps now out of date. . . . The United States is the only large industrial country in the world which has no adequate mapping programme. On account of the lack of complete topographical maps, the United States is handicapped in the whole of its economic development and in its national defence as compared to other nations in the world."

It appears that the work of the United States Geological Survey and the Soil Conservation Service, among others, have been directed towards the needs of the body concerned rather than to a national programme. The lack of topographical maps has led to the careful consideration of a definite programme of air survey in order to rectify the matter in a reasonable time.

Another American writer, [78] quoted by Carpenter [17] of the U.S. Bureau of Public Roads, in discussing their mapping problems says: "Most of the overlooked opportunities, the mistakes and fortunes of the

World are due to making assumptions from half-truths and inaccurate information. For nearly three-quarters of the United States the best available maps are generalized compilations of a heterogeneous lot of poorly co-ordinated traverses, reconnaissance surveys and sketch maps. Very few Americans are acquainted with the accurate detail large-scale maps of the type made in all the leading countries of Europe . . . these countries have found the maps excellent investments. They say that they must have such maps to act intelligently and that only a country as rich as the United States could afford the waste of doing without them."

The position of topographical mapping in the U.S.A. in 1938 is ably summed up by Dr. W. Bowie [8] who was the chief of the Geodetic Division of the United States Coast and Geodetic Survey, and who has for many years advocated a national mapping programme. "Agencies of the federal government, notably in the Department of Agriculture, are to-day making special-purpose maps for their own immediate needs. In nearly all cases these are planimetric maps made from aerial photographs and without contouring. Unfortunately, they cannot be of the greatest usefulness because they frequently lack the horizontal control data needed to give them accurate position, distance, direction and scale. These surveys are well advanced, but much detailed work needs to be done to satisfy the makers of planimetric and other maps. It would be well if the Coast and Geodetic Survey could receive funds in sufficient amounts to provide control data for all special-purpose maps being made, not only by the federal government but also by states and counties."

A plan for completing the base topographic map of the United States was approved by Congress in 1926, the expenditure to be $100,000,000 spread over twenty years. The plan, however, has been held up by the Government pending certain reorganization of governmental activities.

Bowie remarks that more than half of the existing topographical maps covering about fifty per cent of the country are out of date, or not up to modern needs. "In the past appropriations for topographical maps were very small and the need for base maps by geologists was great. In consequence, large areas were covered by reconnaissance maps that served their purpose well at the time they were made, but not to-day. Bare outlines of geological structure will not meet the needs of the scientist nor will generalized contours satisfy the engineer. . . . In spite of the large array of sentiment for a national mapping plan, it has not yet been started. It seems reasonably certain, however, that a mapping plan must be put into effect in the very near future. . . . We are certainly becoming more map and survey conscious, and our engineers, planners and scientists are requiring more and better maps on which to base plans and studies, and

to direct operations. . . . It is only logical then to expect—and this is a great advance beyond the hoping stage of a few years ago—that the older surveying and mapping methods which varied so in quality will soon give way to scientifically devised methods, and that future surveys will be engineering products, giving maximum service to planners and builders."

There is thus some dissatisfaction in the United States about the progress of national survey, chiefly it appears because of alleged insufficient co-ordination between the various departments.

(ii) *Cadastral Surveys.* In addition to topographical surveys, cadastral surveys are required for administrative purposes which are chiefly for the accurate partition of the land and for taxation purposes on those areas. During the last twenty years air survey has been increasingly employed for this purpose, Switzerland being a pioneer country in this respect.

In the annual report of the Department of Lands and Surveys of Palestine for 1936 it is stated that the hope that air survey would facilitate the survey of old towns has not been fully realized. This is largely because the narrow alleyways are deep in shadow which makes stereoscopic interpretation very difficult. Moreover, in many cases the property boundaries are three-dimensional so that the amount of ground work would have been very considerable.

Wolff[99] writing in the *Empire Survey Review* in 1938 records that while several European countries have been using air survey to an ever-increasing extent for cadastral surveys, the Colonies have been somewhat hesitant. Referring particularly to Palestine, he remarks that topography is not shown on the cadastral maps, and when this is required, it must be transferred from the smaller-scale maps. The accuracy required by property registration is the ruling factor of accuracy. After an investigation of Colonial conditions, Wolff has concluded that air survey is eminently suitable for cadastral purposes and can provide the necessary accuracy, while the estimated saving over ground methods may be as much as sixty per cent.

Other authorities in Palestine and South Africa do not agree with some of Wolff's conclusions.

Salmon, writing in the *Empire Survey Review*[84] in reply to Wolff raises objections to air survey for cadastral survey, but these are by no means sufficient to condemn it.

Hendrikz[52] writing in the same issue of the *Empire Survey Review* criticizes Wolff's article with regard to the conditions in South Africa. "In so far as South Africa is concerned the views appear to be based on misconceived conceptions of 'developed colony' conditions. The present

system of land registration in the Union is the direct outcome of our social and economic environment, and the failure to adopt air photographs for cadastral mapping is not essentially because air survey is unsuited to our cadastral requirements."

There seems to be some confusion of terms as to what exactly a cadastral map is. Although it is essentially a map or plan which delineates land boundaries, the form it takes must depend upon the manner in which boundaries are fixed and marked. In Europe boundaries are usually clearly marked and, in almost all cases, are irregular, so that Hendrikz remarks: "A large-scale map based on air photographs might conceivably be sufficient for the pictorial representation of boundaries and the determination of areas sufficiently for taxation purposes."

He points out that on the other hand in South Africa and elsewhere the boundaries usually have "definite geometrical shapes which are defined by corner beacons, usually cairns of stones over pegs. These boundaries are usually straight except where there is a natural feature such as a stream. In such cases the cadastral map becomes an index from which the ownership of definite areas may be established. The whole area is related to a co-ordinate system so that the corners of plots may be established by their co-ordinates referred to a definite triangulation system." He concludes: "The cadastral surveyor (in South Africa) is mainly concerned with the placing and re-establishment of beacons. This is a field operation and cannot be done by aeroplane."

It seems clear that there is a considerable field for cadastral mapping from air photographs in countries where boundaries are clearly defined: in others, existing methods may be preferable.

Extensive application of air survey methods is contemplated to our own nearest approach to a cadastral map—the 1/2,500 plan. This is not actually a cadastral plan because visible boundaries are shown and these are not necessarily actual boundaries. In investigating the possibilities the Ordnance Survey carried out a number of experiments, and were somewhat over-critical of it in the earlier stages perhaps because one or two of the early experiments were made in rather unpromising places.

The chief point of disagreement is the relative cost of ground survey and the cost of ground work necessitated in addition to the cost of photography at large scales when plotting from air photographs.

It is obvious that the choice of air survey must be controlled by the configuration of the area concerned and the detail required to be shown.

The recent report of the Departmental Committee on the Ordnance Survey (Davidson Report 1938)[72] records that air survey is well suited to 1/2,500 revision in areas more or less built up.

## Engineering Applications of Air Survey

It should be noted that the examples given at random are illustrative only and are not intended in any way to give a complete record.

*Preliminary Surveys.*

In Great Britain excellent six inch to one mile Ordnance Survey maps make it possible in nearly every case to select the best site or route, with no special surveys at this stage except revision of detail and possibly additional levels.

Air photography can frequently be used for this revision, which leads to preparation of the preliminary estimates and the Parliamentary Survey if a Bill is being promoted. The scales for Parliamentary Surveys are usually six inches to one mile in open country and twenty-five inches to one mile (1/2,500) where there are buildings.

The rights of the ground surveyor to entry for survey are almost non-existent until the land has been bought, and without permission of the landowner it is often impossible to obtain reliable information for the preliminary plans. Here, again, can be seen the possibilities of air photography, especially at six inches to one mile where contours of fair accuracy can be obtained from the air photographs.

In many countries, however, the engineer has nothing but topographic survey as a basis and sometimes not even that. The problems of preliminary surveys have been mentioned, but it is interesting to compare the engineer's position in this country with that of the engineer in the United States. There, as has been mentioned, the engineer usually has to initiate the surveys required for a particular project.

Bowie[8] may again be quoted in this connection. "Much of the construction work in the United States by federal, state, city, county and private agencies has been done without adequate knowledge of the terrain where the projects were situated. It is inevitable that without topographic maps, the location planning, and design of large-scale engineering projects cannot be done efficiently. This applies especially to highways; hydroelectric projects; pipe lines; telephone, telegraph, and electric power lines; water or sewer systems; and to flood projects. Billions of dollars are spent annually on public and private engineering works. It would seem only common sense that any area in which such works are to be located should be adequately mapped. . . . It would be economy to have these special surveys made with such accuracy that they would supplement the national mapping plan. . . ."

The importance of air survey for this purpose is obvious.

*Acquisition of Land.*

Many problems arise in the acquisition of necessary land for engineering construction schemes. "In countries where the land is open country or in those where the Government acquires it without much thought for the rights of the person, the problem is not very acute." [23]

In this country, local authorities and statutory bodies, such as railway or water companies, have powers to acquire land compulsorily for the purpose of constructing works. The scope of compulsory purchase is controlled by numerous Acts of Parliament, and varies with the type of works proposed. Certain classes, such as allotments and commons, are more or less protected, and it is sometimes desirable in the case of extensive schemes to promote a special Bill in Parliament, particularly where there is danger of contention.

The price payable by local authorities is the assessed market value of land, while statutory companies are permitted to add a small percentage. In some cases local authorities are able to claim betterment value from the landowners where the value of adjoining land rises as a result of the works. The appearance of surveyors in an area has a remarkable effect in increasing land values and as a result compulsory purchase frequently leads to arbitration or litigation.

This difficulty is much less acute if the neighbouring landowners have not had opportunities for mutual consultation. Air survey will solve this problem, as it is possible to produce plans with very little ground control and to take options or purchase land without much publicity. In Canada, to establish the value of land to be used as a basis for subsequent expropriations, it is usual to negotiate "quietly" the purchase of representative "parcels" scattered along the projected route. In their selection and subsequent negotiations vertical air photographs have been found very useful. Hotine [55] quotes the example of a railway company which selected its route from an air survey and sent out a small army of agents on the same day to buy options on all the land required from the unsuspecting landowners.

*Water Supply and Power Schemes.*

Selection and survey of catchment areas, reservoir sites and dam sites have been carried out satisfactorily in numerous instances from air photographs. A correlated ground survey is indispensable in such cases because of the geological problems involved. Geological information may be obtained in some circumstances from air photographs, but must be checked on the ground.

Reconnaissance and preliminary surveys can be made rapidly from the air and approximate capacity of reservoirs can be found by stereoscopic examination and a few parallax measurements. Some ground levels will be required along the line of the stream.

It is almost impossible, without taking a considerable time, to weigh all the factors satisfactorily when making a preliminary ground examination of possible reservoir sites, and on occasions where this has not been done a better site quite near has been found after construction had started.

Air survey has been much used in Canada because the close nature of the thickly wooded country over a great part of Canada makes ground surveying for engineering projects both expensive and slow. In the Lac Seul development, dams were constructed to raise the level of the water in the lake, and the high-water contour was established on the vertical photographs. Sufficient information was obtained to enable the extent of forest to be flooded, to be marked out and the timber felled and cleared.

Another large Canadian power scheme recently developed[11] was surveyed from air photographs. Approximate contours based on a single line of levels were drawn for just under one hundred miles, and with a few ground levels the values from the air photographs were found accurate enough for all purposes except the final construction survey. It was estimated that at least a year was saved in the preliminary stages.

In the spring of 1938, the Canadian Department of Mines and Resources had occasion to make surveys in connection with the raising of the water levels of a series of lakes by varying amounts up to sixty-seven feet for a power scheme.* This area was just south of the Arctic Circle and the only map available was a preliminary one made from oblique air photographs. After a brief examination of the photographs, not only were the sites for the dams selected, but also information gained such as the location of gravel for concrete, route of transmission lines, and availability of timber. It was found that sufficient information could be gathered from the photographs for the work in hand without a single ground measurement; and the dam could be built and power supplied even in a remote district which was previously uninhabited.

Narraway does not claim great precision for this work, but he does claim that the results are adequate for the purpose in hand, and mentions the case of Reindeer Lake, which was recently required to be raised five feet. The Lake is of considerable area, contains a number of small islands and its shore-line is very irregular. The information available was a small-scale map and the original obliques, from which the map had been made. No levels were available so that the work consists of study, comparison

* Information in a letter from Mr. A. M. Narraway, *supra.*

checking and elimination. The Department is satisfied that the results are near the required accuracy, but to make sure certain local measurements will be made at doubtful points.

In using obliques advantage is taken of the fact that most parts of the area appear from different angles in several photographs. Thus a very much needed check is provided.

Photographic methods are used extensively in the U.S.A. for the survey of reservoir and dam sites. Fairchild Aerial Surveys, Inc. of Los Angeles* have carried out a number of such surveys, though some on larger scales were made by ground photogrammetric methods. In a number of cases earthwork quantities have been estimated.

A contoured map of the Boulder Reservoir (Lake Mead), comprising part of the Grand Canyon of the Colorado River was made under contract by the Fairchild Company. A principal object was to provide the basis of hydrographic surveys at intervals, to determine the rate of silting. The contract required a topographic map at a scale of 1/12,000 with contours at a vertical interval of ten feet. The map was produced from vertical photographs by means of the Zeiss Stereoplanigraph. The survey involved highly skilled work owing to the extremely difficult country, which was precipitous and arid and almost without means of communication. [63] Reconnaissance was effected from the air and control was supplied by the United States Coast and Geodetic Survey both for triangulation points and for levels of first-order accuracy. Dr. W. C. Lowdermilk, chief of the Soil Conservation Service pays tribute to the excellent co-operation of the Coast and Geodetic Survey in enabling the former to provide the contractor with certain field information. Contours were carried to one hundred and seventy feet above the top water level of the reservoir.

Some interesting checks were made upon the accuracy and consistency of the survey work. After the gates of the Boulder Dam were closed early in 1935, the rising water line was photographed at contour intervals of twenty feet, and it was found that the photogrammetric contours agreed very closely with those established by the water line.

The Fairchild Company also made an air survey of the San Gabriel dam site for the Los Angeles County Flood Control District. The following are extracts from a letter to Fairchilds from Mr. E. C. Eaton, the Chief Engineer: "The San Gabriel Dam No. 1 represents a $10,000,000 expenditure, involving excavation quantities of over 1,900,000 cubic yards. The importance of accuracy in the original ground surface is therefore apparent.

* The writer is indebted to Mr. Leon T. Eliel, Vice-President of Fairchild Aerial Surveys, Inc., for much of this information.

"In checking your map we had available two independent contour maps. . . . These maps were compiled from approximately 1,000 points per acre, which were established by precise triangulation and levelling. The quantities computed . . . from your map varied less than one-tenth of one per cent from the mean of the other two maps, based on the estimated excavation.

"We consider your map to be of at least equal accuracy to our surveys; you accomplished the work quicker than would be possible by ordinary methods; and your charge . . . was considerably less than the cost of a similar map of this territory produced by ground methods."

Air survey is also being used for locating aqueducts and water power-lines, and for estimating approximate falls and the head of water available.

In one instance, in Northern Rhodesia, the entire line of feed-pipes and canals was fixed from dam to power-house from photographs taken by the Aircraft Operating Company of Africa. The country was broken and wooded and months of ground reconnaissance surveys were eliminated, because the best line was selected from the photographs. On the ground it was necessary only to traverse and level along this route.

The channel for the Lunsenfwa scheme took only two days to fix in the office from air photographs after several wasted months in the field.

Local materials suitable for use on works can often be located from the air and the air view assists the organization of construction work, particularly with regard to suitable lines of communication, positions for depots, etc.

*Transmission Lines.*

In Canada, particularly, use has been made of the air photograph for this purpose.

Vertical air photography was used by the Ontario Hydro-Electric Power Company in determining the location of their route and of the towers supporting the transmission wires, the route extending from near Ottawa to Toronto.

A transmission line from Flin Flon, Manitoba, to Island Falls on Churchill River, a distance of eighty miles over unmapped country, was selected from oblique photographs of a wide belt, the selected route being re-photographed vertically to fix the construction line. Here, again, final examination on the ground was necessary.

The selection of actual pylon positions for spacing, foundations, clearance of wires, etc., can be done on the photographs, with the aid of stereoscopic observation and parallax measurements for the approximate determination of heights.

*Road and Railway Engineering.*

Under this heading it is proposed to consider the application of air survey to construction in England rather than the purely economic aspect which has already been dealt with. Problems of location and surveying for roads and railways are much the same.

Apart from final setting out and levelling, the air photograph, aided by certain ground measurements for control and obscured detail, enables plans to be produced much more rapidly than by earlier methods. One of the first applications of air survey to road engineering was on the London–Southend road just after the War, but technique was then far from perfect, costs being high and standard of accuracy low.

The rapid awakening of the authorities in this country to the necessity of improvements in the road system to conform to modern standards of safe and speedy transport, has led to the Trunk Roads Act, under which the Ministry of Transport on 1 April 1937, took charge of some four thousand five hundred miles of trunk roads.

There is a danger, however, that without extensive employment of air survey by the Ministry it will be impossible to formulate a really balanced and economic national scheme of reconstruction and improvement in time to be of the greatest value. It is desirable for such a plan to be produced with the minimum of delay to prevent certain works from being undertaken without due consideration of their place in a national network.

The Ministry of Transport has charge of only a small proportion of the old Class I roads, and any drastic alterations to standards on these trunk roads which is not carried down in proportion to Class I and Class II roads may lead to an increase of accidents as appears to be the case on the German *Autobahnen.*

This Ministry has employed air survey to a small extent, but there is no doubt that a well-organized scheme of rapid survey, whether by using modern mechanical aids to ground-surveying instruments [24] or by air survey, would enable a sound programme to go forward in a reasonable period of time, so that all work executed would have a definite place in the national scheme.

Existing roads must be assessed on a basis of safety standards—widths, horizontal and vertical sight distances, gradients, cambers, radii of curves, side entrances, ribbon development, type of surface, slipperiness, etc, Measurements involving levels must be taken on the ground, but these can be made in conjunction with plans prepared from air photographs.

Separate prints, photographic and line plans are all used at different stages.

An indication of the awakening interest of local authorities in air survey for engineering purposes is given by the following extract from the *Sussex County Herald*, 14 May 1937: "East Sussex County Council is to experiment in aerial survey work in connection with the planning of new roads.

"The County Surveyor reported that delay occurred in the planning of new roads and diversions of existing roads owing to the fact that extensive field work was required for the purpose of revising the Ordnance Survey, especially in areas where there had been building development since the last official revision of the Ordnance Survey. . . . From air photographs accurate surveys could be made in short periods and the photographic records so obtained were more helpful than the plans obtained by the normal method.

"The cost of an aerial survey in average country was approximately the same as that of a ground survey, and in developed areas considerably less.

"The sub-committee recommended that with a view to gaining experience as to the desirability of a more extended use of that method an aerial survey should be made of a length of about seventeen miles of the route of the proposed new London–Hastings–Bexhill Road."

A section of the mosaic and line map of the Leatherhead–Guildford Road Survey prepared for the Surrey County Council is given in Fig. 9. The original plan was prepared at a scale of 1/1,000, that of reproduction being 1/3,750 approximately. Notice the general "crispness" of detail, and also how it is clearly seen where detail is obscured, as, for instance, in one place where trees obscure the line, thus enabling the ground surveyor to go at once to the points where detail measurements are required. This necessitated ground measurements. It is well to remember that at all stages of plotting, stereoscopic examination of separate pairs of prints will help interpretation considerably.

In numerous instances it has been found possible to effect the efficient lay-out of road crossings and junctions by examination of air photographs showing the main traffic lanes. Islands have been sited successfully by placing temporary islands in position and adjusting them after rephotographing to show traffic lanes.

Information obtained from traffic censuses has been augmented by air photography. Photographs taken at different times of the day and year show clearly the density and type of traffic using the streets, and by making a series of exposures at short intervals, it is possible to estimate the average rate of movement of traffic.

At Los Angeles [66] some ten years ago an air survey four and a half

miles long and one and a half miles wide was made for the purpose of studying traffic congestion. A controlled mosaic was produced at a scale of 200 feet to one inch, the condition of accuracy being that any distance measured between two points not less than nine inches apart, should not be more than three per cent in error. The actual error was usually less. Photographs were taken between 10.30 a.m. and 1.30 p.m. concurrently with traffic counts and flow studies at various points and from this information the rate of movement and density of traffic could be established.

The Texas Highway Department uses air survey methods extensively. Gibb Gilchrist, the State Highway Engineer at Austin, Texas, states [42]: "The usefulness of an aerial survey begins at the first inception of the work and continues until the last shovelful of dirt is in place." Actually, air survey goes farther than this for revision and record of development.

Air surveys have been made for the Natal Provincial Roads Department for the purpose of re-alignments and general improvements to the road system, and it has been employed extensively during the planning and construction of the German *autobahn* system. In the latter case approximate earthwork quantities have been determined from the photographs.

After an air survey had been made for the Leopoldina Railway in Brazil some years ago, the General Manager, Mr. G. W. Bayne, wrote to the survey contractors, the Aircraft Operating Company,* as follows: "In acknowledging the completion of your work for this Company, I have pleasure in informing you that the aerial survey and the 1/15,000 contoured plan of the Petropolis Serra made to our order have been completed to our entire satisfaction.

"From the plan itself it has been possible to decide that the routing and construction of an adhesion line up the Serra, with a maximum grade of approximately two per cent, requiring 41 kilometres of development, is a practicable proposition, whereas the topography of the Serra is such that to have obtained similar information by the ordinary means would have necessitated the employment of several field parties for a considerably longer period, and would have cost at least eight times that of the aerial survey.

"Appreciable advantages have thus accrued both in time and money.

"Whilst the work was being carried out the weather conditions were very unfavourable, but even so it was completed and the plan produced within the comparatively short period of five months."

The London, Midland and Scottish Railway Company commenced a special air-survey plotting office at Euston Station under Major W. H. Christie Clay, the Chief Estate Manager, and large-scale surveys

* Now merged with Aerofilms into Hunting Aerosurveys.

*Courtesy of Aerofilms, Ltd.*

FIG. 9A—RECTIFIED MOSAIC OF PART OF LEATHERHEAD-GUILDFORD ROAD SURVEY MADE FOR THE SURREY COUNTY COUNCIL.
[Original mean scale 1/1,000. Scale of reproduction 1/3,750 approximately.]

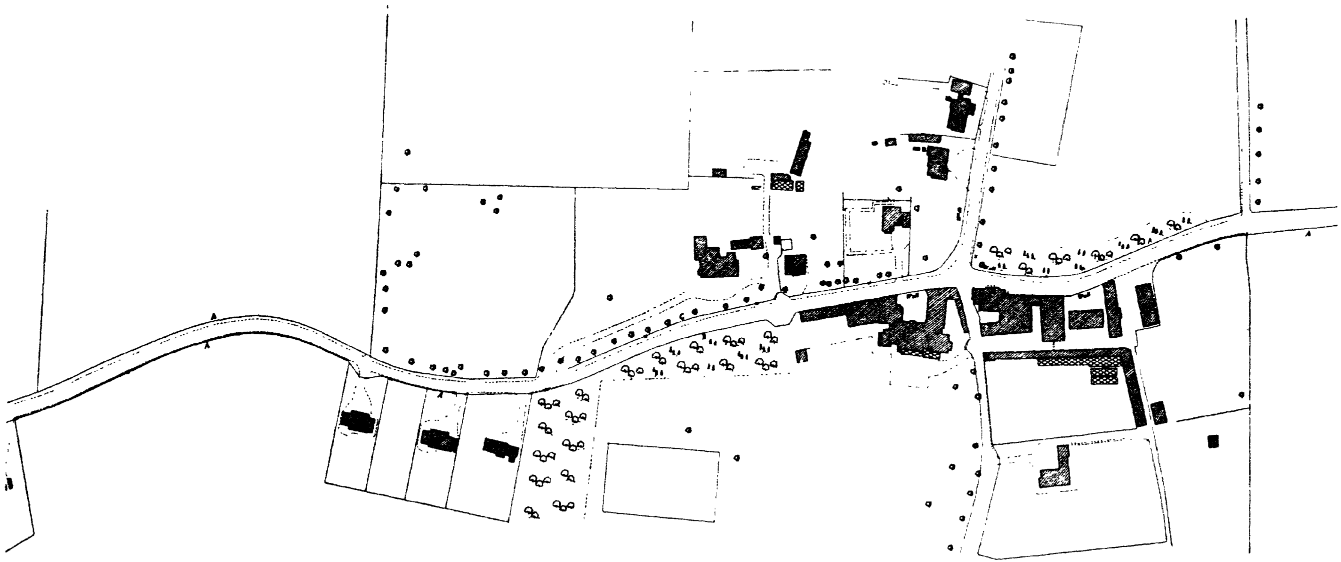


Fig. 9b—Line Plot of the Area covered by the Mosaic on Fig. 9a.
[Original scale 1/1,000. Scale of reproduction 1/3,750 approximately.]

have been plotted from air photographs taken by Aerofilms Ltd. These surveys are made without reference to the Ordnance Survey plans.

Photographic and line plans of a railway station are given in Fig. 10. The photographs were taken on a scale of about 1/3,600, and rectified to the ground control points which are marked by rings. The final average scale of the photographs is that of the plan, i.e., 1/1,584.

The Indian Air Survey and Transport Co., Ltd. (an associate of the Air Survey Co.), has made air surveys on the Bombay, Baroda and Central Indian Railway. A survey of part of the Baroda River facilitated the preparation of plans for draining works and bridges.

With regard to railway electrification, the following is an extract of a letter sent by Mr. F. B. Kitchen the Traction Engineer of the Contract Department of British Insulated Cables Ltd., to Aerofilms Ltd., on 17 October 1938:

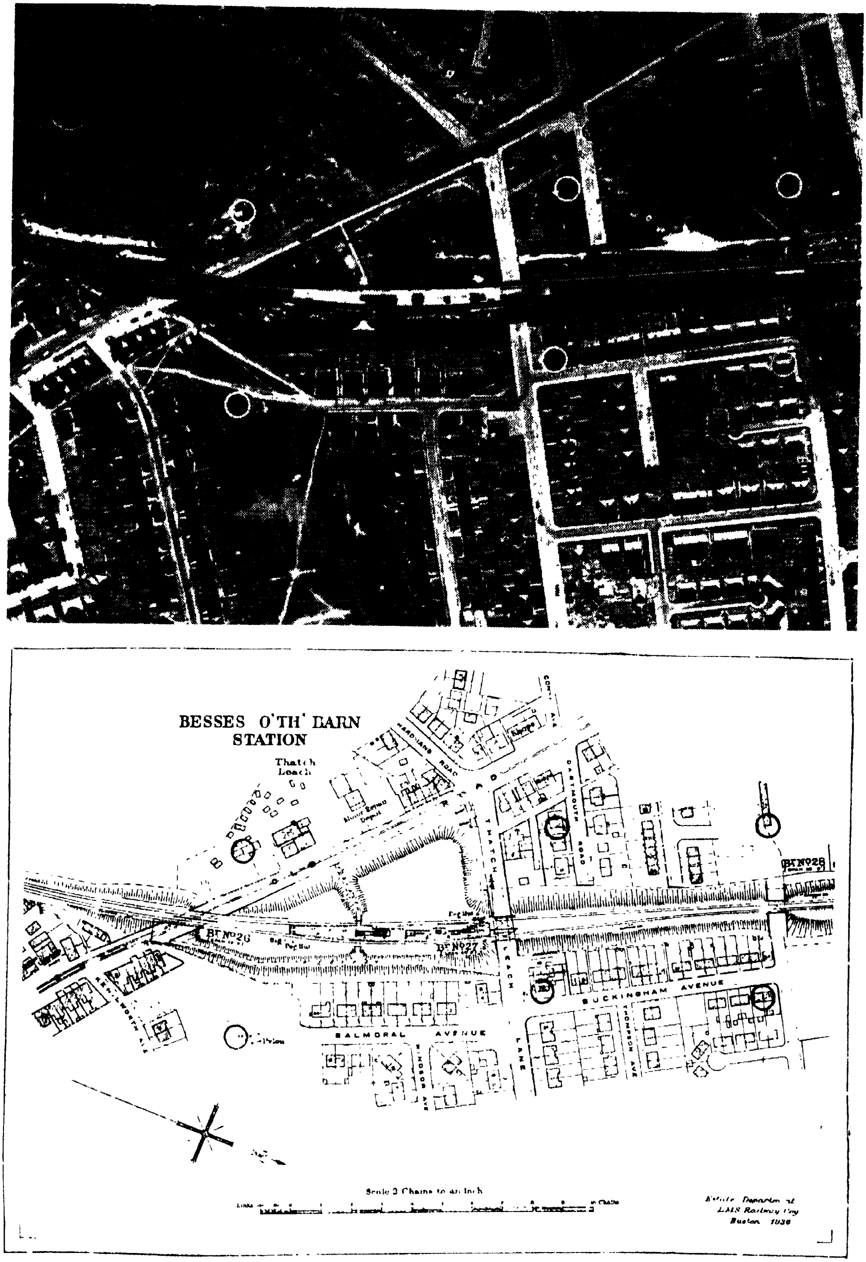
"To determine the position of turn-outs, crossovers, etc., and to enable a rapid survey to be taken of all tracks at important junctions within the area to be electrified between Manchester and Sheffield, it was decided to adopt an aerial survey. The contact prints are enlarged to the required scale of the track plans, and from these enlargements, tracings are made of all the essential features required. Certain selected tie-in points are determined and tracings are pieced together to form the finished plan.

"The above method has worked most satisfactorily and the finished plans have been finished in a fraction of the time and at much less cost than would be possible by any other method. The stereoscopic photographs have proved useful for examining the ground conditions. We intend to continue with aerial survey for this class of work."

*Town and Country Planning.*

It was perhaps in this field that the value of air photographs to local authorities was first appreciated. This probably arose because the necessity for extreme accuracy is not so great as in engineering surveys. Also, the rapid post-War developments in building, and the time-lag of mapping owing to the War, made it imperative to produce some sort of map for town-planning purposes. The addition of various Acts to the Statute Book, culminating in the Town and Country Planning Act of 1932, which require the preparation of planning schemes, made this necessity still greater.

The use of air photography received a pronounced impetus on the passing of the Restriction of Ribbon Development Act 1935. The control gained by the local authorities which administer classified roads over points of entry and building lines made a careful investigation of the conditions




[*Courtesy of Aerofilms, Ltd.*

FIG. 10—LARGE-SCALE RAILWAY SURVEY.

The plan of this station on the L.M.S. Railway has been drawn to a scale of 1/1,584, a corresponding photographic map being shown above. Ground control points are indicated by circles.

necessary. Before this time only those authorities having a planning scheme in operation or a special Act, could control this development. The attraction of producing the information without the "victim's" knowledge, thus saving much unnecessary preliminary argument, has made the use of air survey popular with the authorities. Moreover the contents of back-gardens are not hidden or difficult to obtain as in the case of a ground survey.

Aerofilms Ltd. have made a photographic mosaic near Kingston-upon-Thames to illustrate how by-pass roads are being converted into main streets.

Town planning is an art in which it is necessary to have a general picture of the existing conditions, together with some idea of the possible trend of social and industrial developments. The related studies of soils, vegetation and geology of the district must be considered because they affect the probable utilization of land.

The following are some of the aspects which can be studied by the use of maps and corresponding separate photographs.

(*a*) Soil types and the suitability of various sites for housing estates;

(*b*) Condition of buildings and the detail in back-gardens. Sheds and other structures are often put up without obtaining the necessary sanction from the local authority. (See Fig. 10, the original of which is, of course, larger and much clearer.)

(*c*) Vegetation types and distribution of trees;

(*d*) Improvement and lay-out of parks and recreation grounds;

(*e*) Traffic problems and possible parking places;

(*f*) Transport facilities—lay-out of bus stations, railway stations, local communications;

(*g*) Progress plans for housing estates. (See Fig. 11.)

Here, again, oblique photographs are often very useful in addition to the survey verticals.

The following are a few examples chosen at random:

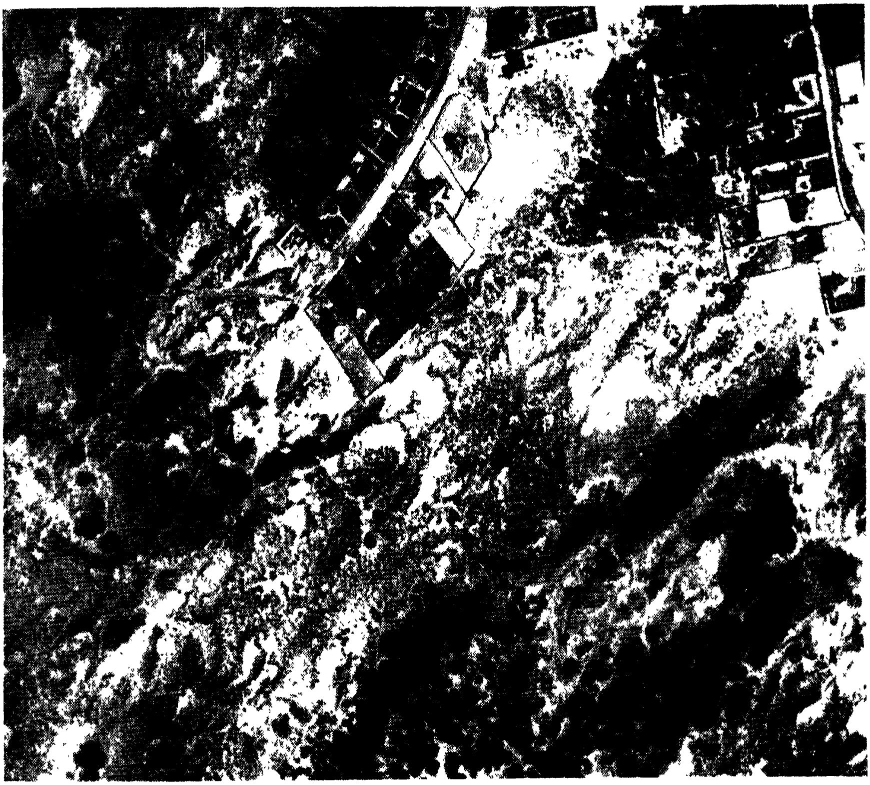
Plans were prepared from photographs taken by Aerofilms Ltd., in 1927, over an area of 12,800 acres, for the application to Parliament of the Manchester City Corporation boundary extension.

An area of some 10,000 acres was surveyed by the same company in 1934 for the Wisbech town-planning scheme; and in 1938 they undertook an air survey of 5,400 acres for the city of Worcester, which included 1/2,500 photographs and revision of 1/2,500 ordnance sheets.

Southall Urban District Council had some 2,500 acres photographed by the Air Survey Company, and plans were prepared to a scale of 1/2,500, while others for allotments and cemeteries were enlarged to 1/640.

Chigwell Urban District Council undertook its own revision plotting from photographs taken by the same company.

Town-planning surveys of Johannesburg, Pretoria, and all the Reef towns, some 450 square miles, plotted at 1 5,000, were undertaken by the Aircraft Operating Company of Africa.



[*Courtesy of Aerofilms, Ltd.*

FIG. 11—HOUSING DEVELOPMENT ON RECLAIMED LAND.

After an earthquake at Bihar in India, in 1934, the Indian Air Survey and Transport Company made reconnaissance flights and prepared large-scale photographic maps, sixty-four inches to one mile, which were used by the reconstruction committee.

Some years ago a controlled mosaic was made of New York City, [66] covering some 600 square miles. The general scale was 1/24,000 and com-

prised 126 sheets, each 14 inches by 21 inches, while a large area in the central districts was produced on a scale of 1/7,200. The main objects of the survey were to plan streets, extensions and improvements of communications and to develop the waterfront facilities.

A line map covering 57 sheets was produced of London, Ontario, from air photographs on a scale of 1/1,000, with contours spaced at 2½ feet. As much detail as possible was plotted from the photographs and additional information was plotted from ground measurements. The completed plan showed all streets, paths, buildings, trench marks, true position of all services, etc.

Many other examples could be quoted from various parts of the world, showing the employment of air photographs for town-planning purposes.

*Land Drainage, Flood Prevention, Irrigation and Soil Conservation.*

As a result of the heavy rainfall during the early part of 1937, flooding in England brought the problems of land drainage rather more to the fore than usual.

In swampy and marshy land it is often extremely difficult to get on to the ground to survey by ordinary methods. An air survey enables a plan to be made and the scheme to be prepared with a minimum of ground work.

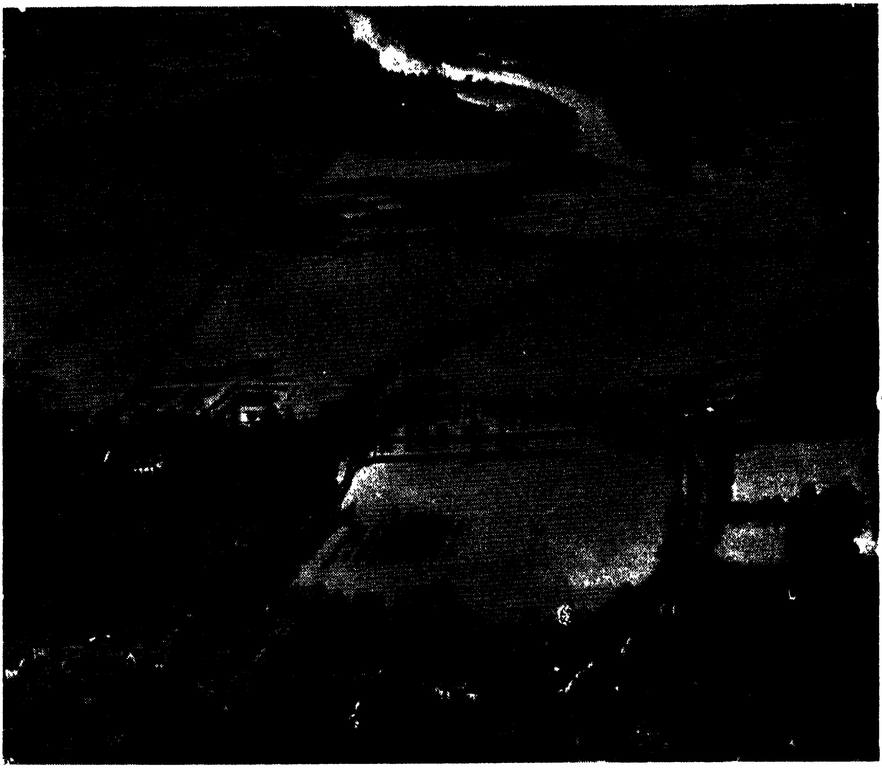
The air photograph has been found particularly useful in indicating limits of flood areas, and in this connection the oblique photograph enables the maximum flood area to be determined cheaply and sketched on to the six-inch map, using buildings, road intersections, bends in streams, etc. to help judge the lines. Weather conditions at such times would frequently be impossible for vertical photography. Fig. 12 is one of a number of obliques taken to determine the extent of Thames floods.

It is interesting to note that Aerofilms Ltd. were called in to map an air survey of some of the flooded area in East Anglia during the early part of 1938. In this class of work the photographs have to be taken with the flood at its highest point. In order to obtain successful photographs it is necessary not only to have the most rapid photographic materials but also a fast aeroplane as insurance against bad photographic weather. In many instances, however, it is possible to trace out flood limits from vertical photographs taken some time after the flood has subsided from its maximum height.

A stretch of some thirty or forty miles had to be relaid on the Central Railway of Africa in 1930 as the result of a very heavy flood, which delayed traffic for several weeks and eventually formed a new lake on the line of the railway. It was considered afterwards that, had the route been

located by air survey, the area liable to flooding would have been apparent and the expense of relaying avoided.

Mr. Eric Haquinius* [49] describes the work of control of the Brazos River Area in the State of Texas, which called for a chain of thirteen major dams on the main stream and its principal tributaries. ". . . As both speed and accuracy were necessary it was decided to use aerial photographs



[*Courtesy of Aerofilms, Ltd.*

FIG. 12—OBLIQUE PHOTOGRAPH TAKEN FOR FLOOD SURVEY ON THE RIVER THAMES.

as the foundation of all the work." The pictures were taken by a private firm at a scale of about 1/15,000.

In preparing schemes for land drainage it is important to know the positions of existing lines of agricultural drains. Frequently no information is available, but the air photograph, by detecting differences of tone can indicate the lines of existing land drains. Several photographic mosaics

* Chief of Survey, Brazos River Conservation and Reclamation District.

were made during the drought of 1934 and agricultural drain lines were easily detected by grass discoloration.

Irrigation problems are in many ways similar to those of drainage. The air photograph can show rapidly where irrigation is likely to be of use, and save much expense by indicating quickly where irrigation will be of no avail. The presence of ancient irrigation channels which have been filled in has been shown from photographs. Ground levels, however, are essential to fix precisely the flat gradients usually required in irrigation channels.

The work of the Soil Conservation Service in the United States has opened up a new field for the application of air survey. The Service commenced its work in 1933 and found almost a complete absence of large-scale maps covering those areas with which it is concerned. It was decided that air survey was the only practicable method of mapping the areas at a suitable scale and since 1933 some 400,000 square miles have been mapped in this way.

The progress made by the Service has been described recently by Dr. W. C. Lowdermilk.* [63] The following information is due to him.

The aim is to carry out the necessary mapping in the combined interests of soil and water conservation, and flood control. For a particular drainage basin the plan must consider the climate, soil characteristics and their relation to such factors as erosion, and cropping, land classification; hydrological characteristics; existing method of farming and its economic condition.

Lowdermilk remarks: "On the basis of such information a master plan is prepared which sets up land-slope classifications, upper limits of safe cultivation, suitable rotations, spacing and grade of terraces, types of terrace outlets and waterways, desirable wild life development, land-use adjustments, run-off retardation, and other such prescriptions for soil and water conservation as may be needed. In turn the detailed plan represents an adjustment of the over-all drainage to a specific farm. Field boundaries may be changed, rotations begun, strip cropping followed, terraces laid out, outlet channels prepared and gullies controlled.

"An inventory of the physical land factors, involving four features was developed. They were (1) soil type, (2) land-slope gradient, (3) degree of erosion, (4) present land use."

The air photographs are usually taken at a scale of 1/20,000 and may be enlarged up to twelve inches to one mile. They are first examined stereoscopically so that drainage lines may be marked in blue. This, together with other information, such as the area to be mapped for land

* Chief of Soil Conservation Research in the Soil Conservation Service.

use, is then available for the ground conservation surveyor, who marks on the four kinds of information according to standard symbols. Also at this time the necessary ground control is fixed for plotting the plan.

The Service is also concerned with certain reservoir surveys, such as Lake Mead (Boulder Reservoir) which must be topographically mapped accurately.

The plans made are available for making surveys for flood control required by the "Omnibus" Flood Control Act. Lowdermilk says: "By this Act the Department of Agriculture was made responsible for the land-use phases of flood control, supplementing downstream engineering works, and the maps are indispensable to this new activity. They will also serve for the evaluation of essential information on a watershed basis. Infiltration capacities of soils, slopes under different land-use, silt-production areas, and rainfall patterns may be applied to the base drainage maps for evaluation in terms of storm run-off and flood flow accumulation."

*Navigation Channels, Harbours and Coast Defence.*

Deepening and widening river channels and estuaries for navigation frequently involves considerable expense. Air photographs will often indicate quickly the presence of sand banks, shallows and rapids by tone variations. They can be used also for the fixation of soundings. It is possible also to detect the movement of silt in estuaries, and tidal behaviour may be investigated simultaneously over considerable areas.

A length of some 450 miles of the Zambesi River was surveyed by the Aircraft Operating Company in order to ascertain the ways in which navigation could be improved.

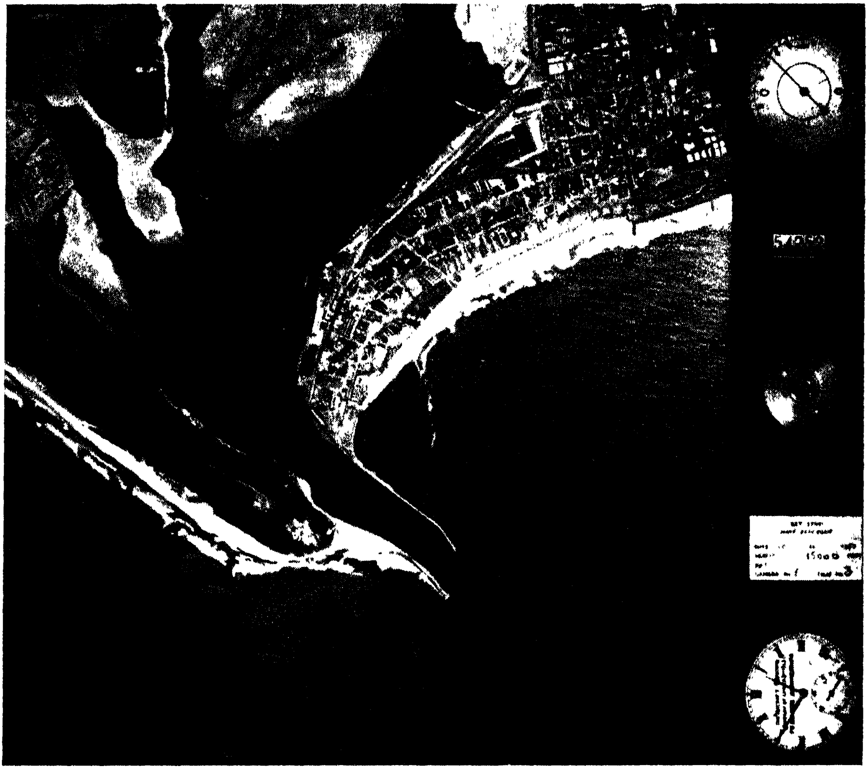
The same company prepared a map of Durban harbour East Africa. Here the water is fairly clear and the photographs gave a fairly good indication of depths. An interesting feature was that deep-water channels, dredged subsequently to the preparation of the chart, were clearly shown in the photographs. Fig. 13 shows clearly how erosion is taking place in the harbour, the lighter colouring indicating small particles in suspension.

In 1930 Aerofilms Ltd. prepared plans covering 20,000 acres for the Aire Navigation, River Humber and Docks.

In the development of the Beauharnois–Valleyfield section of the St. Lawrence River in Canada, it was proposed to construct a large canal for both navigation and power development. A mosaic on a scale of 800 feet to one inch was made of an area about twenty miles long and five miles wide. A base for locating the principal points of the vertical photographs was laid down with a close net of ground-control points. After rectifying the photographs for scale they were assembled in a mosaic, and

this photographic plan was used for selecting the route of the canal and dealing with the land affected.

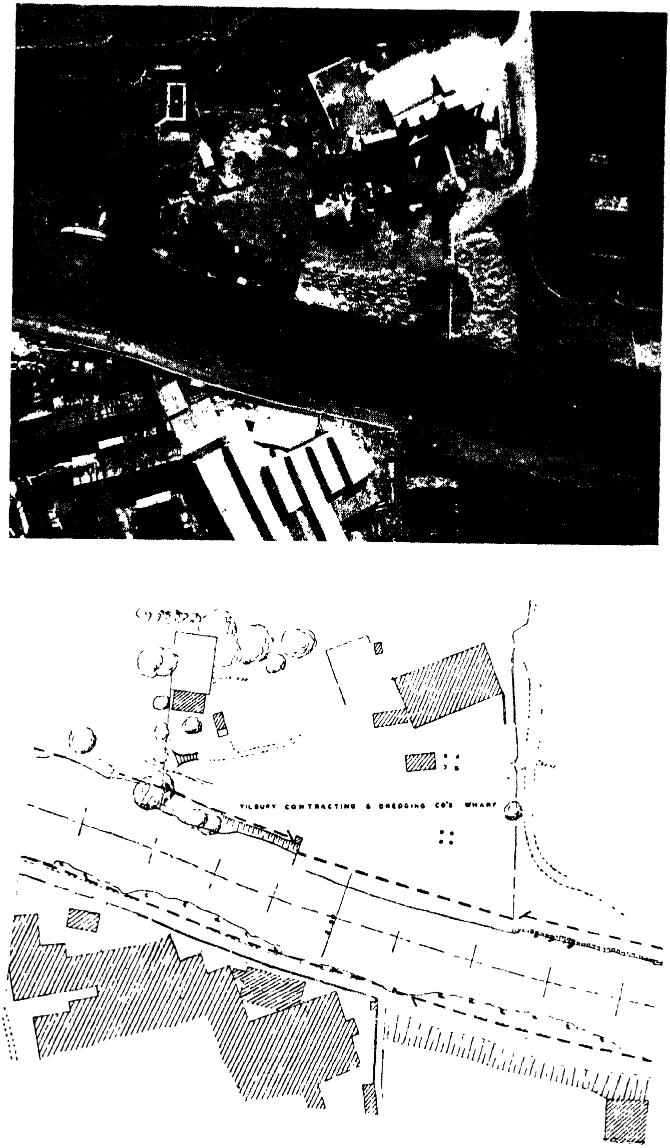
A large-scale survey (1/360) has been made of part of the River Medway for preparing river improvement schemes for widening, cutting



[Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.

FIG. 13—VERTICAL PHOTOGRAPH: DURBAN HARBOUR, SHOWING HOW EROSION IS TAKING PLACE. Scale of print 1/13,000: taken with 7-inch lens from height of 15,000 feet. Scale of reproduction 1/26,000 approximately.]

off corners, etc. The photographic plan and line-plan of a small stretch are shown in Fig. 14. The accuracy has been found to be of sufficiently high standard and the photographs were found useful in Committee. In *The Times*, 5 September 1936, it was stated: "An aerial survey of the River made on a scale of 30 feet to the inch has proved accurate for all practical purposes, and there has been no ground survey."




[*Courtesy of Aerofilms, Ltd.*

FIG. 14—RIVER IMPROVEMENT—PHOTOGRAPHIC MAP AND LINE PLOT—ORIGINAL SCALE 1/360.

Air survey is also proving its value, not only in mapping the form of coast where erosion is taking place so that protection works may be designed, but also in providing progress photographs of erosion and protection and tide records.

Many other instances could be given of the application of air surveys to engineering projects—e.g. in planning the lay-out of aerodromes. In addition to the verticals used for planning, special obliques may be taken so as to enable the designer to site his run-ways in such positions that air obstacles are avoided.

If it is always remembered that co-operation between air and ground personnel is essential, and that air photography is merely a means towards an end, there appear few limits to the scope of its application in engineering works.

### Scientific Interpretation of Air Photographs and their Economic Applications

It has been made clear that the air photograph will give much more information than a topographical map. It is quite wrong to approach the survey of an area incompletely mapped without considering co-operation between such specialists as surveyors, geologists, foresters, botanists and agriculturalists.

Bourne [7] states: "In order to obtain the maximum results from such a reconnaissance, it is contemplated that each field party would be representative of several sciences. Besides the surveyors, there would be an agriculturalist and a forester, both having special knowledge of the local flora and the principles of ecology and soil science. There would also be a geologist, experienced in the geophysical methods of survey. Finally there would be such other specialists as the circumstances might suggest: for instance the presence of a pathologist (in tropical countries) would generally be desirable."

It is now generally recognized that the only way in which this co-operation can easily be attained while mapping easily and rapidly is by means of air photography.

In the survey of some 63,000 square miles carried out in Northern Rhodesia by the Aircraft Operating Company, there was with the expedition an expert forestry officer formerly with the Imperial Indian Forest Service. At the suggestion of the Company the Colonial Office attached a geologist and a botanist to the survey with the duty of studying the economic problems each in their respective spheres. Such work is very

important in a country which is so highly mineralized as Northern Rhodesia.

Bourne [7] continues: "Whereas the incidence of minerals was largely determined in geological times, the distribution of vegetation is subject to the varied influences of climate, geology, soil, disease and man. If nature is left to herself, the ultimate result is nearly always forest. Locally within any climatic zone, the natural forest type tends to change with changes in geological formation and soil conditions. Under the influence of man the character of virgin forest is modified in one way or another, either directly as a result of clearing for cultivation or indirectly through the agencies of fire and grazing animals. Within the Empire there are still vast areas of forest in various stages of development, both progressive and retrogressive. The special study of the changes taking place is the subject of the comparatively young science of Ecology."

Such a study can well be made either by visual reconnaissance from the air or from air photographs, together with some ground checking.

*Ecological Surveys—Forestry; Soil Studies; Agriculture; Geology.*

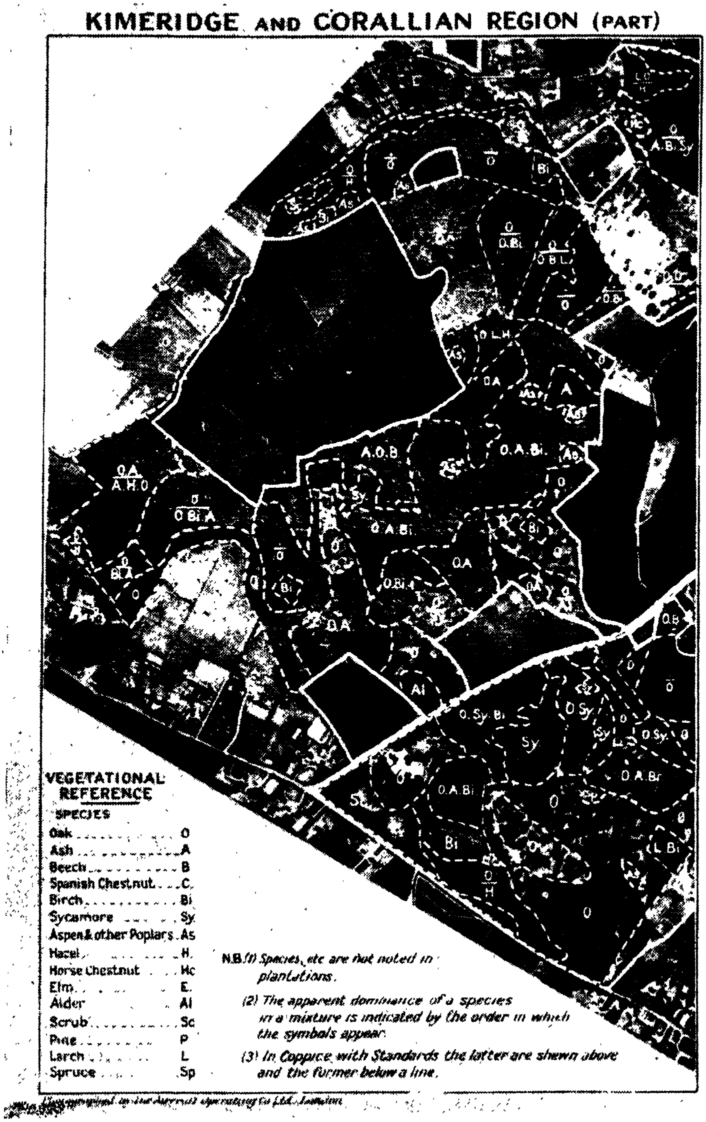
The air photograph is proving of great value in the classification of soils and vegetation. These are related to geological formation and variations of tone are dependent upon the texture and moisture content of the soil, thickness of surface soil, drainage, etc.

It was in connection with forestry that this application of air survey was first applied. Soon after the War, part of the Irrawaddy Delta, almost inaccessible owing to the incidence of swamps and dense vegetation, was surveyed from the air for the Government of Burma.

About the same time air survey began to be employed in extensive forestry operations in Canada, and also later in the United States. Foresters are concerned with the preparation of stock maps of standing timber and for this purpose a proper topographical map is essential, if the forests are to be properly managed. It is impossible to see the whole forest area on the ground and the air photograph, intelligently interpreted and checked on the ground, can provide a much surer method of delineating the areas of varying types.

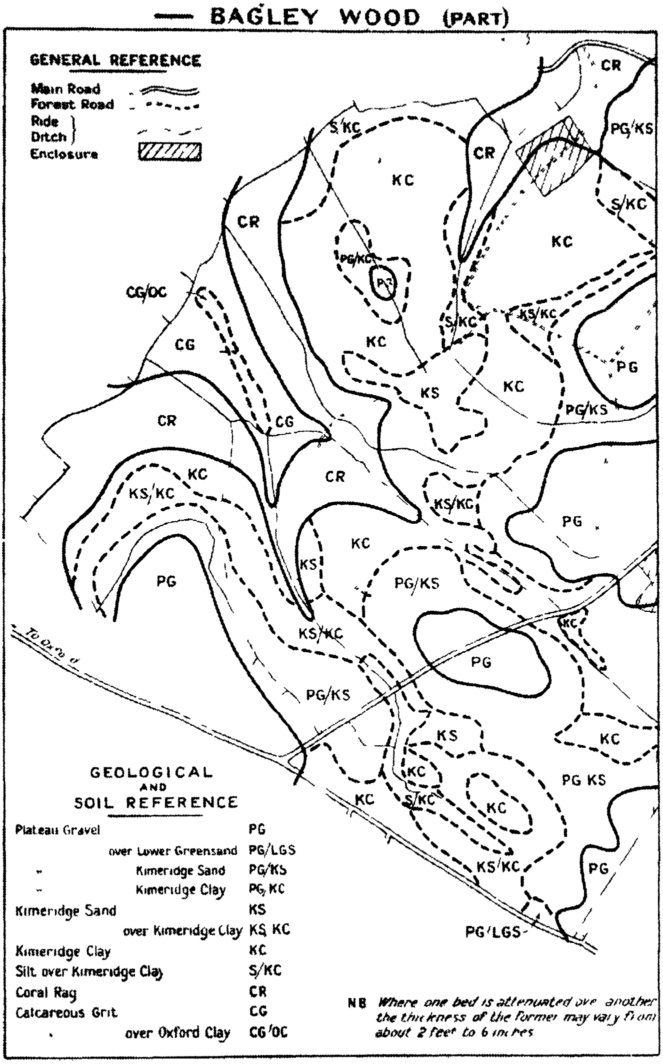
The general object is to find out the location and quantity of the marketable contents. On the ground the denseness of the forest prevents the forester from inspecting anything but the lines along which he travels.

Bourne [7] in describing a pioneer investigation made in Northern Rhodesia in 1927, deals with the interpretation from air photographs of geological detail on ecological lines. Gill [43] remarks that Bourne's




[*Courtesy of Aircraft Operating Co., Ltd*

FIG. 15a—DETAIL VEGETATION MAP.




[Courtesy of Aircraft Operating Co. Ltd.

FIG. 15b—DETAIL GEOLOGICAL MAP.

conclusions regarding the interpretation of photographs did not receive, at that time, unqualified approval from geologists.

Subsequent tests have, however, confirmed and amplified these conclusions and Robbins, who has made a special study of the problems of ecology, described in 1934 another experiment made in Northern Rhodesia. [81]

About 3,000 square miles were covered in three months over the ground where verticals and oblique photographs were available. A large number of types of land and vegetation were distinguished on the ground and the photographs and were transferred to the map. He considered that it would have taken years to fix all the boundaries by ground survey alone. He found that it is possible to pick out with certainty (i) areas which contain the highest proportion of better-class arable lands suitable for first European settlement, (ii) arable land in smaller blocks which may be mixed with sites more suitable for pastoral purposes, (iii) flat forest on limestone, on sand; and hill forest on shales, etc.; i.e., land which is largely useless for agricultural purposes, but where it is possible to define the boundaries of those forest areas which should be protected in the interests of the future water supply, (iv) many types of grassland without trees which are likely to become waterlogged.

Robbins's final conclusions were that the areas of interest could be marked rapidly and at a cost which well justified the employment of expert scientist-interpreters. This compares with the work of the United States Soil Conservation Service described on page 70. In the United States the Agricultural Adjustment Administration determines areas of cultivation from air photographs.

Growth of trees and variations in them are probably the most easily recognized features on an air photograph. It has been established that there is a definite correlation of the types with the moisture-content of the soil and the rock conditions.

The general principle may be seen by referring to Fig. 15. Here on the left is shown a detail vegetational and geological mosaic. Markings have been made along the lines of change of tone, and in the map on the right the distribution of soils and species is shown.

Fig. 16 gives an example of land-classification in Northern Rhodesia from air photographs. [81] The caption below is as given by Captain Robbins from his survey, with the omission of botanical names.

In Canada, the oblique has proved very useful in soil studies in the vast prairie regions, and it is there preferred by many to the vertical because one major soil class can be compared with another several miles away. Soil drifting erosion and burnt-out areas can be easily discovered and analysed.


[*Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.*

FIG. 16—CLASSIFICATION OF LAND.

| | |
|---|---|
| IAi. | Aquatic and reed growth, always submerged. |
| IAii. | Grass swamp, submerged every flood season for some months. |
| IAiii. | Submerged at highest floods only. |
| II A. | Mopane forest. (*N.B.*—Tree found in Southern Africa.) |
| II D. | Flat land; not submerged but badly drained. No mopane. |
| III. | Steep shale hills. |
| VII A. | Foothill type intermediate between III and II D. |

These problems have also become of great importance in such countries as the United States and South Africa. Rapidly spreading soil erosion, due to incorrect agricultural treatment of a region as a whole, demands a speedy solution to avoid denudation of vast areas. Air survey is the only practicable means of mapping the necessary information, where the time factor is of paramount importance.

In Canada, tree heights are being estimated by parallax measurements on air photographs. After allowing for height of undergrowth and other errors, a very fair estimate can be made for height as well as type. It is found that measurements are more accurate when the snow is on the ground, as the base of the tree and its shadow are more clearly shown.

Burns[11] stated before the Conference of Empire Survey Officers in 1935: "Deciduous trees are separated from evergreens by the absence of leaves; species are identified by the tone of the foliage and from characteristics of the tree's crown. . . . Information of this nature that would have cost 20,000 dollars by ground methods was recently supplied on seven hundred square miles for a cost of about four hundred dollars from winter photographs at a height of 10,000 feet. A tree height grid has been constructed to gauge the general height of the stands of timber as shown on oblique photographs. For verticals a time-shadow graph is used. The best results are obtained by a nice combination of photographic maps with supplementary information obtained on the ground."

There is considerable disagreement as to the form which geological maps should take. Geological maps are either "solid" or "drift," the latter purporting to show the surface conditions. Unfortunately the geological mappers classify the various beds in the chronological order of the fossils they contain, but this is not much help to those who wish to make use of the surface material of the earth. For example, Forest Marble, as it is called by the geologist, may outcrop as a clay or a brashy limestone. Even the "drift" map will indicate the marble as a general formation and this is obviously not much use for a soil map. [82] Thus, since the geologist is mainly concerned with his chronological mapping, the results will often give quite a false impression to the cultivator who is most concerned with the top soil.

It appears that the information which is being recorded from air photographs by such bodies as the United States Soil Conservation Service is much more valuable to those who wish to make use of the land surface than that obtained from purely geological maps.

*Geological Surveys.*

The topographical map of an area is usually the basis of the subsequent

geological surveys. By providing such a basis in the early days, the air survey enables the geologist to specialize in his own work so that his observations can be plotted direct.

Gill,[43] in his paper *Air Survey in Relation to Economic Ecology* in 1932, has dealt with the geological aspect very thoroughly, and he gives a very complete bibliography of air survey publications up to that time.

Small surface and vegetational differences are shown by the air photograph to the geologist with a knowledge of geomorphology and experience of interpretation. Experience has shown that features previously missed on the ground have been shown up by the air photograph.

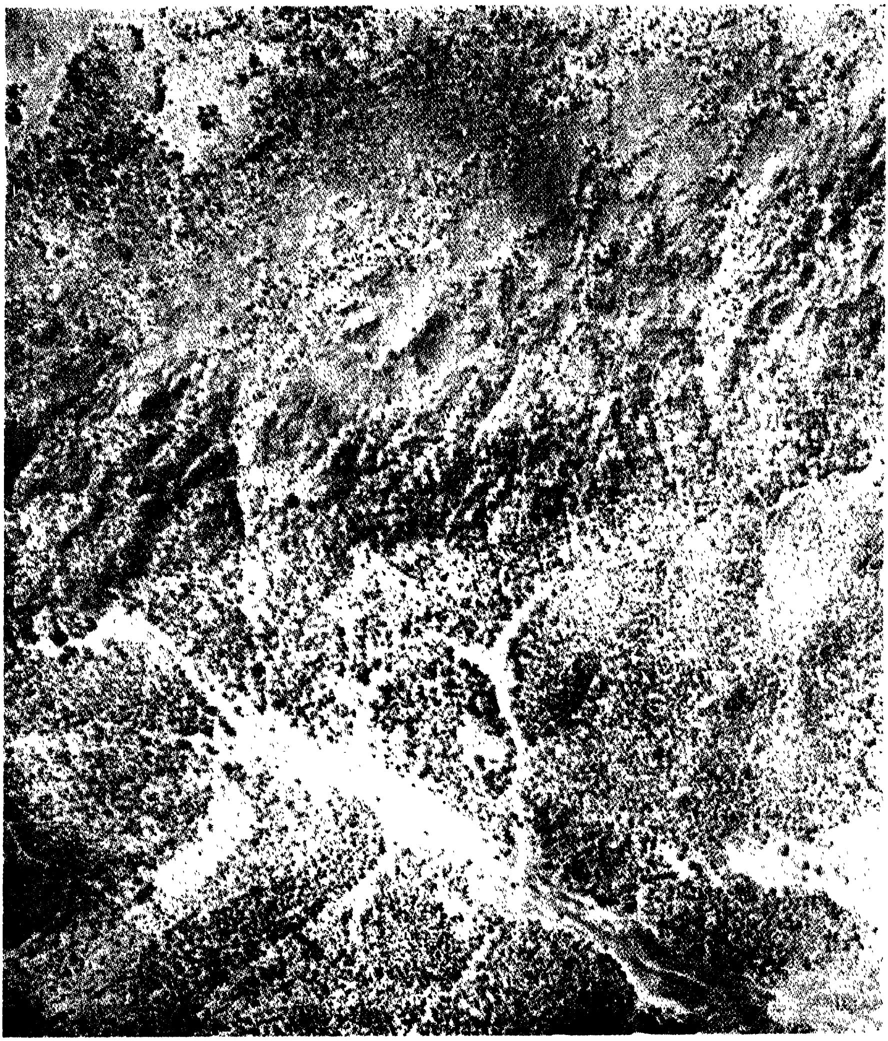
To interpret air photographs satisfactorily from a geological aspect, it is necessary that the geologist should have some experience in interpretation from the ecological aspect.

The main geological features of structure and contact can often be determined in the office before field work is commenced. The system of faults, fractures, dykes, shear zones, etc., is frequently apparent in the photographs and careful investigation is sometimes necessary on the ground to establish the fact, even when the site is known, because of surface covering.[83] In one instance in Western Australia, the probable line of a fault appeared on a photograph, but could not be found by inspection on the ground. Later the suspected position was located by measurement on the photograph and the fault was found below the surface as indicated by the photograph. Where faults, dykes, etc., are covered by soil or other surface material, the local movements or drainage which affect the moisture-content of the soil, have an effect upon the vegetation which is changed in tone due to variations of colour, type and growth.

The fact that the air photographs give a view many times greater in area than can usually be obtained from a ground view, aids the general geological interpretation of the district.

Robbins[82] states that he has known cases where areas of highly sheared rocks covered by soil, could not be identified on the ground. The areas were indicated by variation in soil tone. Subsequent petrological examination confirmed that in one case the rock was sheared granite and in another a sheared grit. He also quotes a case where a faulted dyke in dolomite was invisible on the ground as there was no outcrop of the dyke. Later the photograph was proved correct.

It is easy for the geologist working on the ground to join up the wrong features. "The greatest value is obtained from aerial or ecological-geological work in areas that are the most difficult, in areas where the rocks of importance are completely obscured. Even to know whether an area is clean or much faulted is worth a great deal."[83]



[Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.

FIG. 17--GEOLOGICAL CONTACT SHOWN BY VEGETATION. S. RHODESIA

A photograph taken by the Aircraft Operating Company in Brazil for the Leopoldina Railway Company during a survey of the Petropolis Serra shows a remarkable effect. The height of the trees is from 100 to 150 feet and across the photograph can be seen a strip where the trees

[*Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.*

FIG. 18—PITCHED FOLD SHOWN BY TOPOGRAPHY. STREAM FOLLOWING MAIN FRACTURE. NO FAULT ALONG FRACTURE, BUT VERY IMPORTANT FROM THE MINING POINT OF VIEW. SWAZILAND.

are lighter in tone; actually they are in flower. From that photograph was found the line of a terrace fault, previously unsuspected, the difference in tone being largely due to effect on vegetation of local variation of soil moisture-content.

A vertical photograph of an area of bush country in Southern Rhodesia is given in Fig. 17. The type, density and tone of vegetation indicates variation of soil or formation and here the variation of tone running across

the photograph was found to be a pre-cambrian contact. It should be noted that detail is much clearer on the larger, original photograph.

A more striking case from Swaziland is shown in Fig. 18, which illustrates how the topographical forms aid interpretation. Here a pitched fold is shown by the configuration. The photograph was taken during a gold-prospecting survey and the fractures where mineralization has occurred are indicated by the stream lines. In this particular instance the broken type of country made appreciation difficult on the ground.

The geological mapper makes use of existing maps when possible, but in the field there are always many measurements to be taken. If the survey is made from air photographs and a photographic map and separate prints are available, the geologist has only to study the geology and uses the pictorial records for marking the information he obtains. It has been estimated that by this method the geological map can be produced in one-fourth the time that would be required by ground methods only.

*Applications of Geological Interpretation to Prospecting and Mining.*

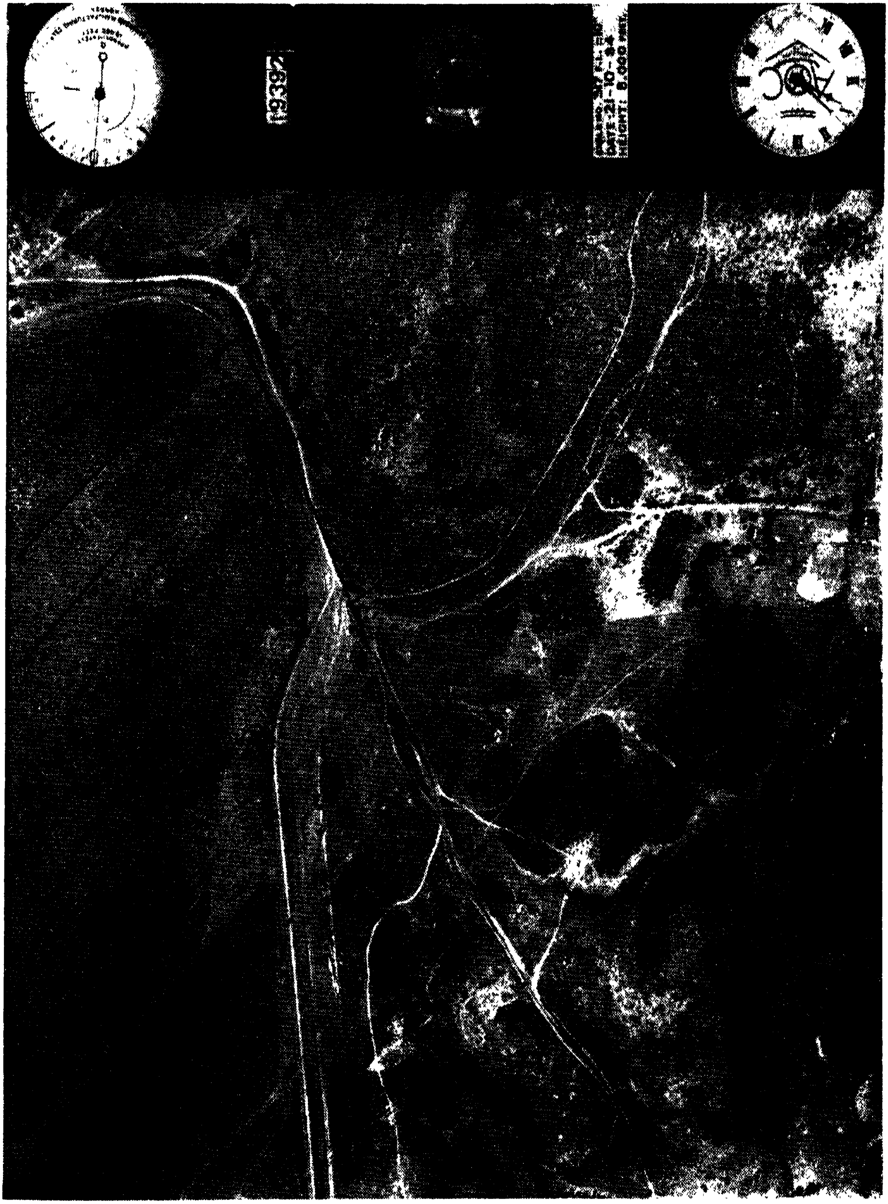
There have been a number of instances where air photographs taken for some other purpose have by accident proved of great value; for instance Hemming[51] recalls, how a few years ago a petroleum technologist happened to be examining some air photographs taken for mapping purposes. He noticed that certain characteristics were shown which looked interesting. When this area was checked geologically on the ground the result was the discovery of a new oilfield.

Most surveys for oil companies are of a confidential nature, and it is probably not generally realized that no oil company would go forward with a project on a new area without first making a thorough examination of air photographs, and of the ground in conjunction with the photographs. This is particularly useful in the examination of sedimentary rocks which promise the presence of oil.

In mineral prospecting from the air the interpreter is helped considerably by the fact that mineralization is often connected with major faulting. These faults are frequently indicated by changes in tone of the ground or by variations in the vegetation. Here again, the confidential nature of most of these surveys, prevents them from becoming generally known.

A number of economic geological surveys have been made in South Africa by the Aircraft Operating Company of Africa and an example was the survey of some 2,000 square miles of the Witwatersrand Reef in 1934.

Fig. 19 shows a marked-up geological photograph from a mineral-prospecting survey in South Africa. (Note the line of fault running diago-



[*Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.*

FIG. 19—GEOLOGICAL PLOT ON AIR PHOTOGRAPH.

nally across the photograph from the top left hand corner.) The lines of contact were marked on the photographs where possible and later checked on the ground.

An application of the air photograph to alluvial gold prospecting in Africa is shown in Fig. 20. It will be noticed that the old bed of the river shows up quite clearly, the site not being very apparent on the ground. About half an inch below and running parallel to the white line (a road!) cutting across the top left-hand corner of the photograph, can be seen a thick black line about two inches long indicating dark vegetation. This at once suggested that the ground should be examined along this line and, as a result a fault was discovered. Also the silted-up ox-bow, seen quite clearly on the left of the existing stream about half-way up the photograph, is actually not very obvious on the ground. This is a place which promises the presence of gold. (The dark rings which can be seen in the photograph are native kraals.)

It is impossible to give in these reproductions of air photographs a clear impression of the detail to be seen in the originals and the clear-cut view under the stereoscope.

H. Hemming and Partners specialized in this class of work and have contributed considerably to the development of the use of air survey in prospecting for gold and other minerals. Surveys have been made in New Guinea for the Bulolo Gold Dredging Company and in Papua an air reconnaissance was completed of some 22,000 square miles of virgin territory covered with thick forest and entirely unmapped. In Western Australia a very extensive survey of 88,000 square miles was commenced in 1933 for the goldfields of the Western Mining Corporation. By September 1934, 20,000 negatives and 60,000 prints had been prepared. Mosaics covering 6,600 square miles had been produced from these photographs at scales of 1/48,000; 1/31,680 and 1/14,400.

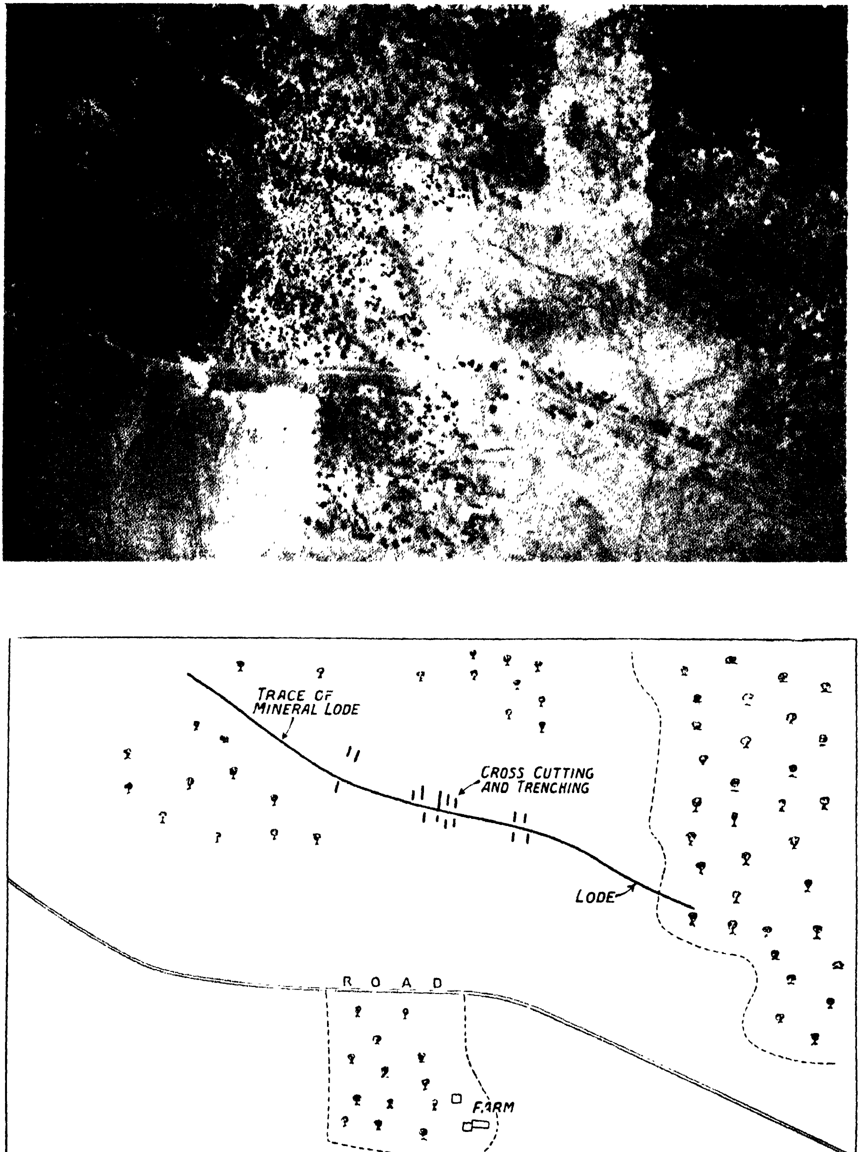
An air photograph taken during this survey is shown on a very much reduced scale in Fig. 21. It shows an established mineral lode, which is disclosed by its effect on the vegetation and soil, and makes it appear as a continuous line running across country. The sketch map shows the direction of the lode.

In Australia, also, several thousands of square miles have been photographed by the Australian Air Force in Queensland, Western Australia and the Northern Territory. This was largely in the nature of a reconnaissance survey and was directed particularly towards investigation of geological and soil characteristics.

Extensive work has been done in Canada where mineral deposits have been located from air photographs even where there is forest.

[Courtesy of Aircraft Operating Co. of Africa (Pty.), Ltd.

FIG. 20—KENYA, NEAR VICTORIA, NYANZA, showing present and old river beds. Photographed for alluvial gold prospecting. There is a dyke running parallel to and just below the road in the top left-hand corner. This is shown by the line of dark vegetation.




[*Courtesy of H. Hemming and Partners, Ltd.*

FIG. 21—WESTERN AUSTRALIA: MINING SURVEY.

Many thousands of square miles have been mapped for economic purposes by Professor Schermerhorn of the Technical High School at Delft in Holland. In particular he has mapped large areas in the Dutch East Indies and elsewhere for the Dutch-Shell Oil Company.

In general, mining and prospecting work is undertaken at the instance of private concerns which may be somewhat diffident in putting this work in the hands of a survey company, the operatives of which will have very valuable information before the initiators themselves.

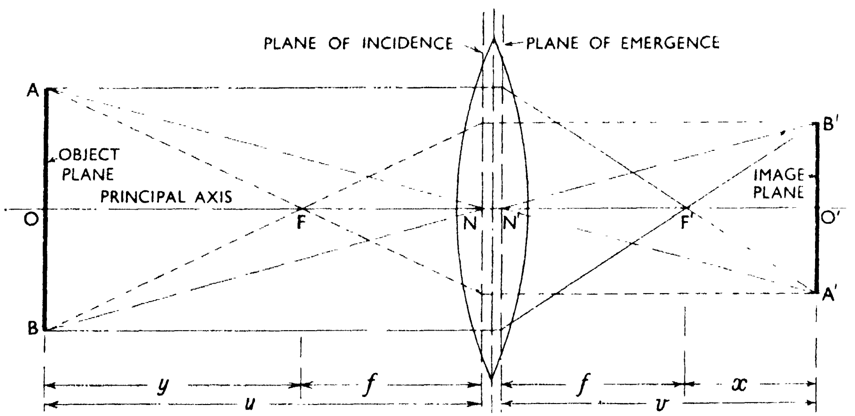
Such a firm must be able to be trusted implicitly to supply reliable information without possibility of leakage; in fact the air survey company must bear the same confidential professional relationship to the client as for instance a consulting engineer, or a lawyer.

CHAPTER IV

# PRINCIPLES OF AIR PHOTOGRAPHY: THE SURVEY CAMERA: PROBLEMS OF AIR PHOTOGRAPHY: FLYING FOR AIR SURVEY PHOTOGRAPHY

## Geometrical and Optical Principles of Photography

The essential fact of photography is that an image, projected through a converging-lens system, is recorded on a sensitized surface upon which it can be developed and fixed so that a permanent record is available.




RELATION OF OBJECT AND IMAGE FOR A CONVEX LENS

Fig. 22.

There are certain optical terms and definitions which from time to time cannot be avoided and each will be described as it is met. In Fig. 22 a diagram is given of the formation of an image by a simple convex lens. An image is formed in the same way as in a camera lens system. The *optical axis*, or *principal axis* (OO′), of a lens may be defined as that line along which a ray of light will pass without deviation or displacement and which will pass through the centre of the lens normally to its surfaces. Any other ray such as BB′ passing by the most direct route BNN′B′ from

the object B to the image B′ will meet the principal axis at N and leave it at N′ parallel to the original direction, i.e., any ray which appears to converge to N, appears to diverge from N′. These points, N and N′ are known as the *front* and *rear nodal points* or *nodes* respectively. A ray through A in the object space in a direction parallel to the principal axis, will be refracted on meeting the lens and will pass through the rear *principal focus* (F′), also on the principal axis, and will meet the image plane at A′. Similarly a ray in the image space which passes through A after emergence, being incident parallel to the principal axis, will have passed through the front principal focus (F) which is also on the principal axis. The plane which is perpendicular to the principal axis and which contains the front node (N) is called the *plane of incidence*, while that containing the rear node (N′) is called the *plane of emergence*.

All rays passing through the lens by the shortest geometrical route apparently travel along the principal axis for the length of the *inter-nodal* distance (NN′) which may be of the order of three millimetres. The one ray which is not refracted is that along the principal axis.

The image A′B′ is formed in a plane perpendicular to the principal axis at a distance $v$ from the rear node N′, by the projection of an object AB from a similar plane at a distance $u$ from the front node. The *focal length* of the lens is defined as the distance from a principal focus to the corresponding node. In this case, $FN = F'N' = f =$ focal length.

It can be shown that for such a lens or lens system, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$*.

In an ordinary camera the image is brought into clear focus by adjusting the distance $v$. In surveying cameras used in air photography, $u$ is not likely to be much less than 3,000 feet, while the focal length is not generally greater than 21 inches; hence $u$ may be assumed to be infinitely great compared with $v$, so that $f = v$ approximately. The camera is therefore not provided with any focussing arrangement and is constructed in a box, all images being formed in the *focal plane* which passes through F′ and is perpendicular to the principal axis.

Assuming that the lens does not distort the rays in any way, the image formed upon a camera plate is a true perspective picture.

Since the direction of emergence of the shortest geometrical ray from the lens is parallel to that at entry, it will be noticed that the inter-nodal distance has no effect upon the perspective construction.

The front node is often called the *external perspective centre* and the rear node, the *internal perspective centre*.

* Here the expression has been taken as an arithmetical one, i.e. there is no change of sign on opposite sides of the lens, which is a usual convention.

When the object is at infinity, i.e. when $f = v$, the ratio

$$\frac{\text{Length of image A'B'}}{\text{Length of object AB}} = \frac{f}{u}.$$

The small internodal distance is quite negligible when compared with the object distance $u$, so that for all practical purposes it is sufficiently accurate to assume that the internal perspective centre N′ is the perspective centre for both the object and image spaces. It is usually assumed that all rays passing through this point proceed undeviated.

A diagram of a conventional ground photograph is shown in Fig. 23, in which the centre of the optical system is at the rear node N′ of the lens

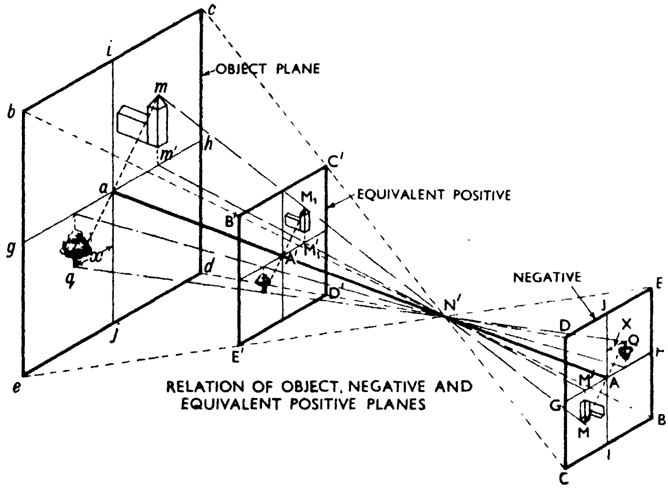



FIG. 23.

system. The principal axis, A$a$, passing through N′ is taken as horizontal in this case. A vertical plane in space, normal to the principal axis and containing the top $m$ of a church spire and the centre $q$ of the base of a tree, is represented by $bcde$. Images of these points projected through the lens system are seen at M and Q respectively in the vertical image plane BCDE which is also normal to the principal axis. The paths of these rays are indicated by the dotted lines.

Horizontal and vertical collimating marks GH and IJ are placed in the focal plane either as hairs or lines etched on glass, so that they appear in the photograph. These lines intersect at A, the plate-centre of the photograph. Corresponding points in the object plane are $g$, $h$, $i$, $j$, and $a$.

The horizontal plane containing $gh$, GH, and A$a$ is the plane of projec-

tion for the plan. The projections of M and $m$ into this horizontal plane are M′ and $m'$ respectively.

The bundle of rays in the object space is exactly similar to that in the image space, and a variety of methods may be used for reproducing the detail seen in the photograph to a desired scale. Thus the projected angle in the image space between the principal axis N′A and the ray N′M′ will be equal to the corresponding angle in the object space, i.e. $\angle m'N'a = \angle M'N'A$. This is equally true for all pairs of similar points.

The focal length ($N'A = f$) of the camera lens is known and the horizontal or $x$-ordinate (AM′) of the point can be measured from the vertical collimating line IJ. Thus the direction of the point in space from the camera station is known if the angle between the camera axis (i.e. the principal axis), and a base line of known length is measured.

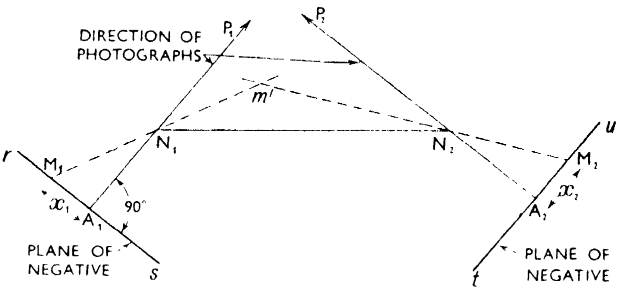



FIG. 24.

The principle is illustrated in Fig. 24. Photographs are taken from $N_1$ and $N_2$ at the ends of a measured base line, so that images of a point $m$ appear in the photographs at $M_1$ and $M_2$ respectively. The camera or photo-theodolite is set up so that the plane containing the perspective centre, plate centre, and horizontal collimating line are in a horizontal plane. The $x$-ordinates $x_1$, $x_2$ are measured from the photographs. $N_1N_2$ is plotted to scale and the known directions of $P_1$ and $P_2$ relative to the base line are set off. $P_1N_1$ is produced back to $A_1$ so that $A_1N_1 = f$, the focal length. The plan of the vertical plane containing the image of point $m$ is $rs$ perpendicular to $A_1N_1$. $M_1$ is plotted by measuring off $x_1$ from $A_1$ along $A_1r$. The plan direction of the point $m'$ is determined by producing $M_1N_1$. Similarly the line $M_2N_2$ is produced and the rays intersect at $m'$, which is the position of the point in plan to the scale of the base line. This is a usual method when plotting a survey from ground photographs taken by photo-theodolite. From Fig. 23, where the photo-plane and the object plane are each perpendicular to the

principal axis, it follows that the bearing of the point *m*, from the ground principal point *a*, measured with reference to a standard direction (say *ai*), is equal to that of the corresponding line AM in the image plane measured from AI. Similarly the direction *aq* from *ai* corresponds to that of AQ from AI. Hence $\angle qam = \angle QAM$, and similarly for any other pairs of corresponding points contained in parallel planes. This property is of considerable value when plotting from vertical air photographs because when the planes *bcde* and BCDE are horizontal instead of vertical, and $\angle QAM$ is measured on the photograph, this angle will be equal to $\angle qam$ as measured on the ground with a theodolite. If two such photographs are taken from opposite ends of a horizontal base line, and an image of a point appears in each, then the intersection of rays drawn from opposite ends of the "air-base," plotted to scale, fixes the point in plan in a manner similar to that given above. This is the basis of the "radial-line" method of plotting from air photographs.

In both cases it can be seen that the principle is analogous to that of simple plane-tabling, where rays are drawn on the table from two stations at opposite ends of a measured base. The employment of photographic surveying enables these rays to be drawn in the office from the photographs, instead of on the ground on to the plane-table.

## The Surveying Camera

The camera is the instrument by which the perspective picture is recorded, and if it is hoped to obtain precision in the preservation of the correct angular relationship of the "bundle" of rays in the object space when they are brought in to the image space and recorded on a photograph, the greatest care must be taken in design of apparatus and in selection of photographic materials.

The transference of the correct picture on to the photograph represents the joint efforts of the optical designer, instrument maker and photographic chemist. Obviously any increase in the scope of air survey must be dependent upon increasing precision of reproduction. Thus the surveying camera must be constructed and adjusted to the standard that would be expected in a ground-surveying instrument.[75] There are certain parts of the camera which contribute to the standard of accuracy of the result and these will all be considered briefly after the essential optical definitions have been mentioned.

### *Requirements of the Camera: Optical Definitions.*

Although roll-film is now generally used for air photography, it is convenient to retain the term "plate" in this connection.

The first essential is that the optical axis of the lens system should be perpendicular to the plane of the plate.

The *plate centre* is the centre of the plate as defined either by collimating hairs mutually perpendicular as in the usual diaphragm in a theodolite, or by collimating marks engraved on a glass pressure-plate, which keeps the film in the focal plane during exposure and transfers the marks to the negative.

The *plate perpendicular* is the perpendicular from the rear node to the plane of the plate. Its length is the *principal distance* and it meets the plate at the *principal point*. When the camera is accurately calibrated, the principal point and plate centre will become coincident: if not, either a calibration template is used to make the necessary allowance, or the camera must be adjusted.

The actual principal distance is increased somewhat by refraction of light rays in passing through the glass pressure-plate, the amount being $\frac{t}{\mu}(\mu - 1)$, where $t$ = thickness of plate and $\mu$ = coefficient of refraction of the material of the pressure-plate.

The plate perpendicular coincides with the optical or principal axis when the instrument is in adjustment.

*Lenses.*

Although, theoretically, a photograph represents the true perspective of the landscape projected into the image plane, there are many sources of error, optical and otherwise, which may adversely affect the accuracy and quality of the image. The lens, in particular, is liable to produce aberrations of several types. The lens fitted to a survey camera must preserve maximum accuracy over the whole area of the working field, and the problems of its design are very different from those arising in the design of lenses which are required to define an accurate line of sight in surveying telescopes.

The most important geometrical aberrations are:

(*a*) Spherical aberration, coma, and astigmatism, which cause the rays from a single object point to meet the image surface in areas of various sizes and characteristic shapes instead of in a point.

(*b*) Curvature, which causes the sharpest image to lie on a curved surface (such as a sphere) instead of on a plane, and

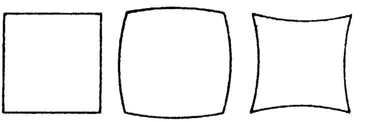
(*c*) Distortion of the image, giving a pin-cushion or barrel effect as seen in Fig. 25.

When a ray of white light enters a lens the rays of the different consti-

tuent colours are deviated to different extents just as with a prism, and a blurred image results. This is known as *Chromatic Aberration*; the effect can be reduced to a small amount by making the lens system of two or more kinds of glass.

The use of a suitable combination also reduces, or balances out, geometrical aberrations.

In 1840, Ross in England first produced a system of lenses which reduced chromatic aberration. This was followed by Dallmeyers's "Rapid Rectilinear" (1867) and Jena glass (1886) which reduced astigmatism. Since then improvements of design have reduced distortions and have increased illumination at the edges. A full bibliography of the development of photographic lens systems is given in the English translation of Gleichen's *Theory of Modern Optical Systems*, by Emsley and Swaine. [44]



CORRECT SHAPE BARREL EFFECT PIN-CUSHION EFFECT

DISTORTION OF LENSES

FIG. 25

Theoretically a perfect projection of only *one* object plane can be produced through a lens. The images of points in any other plane which are shown on the plate are actually small circles known as "circles of confusion." It is possible, however, to design lenses which have a suitable depth of focus, in which the images may be considered to be sharp, and, in air photography there is not much difficulty because the object plane is for practical purposes at an infinite distance.

Winchester and Wills [97] point out that a lens of narrow field is comparatively straightforward to design because only spherical and chromatic aberrations have to be allowed for, but when the field and aperture are increased a number of other sources of error become of importance, and the design becomes more complicated.

These writers also mention the difference between the high-class pictorial anastigmat lens which aims at a wide field with some sacrifice of image definition at the edges, and the survey lens where good and accurate definition over the whole field is of greater value than an ultra wide-angle which sacrifices geometrical accuracy.

During the War in the early days of air photography it was soon realized that a lens giving greater accuracy over a wider field would be desirable and a number of lenses were designed and improved.

As a result of this requirement the Ross Wide-angle Xpres lens,* having an aperture of $f/4$, was developed. The diagonal field is 70°, the

* There are suitable lenses produced by other manufacturers. The Ross series has been chosen to show the normal range of lens types obtainable.

focal length being either 5 inches or 7 inches for photographs 5 × 5 inches or 7 × 7 inches respectively. This lens has been used extensively by the Royal Air Force and commercial air photographers.

A diagram of this lens is shown in Fig. 26. It will be noticed that the system is very different from the simple convex lens. The lens system was designed for use with all colours, four different kinds of glass being used in its make-up. It has proved very satisfactory in use, and although infra-red photography employs a part of the spectrum beyond that for which it was designed, the lens has been found to be equally suitable for this. A report on a lens of this type made by the National Physical Laboratory and quoted in the report of the Air Survey Committee 1935, [6] shows a maximum distortion of image, due to the lens, of 0·03 mm. in the

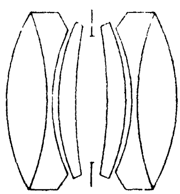



FIG. 26.

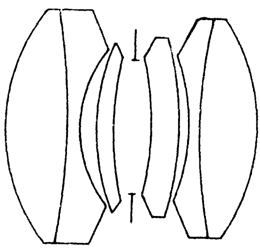



FIG. 27.

corner of a photograph where the angular field is 71·2°. At a field of 70° this distortion is of the order of zero for practical purposes.

The amount of light reaching the edge of the plate from an object on the earth's surface is much less than it would be if the image were at the centre of the plate. The increased effect of haze or atmospheric absorption is trivial in comparison with other factors. The chief of these are the geometrical factor, arising from the laws of light, which affect all lenses similarly; and the reduction in the effective aperture of the lens for rays of appreciable obliquity owing to the axial separation of the extreme component lenses. At an angle of 45° to the vertical the first factor reduces the intensity to one-quarter of that at the centre, and with most lenses the second factor reduces the illumination to zero for an angle in the neighbourhood of 30°. To increase the marginal illumination the diameters of the lenses more distant from the stop must be increased, and this may involve extensive re-designing of the lens to ensure the necessary correction of aberrations. This procedure has led to the development of the Ross Wide-angle Xpres E.M.I. (extra marginal illumination) lens to cover the

same angular field as the Wide-angle Xpres. The characteristic difference of this lens, which has proved very successful, from the other form can be seen from Fig. 27. Experiments are being made with a multi-lens camera fitted with lenses of this type.

The following is a typical lens report*:

## REPORT

### ON EXAMINATION OF A PHOTOGRAPHIC LENS BY MESSRS. ROSS LTD., FOR MESSRS. ROSS LTD., CLAPHAM COMMON, LONDON, S.W.4.

*Description.* Wide-angle Xpres $f/4, f = 7$ ins., No. 114223, Diameter 4·4 cm.

*Size of Plate.* 17·8 cm. × 17·8 cm. (7 ins. × 7 ins.).

*Number of Glass-air Surfaces.* Eight.

*Flare Spot.* None.

*Visible Defects—Striæ, Veins, etc.* None.

*Relative Centering of the Surfaces.* Satisfactory.

*Principal Focal Length.* 17·54 cm. (6·91 ins.). Distance of the focus from the point where the lens-axis cuts the back surface 15·04 cm. (5·92 ins.).

*Angular Field.* The diagonal of the plate is 25·14 cm. (9·90 ins.) requiring a field of semi-angle 35·6°.

*Definition* at the centre with the largest stop is very good. In wave-lengths of the mercury green radiation the maximum path difference is 1. The stop marked 4 gives satisfactory definition over a field of semi-angle 33°.

*Central Spherical Aberration.* The plate having been focussed at the centre of the field, with the largest stop, for an infinitely distant object, the movement necessary to bring it into focus when the aperture employed is limited to a zone of the lens, lies between +0·02 cm. and −0·07 cm.

*Coma.* The lens is well corrected.

*Achromatism.* The plate having been focussed at the centre of the field, for white light, the movement necessary to bring it into focus for blue light (approximate wave-length 0·450$\mu$) is 0 cm.; and for red light (approximate wave-length 0·665$\mu$) is −0·01 cm.

*Astigmatism and Curvature of the Field.* The subjoined table gives the distances from the focal plane of the positions of best focus ($D_1$, $D_2$) for radial and transverse lines, and of best general focus ($D_3$) respectively, at different angles of obliquity, measured from the lens axis. The focus is for an infinitely distant object, the stop marked 4 being employed.

* This report and that following were kindly supplied by Messrs. Ross. Ltd.

*Distortion.* The displacement—positive, if towards the centre of the plate, negative, if away from it—of a point in the image from the correct position it should occupy is given, for different angles of obliquity, in the table.

| *Angle Degrees.* | *Astigmatism.* | | *Curvature.* | *Distortion. mm.* |
|---|---|---|---|---|
| | $D_1$ *cm.* | $D_2$ *cm.* | $D_3$ *cm.* | |
| 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | —0·01 |
| 10 | +0·01 | +0·01 | +0·01 | —0·02 |
| 15 | +0·04 | +0·03 | +0·03 | —0·02 |
| 20 | +0·08 | +0·04 | +0·06 | —0·02 |
| 25 | +0·09 | +0·04 | +0·06 | 0 |
| 30 | +0·02 | 0 | +0·01 | 0 |
| 35 | —0·22 | —0·09 | —0·15 | 0 |
| 35·6 | —0·28 | —0·13 | —0·20 | —0·03 |

*Effective Aperture of Largest Stop.* 4·35 cm. *f*/number 4·0.

*Note.*—Distances measured from the focal plane towards the lens are reckoned positive and away from the lens negative.

*Date:* 27 *March* 1928. (*Signed*) J. E. PETAVEL.
*Reference* A: 222. *Director.*

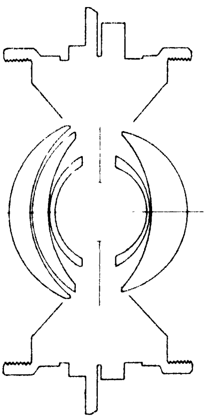
The focal length and characteristics of lenses used will naturally vary with the requirements, and the length of lens cone supplied with the camera depends upon the lens to be used.

A number of firms on the Continent, in the United States and elsewhere have produced lenses suitable for air survey photography.

Zeiss have produced the Topogon lens which has an aperture of *f*/6·3. It is claimed that although the angle of view of this lens is 95° (against 57° of earlier Zeiss lenses), it unites high-light transmission with practical freedom from distortion. The standard lens has a focal length of 10 cms. (4 ins.), and the photograph is 18 × 18 cms. The image is shown as a circle of 8¼ ins. diameter on the photograph, with the clock, bubble and counter in three of the corners.

Recently Ross have introduced a new ultra wide-angle lens with an

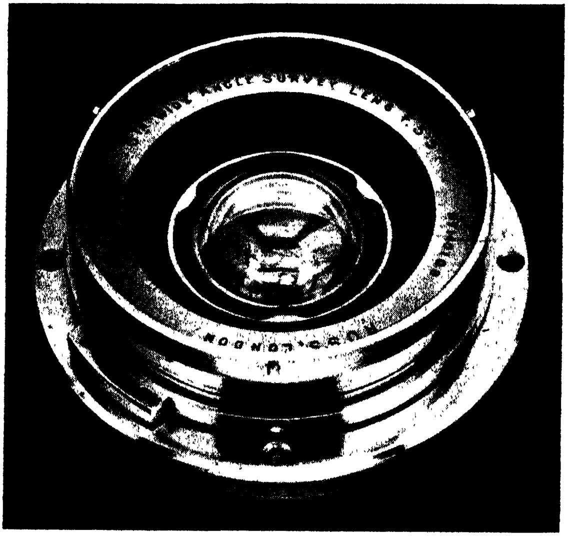
aperture of $f$/5·5 and covering approximately the same field as the Zeiss Topogon. This Ross lens which is illustrated diagrammatically in Fig. 28 is proving very successful. It has not yet been given a specific name and will, with the Topogon, be referred to in future as an ultra wide-angle lens. That illustrated has a focal length of 4½ inches. In Fig. 29 is shown a photograph of a 5-inch lens of this type. Note that the front surface is almost hemispherical. Photographs 5 inches square taken with a lens of this type having a focal length of 3½ inches, have been taken for an experimental plot for the Ordnance Survey. This lens was tested by the National Physical Laboratory, and was found to possess remarkable freedom from distortion; that at 40° from the principal axis (80° field) being 0·01 mm., and that at 45°, 0·06 mm.



[*Courtesy of Ross, Ltd*

FIG. 28—ROSS ULTRA WIDE-ANGLE SURVEY LENS.

The following is a report of a test carried out by the National Physical Laboratory on one of these lenses:



[*Courtesy of Ross, Ltd.*

FIG. 29—ROSS ULTRA WIDE-ANGLE SURVEY LENS.

## REPORT

### ON THE EXAMINATION OF A PHOTOGRAPHIC LENS FOR MESSRS. ROSS LTD., CLAPHAM COMMON, LONDON, S.W.4.

*Description.* Ross Wide-angle Survey Lens $f/5{\cdot}5$, $f = 4\frac{1}{2}$ ins., No. 141629, Diameter 4 cm.

*Principal Focal Length.* 11·61 cm. (4·57 ins.).* Distance of the focus from the point where the lens-axis cuts the back surface 8·89 cm. (3·50 ins.). When the glass plate (of thickness 3·77 mm.) is interposed between the lens and the focal plane, the latter distance becomes 9·02 cm. (3·55 ins.).

*Angular Field.* 95°.

*Filter.* A Kodak K2 filter was mounted in front of the lens.

*All the following measurements were made with the glass plate placed between the lens and the focal plane, the surface of the plate farther from the lens being in the focal plane.*

*Distortion.* The displacement—positive, if towards the centre of the plate, negative, if away from it—of a point in the image from the correct position it should occupy is given, for different angles of obliquity, in the following table.

| *Angle Degrees.* | *Distortion mm.* | *Resolving Power Lines† per mm.* | |
|---|---|---|---|
| | | *Horizontal Lines.* | *Vertical Lines.* |
| 0 | 0 | (250) | (250) |
| 5 | +0·005 | (250) | 220 |
| 10 | +0·005 | (250) | 133 |
| 15 | −0·005 | (250) | 100 |
| 20 | −0·02 | 249 | 68 |
| 25 | −0·035 | 222 | 54 |
| 30 | −0·05 | 200 | 55 |
| 35 | −0·06 | 173 | 54 |
| 40 | −0·055 | 138 | 53 |
| 45 | +0·035 | 104 | 36 |
| 47·5 | +0·09 | 91 | 21 |

* In obtaining a value for the focal length the distortion over the whole field has been taken into account and that value has been adopted which results in minimum distortion.

† By a "line" is to be understood a distinguishable image of one clear space and one opaque line.

*Resolving Power.* Reduced images of half-tone screens, with the clear spaces and opaque lines of equal width, were used to measure the resolving power. The limits of visual resolution in the focal plane through the position of best central definition are given, for different angles of obliquity, in the table. Where the values are less than 180 the observations were made with a microscope giving a linear magnification of about 30: for values greater than 180 a magnification of about 100 was used.

It was clear that finer screens could be resolved visually in the positions where the values in the table are enclosed in brackets.

*Date:* 23 *September* 1938.
*Reference:* A: 362.

W. L. BRAGG,
*Director*
T. SMITH,
*Superintendent Optics Department.*

It will be noticed that the distortion is greater than with the Wide-angle Xpres Lens. Hence the employment of a reseau becomes of importance, when plotting from photographs taken with this type of lens. In general for the larger scales, lenses of narrower field will be used, so that distortion due to the lens will be negligible.

These lenses have been introduced in competition with multi-lens cameras, which seek to widen the field normally obtained with a single lens. A number of multi-lens cameras have been tried whose prints, after rectification, produce an equivalent vertical. It has, however, been only since the improvement of illumination at the edges of photographs that real progress has been made with these cameras. The issue now is whether a single ultra wide-angle lens should be used, or a multi-lens camera. Both appear to have their uses and the surveying problems will be discussed later.

Irvine C. Gardner of the United States National Bureau of Standards sums up the present trend in lens requirements: [41]

"As methods of mapping become more precise, and particularly as the stereoscopic method of plotting contours becomes the more usual method of treating a series of photographs, the testing of the lenses becomes more important and greater precision is required. This tendency is made very evident by the character of the tests that are now being requested and by the increased interest in specifications for lenses and cameras."

Tests which are carried out by the Bureau of Standards are described in Research Paper RP984, *National Bureau of Standards Journal of Research,* April 1937.

Pendleton [75] mentions the difficulties which arise in calibrating a

survey camera when, as sometimes happens, the various elements of the lens are not perfectly centred by the manufacturers.

Accurate determination of focal length and inter-nodal distance is a laboratory measurement, as are the amounts of any errors due to distortion. Any such distortion is found to increase radially from the centre, the maximum amount due to a modern lens being of the order of 0·1 mm. at the edge of a photograph. Where great precision is required, it has been found that the errors are evenly distributed by making a slight adjustment to the focal length.

The results of lens tests by the United States National Bureau of Standards are given for each 5° of field from the centre up to 35°, for distortion, definition and resolving power.

*Shutters.*

The Air Survey Committee reported in 1923[5] that the focal plane shutter was in general use in this country. This type of shutter which resembles a roller-blind with a slit in it, is positioned just in front of the plate, the slit moving across the focal plane during exposure.

Owing to their simplicity and comparative reliability of operation, focal plane shutters have had a long period of use for air photography in this country. Other errors and distortions made the displacement of image due to relative movements of the aircraft and the traversing slit of minor importance at first, but reduction of these errors brought this problem to the fore.

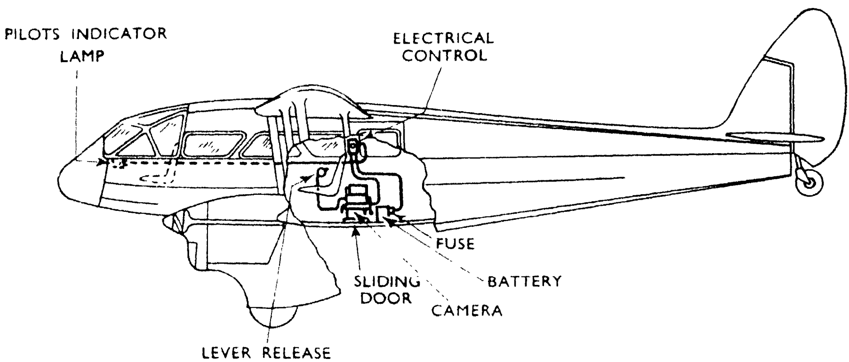
The time of exposure of a plate in this country is generally between 1/50 to 1/120 second although elsewhere shorter exposures are frequently used. In their second report in 1935, the Air Survey Committee[6] stated that the best practical results for this type of shutter were obtained with a slit one inch wide taking 1/20 second to traverse a five-inch plate for an exposure of 1/100 second at every point. The relative scale displacement of the images of two points at opposite edges of the photograph will be equal to the movement of the aircraft in 1/20 second. If the photographic scale is 1/5,000 and the speed of aircraft 125 miles per hour, the image distortion will be 0·021 inches, which is obviously not negligible.

At first, the rotating-disc shutter operating between the lens was also available, but was considered in this country, though accurate, to be unreliable and unnecessarily complicated, so that the focal-plane shutter was preferred by the Royal Air Force.

On the Continent and in the United States, however, where air photography for mapping was employed extensively for some time before it was considered seriously in this country, the rotating-disc shutter has been brought to a high state of development. The quadruple blade shutter

fitted with the Zeiss Automatic Film Aircraft Camera has an efficiency of 87 per cent, i.e., 87 per cent of light value is registered on the film. A "multiple-leaf" shutter is fitted by Zeiss between the two halves of the lens. The total field is exposed simultaneously and it is claimed that no distortion arises.

For a number of years the Royal Aircraft Establishment, in collaboration with the Williamson Manufacturing Company, has carried out research work with the Stringer Louvre Shutter which is considered to offer a better solution than the others. This shutter consists of a number of thin strips of metal between lens and focal plane, which can be folded up or down, like a "Venetian" blind. All the strips operate simultaneously,




[*Courtesy of Williamson Manufacturing Co., Ltd.*

FIG. 30—DIAGRAM SHOWING INSTALLATION OF WILLIAMSON "EAGLE" AIRCRAFT CAMERA FOR AIR SURVEY WORK IN DE HAVILLAND RAPIDE AIRCRAFT.

so that there is no distortion of image, and it is claimed that the loss of light efficiency is no more than twenty per cent. While this is greater than would be the case with a rotating disc, it is considered that by having the shutter clear of the lens system, the great advantage of freedom of choice of lens is achieved. The louvred shutter is now fitted to the "Eagle Aircraft" cameras made by the Williamson Company.

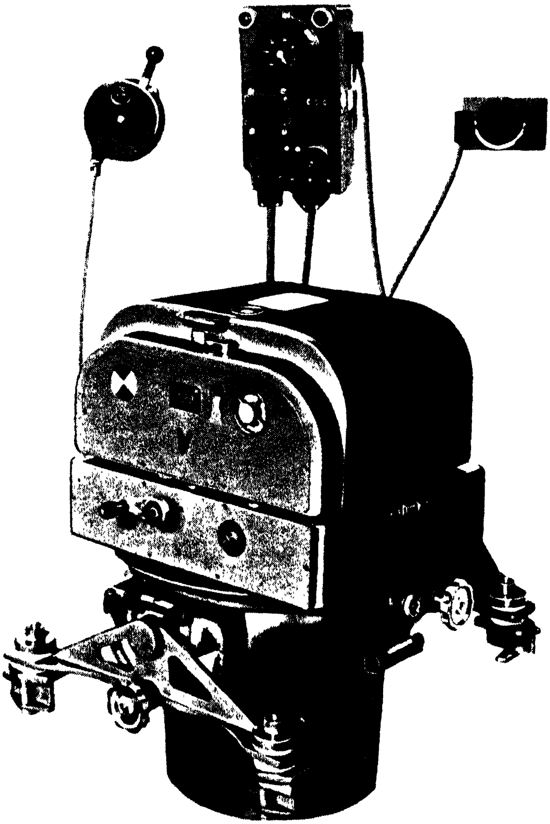
The rotating-disc shutter, even with recent improvements, has not found as much favour in this country as the louvred one. Recently, however, with the introduction of the ultra wide-angle lens, the latter type has proved unsatisfactory since the width of the slats tends to affect photographic accuracy at the edges where the rays are oblique. As a result cameras fitted with ultra wide-angle lenses have a between-the-lens rotating disc or leaf shutter.

*Cameras.*

Cameras used for some of the earlier oblique photography were designed

to be held in the hand, but now the camera is almost always attached to the aircraft (which acts as the platform) and is mounted so that it points through the floor of the aircraft, being supported on gimbals to allow of adjustment for tilt and drift. A diagram of an "Eagle" camera installation on a vibration-free support in a de Havilland Rapide Aircraft is shown in Fig. 30.

The "Eagle" cameras made by the Williamson Manufacturing Com-



[*Courtesy of Williamson Manufacturing Co., Ltd.*

FIG. 31—WILLIAMSON "EAGLE" IV AIRCRAFT CAMERA IN SUSPENSION MOUNTING WITH ALTERNATIVE METHODS OF CONTROL.

Total Weight of Outfit as shown above, without Lens: 46 lb. (20·8 kilos). Overall Dimensions of Camera: $12\frac{3}{4} \times 11\frac{1}{2} \times 19$ in. ($32 \times 29 \times 48$ cm.).

pany are generally used in Britain. Those supplied to the Royal Air Force for survey purposes are often fitted with a Ross lens of 7-inch focal length, the photographs taken being 7 × 7 inches. For other survey purposes the usual size of photograph is 226 × 165 mm. (9 × 6½ inches), the lens range being as stated above. An illustration of this type of camera is given in Fig. 31.

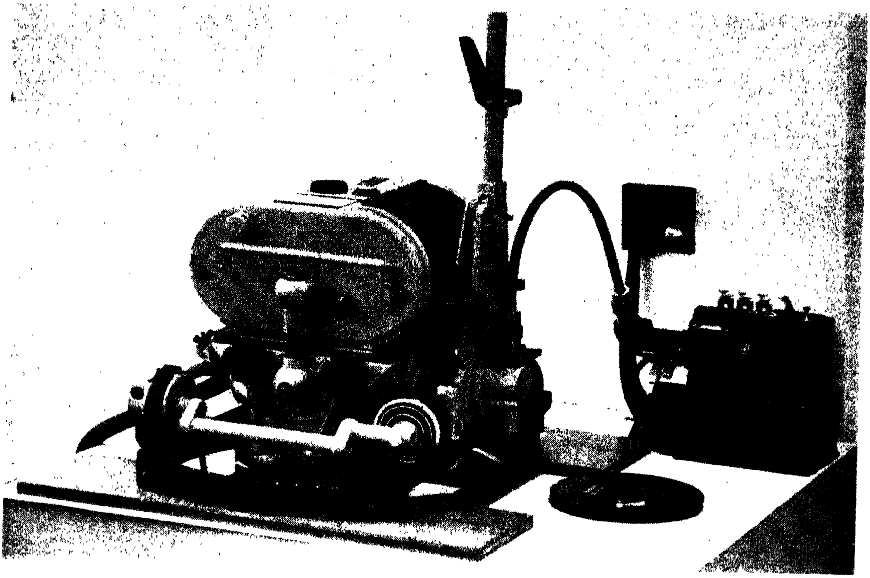
On account of the inconvenience of loading plates, film is used in rolls sufficient for two hundred exposures, it being possible to change a magazine in three seconds. The length of exposure can be set according to weather conditions, and the interval between exposures adjusted to allow for aircraft speed and height. The film is moved round and the shutter operated by means of an electrical drive, and a glass pressure-plate ensures that the film is in the correct position when the exposure is made. Collimating marks engraved on the plate appear in the photograph. A warning lamp fitted in the pilot's cockpit gives him five seconds warning before an exposure is made, so that if manual control is used he may steady up the aircraft. A handle and a windmill geared to the mechanism and operated by the slip-stream are provided as alternative methods of drive if the electrical gear breaks down during a flight.

The Zeiss Automatic Surveying Camera is illustrated in Fig. 32. It is fitted with a Zeiss Orthometar lens with a focal length of 21 cm. and an aperture of *f*/4·5. The size of photograph is 18 × 18 cm., and the film spool which allows of 300 exposures is driven electrically with alternative methods as above. In order to avoid the small distortions which arise due to the refraction through a pressure plate on to the film, in this case the film is pressed against a plate at the back and held in position by means of compressed air. Exposures may be 1/75, 1/120 or 1/150 second, and the time interval between exposures may also be regulated.

Another Zeiss camera is the RMK.P.10 which is designed for use with the Zeiss Topogon wide-angle lens. It is illustrated in Fig. 33. Although the picture size is still 18 × 18 cm., the focal length is reduced to 10 cm. (about 4 inches). This camera is fitted with a seat and drive-connection so that a small camera may be attached for recording the horizon simultaneously with the vertical exposure. This enables the amount of tilt to be computed. Other cameras are constructed to take Topogon lenses of larger focal length, which enables picture size to be increased. Details of operation are very similar to those for the previous camera.

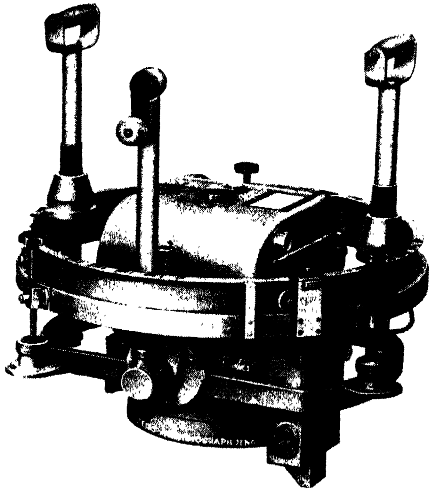
The air-camera made by the Swiss firm of Wild, is somewhat similar, but plates are used instead of roll film.

A number of multi-lens cameras have been designed, such as the



[Courtesy of Carl Zeiss (London), Ltd.

FIG. 32—ZEISS AUTOMATIC SURVEYING CAMERA RMK P21.



[Courtesy of Carl Zeiss (London), Ltd.

FIG. 33—ZEISS WIDE-ANGLE AIR SURVEY CAMERA RMK P10.

Barr and Stroud Seven-lens Camera, and these will be discussed later on, although it may be mentioned that the general purpose has been to cover a wider field than with a single lens.

*Collimating Marks.*

The method of ensuring that collimating marks appear on the photograph must be considered in relation to possible loss of accuracy by its use.

In British practice it is usual to have an optically flat glass pressure-plate which presses against the film at the moment of exposure, and which transfers to the photograph the collimating lines which are etched on it. It is thus possible to fix the optical centre of the photograph. Recently glass pressure-plates have been engraved with a one or two centimetre reseau, i.e. squares of one or two centimetre side with lines 0·001 inch thick, so that distortions over the final photograph can be determined.

Even though the pressure pad is optically flat, there will still be distortion due to refraction of rays in passing through it in the image space before reaching the sensitive surface. The effect of refraction is a general reduction of scale, and Hotine [55] states that this corresponds approximately to an apparent variation in principal distance, which is increased by an amount depending upon the refractive index and thickness of the plate, the remaining uncompensated error being very small (p. 95). He remarks "In precise processes, this can be corrected or it can be cut out by re-projecting the photograph through a similar sheet of glass contained in a similar perspective; in other cases it can be neglected." Pendleton, [75] writing more recently, does not agree that the effect is negligible and says that wherever a pressure-plate is used, either a special lens must be designed to eliminate the resulting distortions or the photograph must be projected through a compensating device in printing. He concludes: "The best practice will avoid the use of such plates in cameras to be used for stereoscopic mapping."

It is common practice outside Britain to use compressed air or suction to ensure the image being formed in the focal plane, the register pad being in this case behind the film. The collimating marks are now impressed onto the photographs by an optical or mechanical process, e.g., in one case of an optical projection, minute dots are projected into the focal plane, and the collimating lines drawn by joining up these points: in another shadows are projected. These collimating lines usually intersect at 90°, a tolerance of $\pm 30$ seconds being suggested. [75]

*Accessories.*

*Camera Aiming Sight.* In vertical photography, careful sighting and

setting of the camera is essential if all the area is to be covered with the correct lateral overlaps. Hence a special sight is fitted, such as the Aldis Camera Aiming Sight. This consists of a vertical sighting line with a graticule capable of being rotated to allow for the angle of drift of the aircraft, which can be measured and used to adjust the camera orientation. In order that the line of sight can be checked both fore and aft, a movable prism is also fitted. In addition, the graticule shows the area covered by a single exposure, the centre point of the sight and two reference points, to aid longitudinal setting for correct overlap since the ground speed is affected by wind velocity. Improved accuracy of sighting when an automatic pilot is fitted is obtained by a sight which is attached to the camera.

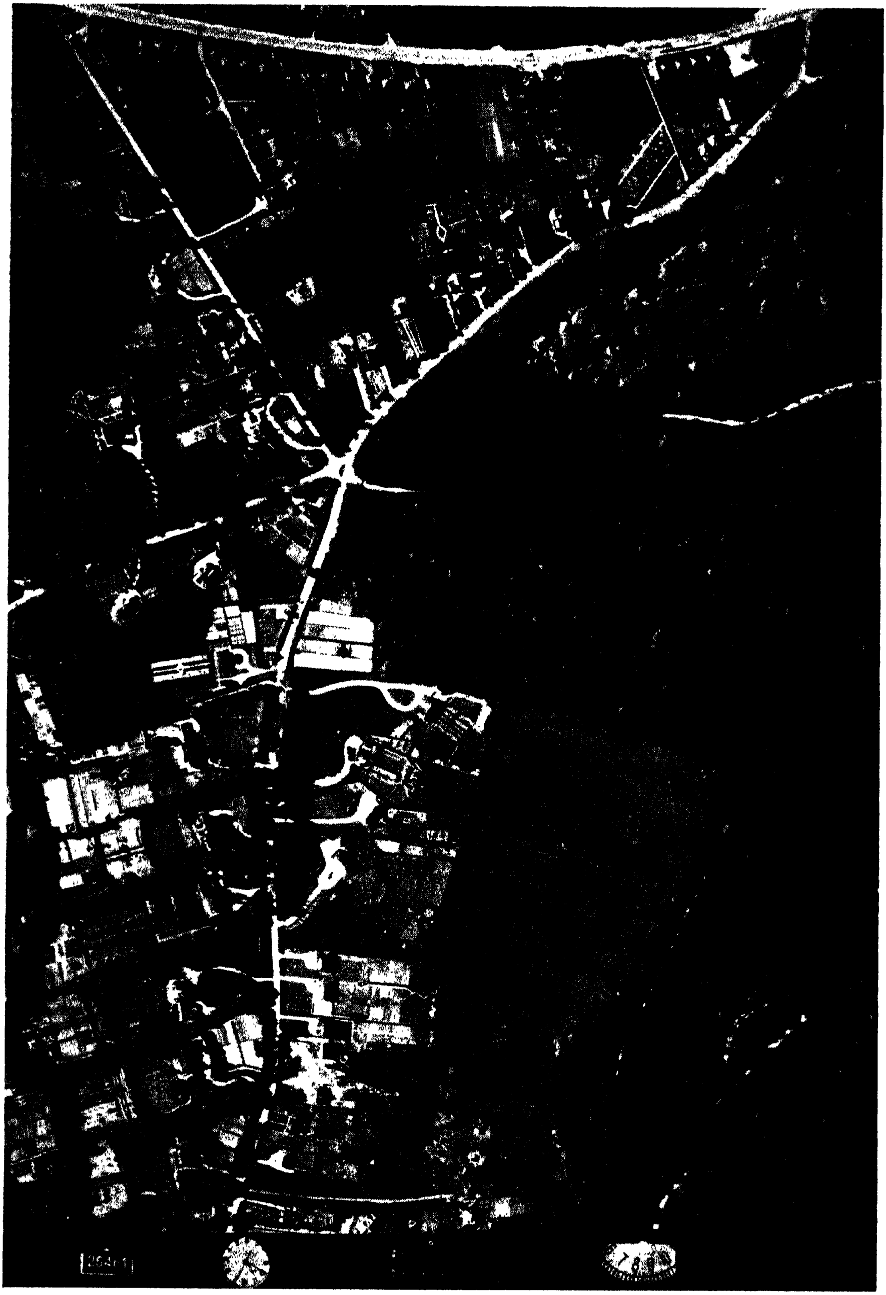
*Instrument Box.* Most air survey cameras at the present time are fitted with recording instruments which are photographed at the same time as the area covered, and the readings appear on the edge of the photograph (see Fig. 34). These instruments consist of an altimeter, clock, counter, and spirit level. The altimeter is too small to be of much value and much research work is being done to improve its accuracy. If a larger instrument or a statoscope is fitted, this must be photographed separately from, but simultaneously with, the ordinary exposure. The spirit-level, useful for calibration purposes gives little indication of the horizontal in the air because it is affected by random accelerations of the aircraft. The clock, fitted with a second-hand, acts as an auxiliary to the exposure counter.

*Filters.* Lenses which allow all the light to reach the plate produce photographs which are lacking in contrast. For good results it is necessary not only to have plates colour sensitized (usually panchromatic), but also a filter of suitable colour between the lens and plate. Moreover, since there are few days when air survey photography is possible and often then there is haze (which may be either water-mist or dust haze), the filter selected must help to reduce this effect.

Wills [96] points out that filters which render all the colours according to the actual amount of light which the object reflects to the plate must also be avoided because, owing to the similar brightness of many objects in summer, they would all appear dark in the photograph and the result would be "flat."

The filters most commonly used are yellow or orange which will absorb blue light, penetrate haze and show up the maximum of detail and contrast. Williamsons recommend gelatine filters which can be fitted between the lens combinations; while Zeiss supply filters of glass which is coloured throughout and which is guaranteed parallel faced to within ten seconds of arc.

Pendleton [75] stresses the necessity for optical flat glass in filters to



[*Courtesy of Aerofilms, Ltd.*

FIG. 34—VERTICAL AIR PHOTOGRAPH.
Approximate scale of reproduction, 1:6,200.

eliminate refraction and mentions that only with such glass can a filter be changed without disturbing the calibration of the camera. It is desirable that surveying cameras should be calibrated with the filters in position.

*Calibration of Surveying Cameras.*

The essential requirements of a surveying camera are as follows: (i) The plane of the plate at exposure should be perpendicular to the principal or optical axis of the lens. (ii) The principal point, at which the principal axis meets the plane of the photograph must be accurately known. (iii) The principal distance must be accurately known. (iv) The collimating marks should intersect near the plate centre at the principal point and be mutually perpendicular.

The effective principal distance must be accurately determined when the lens is fitted in the camera and allowances made for distortions such as those due to the pressure plate and the shutter. Care must also be taken to allow for the shrinkage of the film base and printing paper.

It follows that a surveying camera should be provided with certain adjustments. The lens must be in a mounting which will allow not only the angular relationship to the photographic plane to be established, but also a small range of adjustment in principal distance so that this may agree with the focal length. Also camera design should be such that the effect of very low temperatures at great altitudes and corresponding shortening of principal distance by contraction, can be controlled by regulating the temperature of the lens cone. In addition the possibility of humidity control for reduction of film shrinkage should be considered. [75]

In some existing cameras, if it is found that the optical axis of the lens does not coincide with the intersection of the collimating marks, a correcting mark is engraved on the pressure-glass, where the lens is not adjustable. This is obviously undesirable in an instrument of precision.

Pendleton claims that bench tests in the United States will enable the principal point to be located to 0·025 mm. He remarks that laboratory methods of calibration will probably be more accurate than field methods for modern precision cameras.

In the United States, these tests and calibrations are undertaken by the National Bureau of Standards, which has laid down standard charges for lens tests. In this country similar work is undertaken by the National Physical Laboratory.

It is not intended here to go into exact details of the calibration of such cameras, which becomes more exact and specialized as the scope of air survey photography is increased. The camera will normally be calibrated and limitations and errors of component parts determined by the

National Physical Laboratory or similar institution. It is the responsibility of those specifying the requirements of the photographs to bear in mind the camera particulars.

In a large survey organization, camera adjustments will probably have to be made, and for further details the reader is referred to the Second Report of the Air Survey Committee; [6] to Hotine's paper "Calibration of Surveying Cameras"; [57] to his book *Surveying from Air Photographs*; [55] to Salt's *A Simple Method of Surveying from Air Photographs*; [86] to Von Gruber's book *Photogrammetry*; [46] and to Research Publication RP 984, *United States National Bureau of Standards Journal of Research.*

Also details of specifications for precision-mapping cameras are given by Gardner in an article in the Report of the American Society of Photogrammetry [1] for the International Conference at Rome 1938.

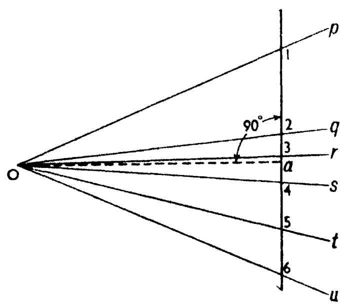



FIG. 35.

In order, however, that the significance of such tests may be understood the following will show the basic principles, but it must be realized that great precision in measurement and computation instead of drawing is called for.

The camera is mounted in a holder so that the principal axis and one collimating line can be adjusted into the horizontal. Six points are chosen so that they will all appear approximately along the collimating line and distributed across a photograph with three points near the centre, one near each edge and the sixth in some intermediate point. Angles are read between these points with a theodolite and a photograph is taken. The measured angles to the points such as *p*, *q*, *r*, *s*, *t*, *u* are plotted (Fig. 35), and, since the corresponding angles between the images of these points subtended at the perspective centre and those on the ground from this point are equal, the camera adjustment may be checked. The distance should be measured along the collimating line on the photograph from the marked-principal point to the projections of *p*, *q*, *r*, *s*, *t* and *u* on to the collimating line. These distances are then plotted on a piece of tracing paper. For precise determinations some form of co-ordinate measuring machine would be employed. If the tracing paper is now fitted over the drawn rays so that each image position lies on its own ray, then the angular condition is satisfied. The plate perpendicular drawn from O should pass through the plate centre and O*a* is the effective focal length of the lens.

If it be assumed that the collimating marks have been accurately engraved, then the camera is rotated through 90° so that the other collimating line is horizontal, and another photograph taken. From a consideration of these photographs, either graphically or by calculation, the coincidence of plate and principal centres are examined and the principal distance determined. This latter may be compared with the stated value of the focal length to check the equality of the principal distance and the focal length.

To check the inclination of the plate to the principal line, a test such as that above is made for about four high and four low points in a single photograph. The lengths found corresponding to $Oa$ should all be the same and be equal to the principal distance. If that at the top and that at the bottom differ slightly then it is clearly indicated that the plate is tilted out of the focal plane.

If when measuring horizontal ordinates to the various points, the position of the collimating line is also measured and recorded on the tracing paper, then the plate perpendicular $Oa$ should pass through this point. Any discrepancy will generally indicate an error in lens setting, whereby the directions of optical axis and principal line do not agree. From the number of tests suggested above, the exact position of the principal point may be found in relation to the point of intersection of the collimating lines.

A direct method of fixing the principal point of a camera is by means of the "Auto-collimator." An optically flat mirror is levelled (or trough of mercury used), and the camera set up over it with its axis vertical. A well-defined object is chosen on the photographic plane, and by illumination, the image is reflected from the mirror back to the focal plane again.* If the point is adjusted until its image is superimposed on itself, then this must be the principal point. Hotine [55] in mentioning this method points out that errors in centring of the lens will introduce errors, which may be appreciable. He considers that the method is of assistance for determining the position to engrave collimating marks. The Auto-collimator itself, has a sighting telescope and is a refined form of the simple method, thus making the method more suitable for general calibration.

The success of air survey depends upon great accuracy in plotting and measuring from the photographs, and in eliminating as far as possible the many possible distortions and inaccuracies. It is therefore important that the calibration of air cameras should be undertaken with the utmost precision in order to avoid errors in the plot arising from these causes.

* By the laws of optics, the reflected ray is at the same angle to the normal to the mirror as the incident ray.

## Photographic Materials

Although progressive improvements in the accuracy of projection of photographic images through lens systems had been made over a period of years in connection with ground photogrammetry, much less had been done until recently to increase the accuracy of the final record after the photograph had been taken.

It is necessary that the rendering of detail in the photograph should be as fine as possible on photographs from which measurements must be made. The sensitive emulsion employed must be handled with care during the developing and fixing stage, and the result should be such that the image may be enlarged a number of times without showing the grain.

Progress has been such during the last few years that, provided the photographic processes are carried out under standard conditions conducive to minimum distortion, it is safe to say that inaccuracies of actual measurement on the photographs may be greater than photographic errors due to the distortion of emulsions or their bases. As a result the scope of air survey is widened in the larger scales.

Some consideration may be given to the standard to be expected in the comparison of the final photographic print and the image which was projected through the lens on to the plate.

### *Distortion of Film.*

For many years glass plates were in use for all photographic work which aimed at accuracy, and in many cases this custom was continued for air survey, although the plate has grave disadvantages for this purpose. It is not economical to take air photographs in small numbers, so that when plates are used, there must be provided, either arrangements for storage and automatic changing, or for changing plates in the air.

Although the tendency of the base of roll films to distort was realized, it became the general practice in this country to use the roll film before it was adopted to any extent elsewhere. This was largely on the score of convenience, and again it should be noted that the early developments here were for military surveys for which convenience and ease of operation is sometimes more important than extreme accuracy.

The Report of the Air Survey Committee of 1935 [6] records the progress during the last ten years:

"In 1928 it was stated by a technical expert that there was not the slightest chance of obtaining a flexible film base that would not shrink during development to an extent far too serious for accurate air survey photography. Since that date, however, manufacturers have made efforts

to meet the requirements of the Committee and considerable progress has been made in reducing the shrinkage of the base."

This report gives a very full statement of the improvements in photographic accuracy between 1923 (the date of the first report) and 1935 and some of the instances given below are taken from it.

Experiments were made at the Admiralty Research Laboratory in 1929–30 on standard Royal Air Force film, by making prints on the film of an accurate graticule of lines and checking its dimensions after developing, fixing and drying under the standard conditions. As a result it was concluded that the shrinkage was uniform in particular cases, the distortion being between 1 in 600 and 1 in 1,000, i.e. 0·1 per cent. This represented a movement of from 0·06 to 0·17 mm. over the 10 cm. test length, the average being 0·12 mm. A specially tested film showed a movement of only about one-sixth of this amount.

The firms of Ilford in this country, Kodak in the United States, and Zeiss-Agfa in Germany are all working to produce improved materials. Imperial Chemical Industries have been studying the properties of a new base which will reduce the movement to an extremely small amount.

Safety or non-inflammable film bases were found to distort appreciably and until comparatively recently the nitrate film was used for air survey. This suffers from the disadvantage that it is extremely inflammable. Tests* in the United States by the National Bureau of Standards have shown that the modern safety film produces an image almost as accurate as the nitrate film. It was found that ten samples of film (five of each kind) gave a shrinkage of from 0·01 to 0·05 per cent, with a differential shrinkage of 0·00 to 0·03 per cent under conditions of controlled temperature and humidity, at the optimum conditions of 72° F. and 65 per cent relative humidity. If the relative humidity varied by 15 per cent, then the shrinkage varied between 0·08 and 0·12 per cent.

*Distortion of Paper.*

In many cases, measurements in air survey are made upon the prints, so that one is concerned with the distortion of the paper during processing after printing. A silver bromide emulsion is used almost exclusively for photographs which are enlarged or rectified. Ordinary paper is not waterproof, and the shrinkage lies between 0·2 and 0·8 per cent while differential shrinkage is up to 0·36 per cent. [1]† This is a distortion of

* Davis, R.; Stovall, E. J.; Pope, C.I.: *Dimensional Changes in Aerial Photographic Papers and Films*, results of tests by the United States Bureau of Standards, 1937, quoted in report of American Society of Photogrammetry to International Conference, Rome, October 1938. [1]

† ibid.

considerable magnitude, and it has also been found that humidity distortions are rather greater than for films, and vary in different directions according to the manufacturing process.

The British firm of Ilford produced a "waterproof" paper in which the shrinkage was reduced to about 0·1 per cent, [6] and since then this type of paper has been improved and recent tests in the United States [1]* give a shrinkage of 0·01 to 0·06 per cent, with differential shrinkage 0·00 to 0·05 per cent. Variations due to humidity changes of 15 per cent may result in distortions between 0·04 and 0·19 per cent according to the direction of grain of the paper, and it is recommended that manufacturers should indicate the machine directions of printing paper so that allowances can be made.

Recently the firm of Gevaert have produced a cellulose-sprayed paper, the spraying being done before putting on the sensitive emulsion. Thus the paper is truly waterproof and does not become soaked with water during processing.

In the United States, the Positype Corporation has introduced "Air Map Special Paper" on similar lines at the request of the United States Coast and Geodetic Survey. This paper satisfies the specification of the American Society of Photogrammetry.

In some cases special materials have been used for the photographs, such as celluloid and metallic foil "sandwiched" between sheets of paper. The latter type has been tried by the Ordnance Survey. In Germany "Correctostat" aluminium laminated paper has been found to give a regular surface shrinkage of about 0·05 per cent and a uniform differential shrinkage of about 0·015 per cent. [1] There are obvious advantages in a uniform distortion in all directions.

The distortion may be checked by the use of a reseau such as that now used by the Ordnance Survey.

*Photographic Emulsions.*

The most usual type of emulsion is the panchromatic which, in conjunction with yellow or orange filters, produces a photograph which has good contrast and reliable tone differentiation. Recently emulsions have been produced which are much more sensitive to red, are faster and allow use of a wider range of filters in order to differentiate the detail more surely.

Infra-red emulsions have also shown results in giving a clear view in

* Davis, R.; Stovall, E. J.; Pope, C. I.: *Dimensional Changes in Aerial Photographic Papers and Films*, results of tests by the United States Bureau of Standards, 1937, quoted in report of American Society of Photogrammetry to International Conference, Rome, October, 1938. [1]

cases where the haze was impenetrable to the human eye. They are slower than ordinary emulsions and although speed has increased, at best they are about four times slower than standard material. Infra-red film cannot be kept in good condition for any length of time and infra-red light cannot penetrate cloud or rain. It is not anticipated that its use in vertical photography will be extensive except possibly for detecting camouflage.

The desirability for humidity and temperature control has been shown by the distortion values given above.

In developing, special precautions must be taken in order to obtain the greatest clarity of detail. Restrainers are used in order that the darker parts of the detail can be controlled. The printing paper is usually bromide. Although greater clarity of detail is obtainable on "glossy" prints, "matt" or "semi-matt" surfaces were used until recently, because of the difficulty of drawing on glossy paper. A new type of glossy paper upon which pencil lines may be drawn is now being used.

### Some Imperfections of Air Survey

While reserving for a later chapter discussion of the methods employed to overcome difficulties arising from the shortcomings of air survey, it is desirable to enumerate the more important of them here so that the flying problem can be considered.

A photograph of level ground is a true perspective picture if all distortions due to the lens and photographic materials are neglected, and, in the case of a truly vertical photograph taken from a known height, a true-to-scale pictorial plan would be obtained.

Unfortunately variations of ground level, uncertainty of scale due to variation of height of aircraft, difficulty of measuring this height, and random tilts of the aircraft make it impossible ever to obtain a photograph which is a true and undistorted perspective.

*Distortion due to Height of Object.*

In Fig. 36 *xy* is supposed to be a level stretch of ground, which is photographed vertically, the true plan being represented by XY in the negative. A vertical chimney *ab* has its base *b* in this level plane, the image of the top and base appearing at A and B respectively in the photograph. The rays in each case are assumed to pass straight through the inner node which is taken as the perspective centre, O. Distance AB in the photograph is the distortion out~~wards of the im~~age of the top of the chimney due to its height above the datum plane, and A is therefore coincident with the image of a point *b*′ on the ground.

Similarly it may be seen that the image of a point $c$ below the level plane will be displaced *inwards*.

If $f$ is the focal length of the lens and $H$ is the flying height above the datum, it is clear by proportion that the scale of the image of the level plane xy is $\frac{f}{H}$. This, however, is true *only for the datum*, since the scale at the height of the top of the chimney of height $h$ must be $\frac{f}{(H - h)}$. It follows that the scale of the photograph varies from point to point according to the height of the point above the datum, so that each contour will be at its own particular scale. For this reason mosaics can only be a compromise as regards scale.

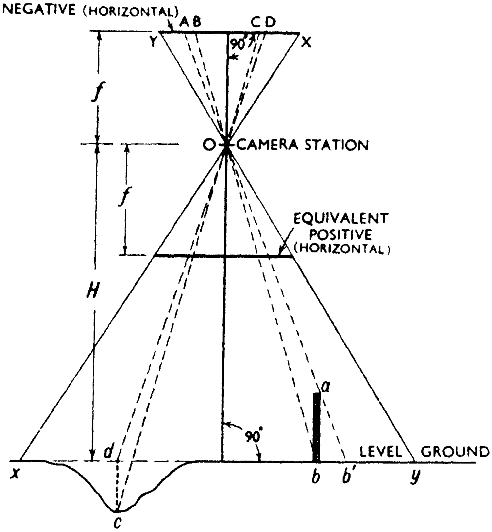



FIG. 36.

In the case of oblique photographs, this height distortion will be even greater in the background.

*Unknown Flying Height.*

In ground survey it is usual to know, or to be able to determine with precision, the position of the observing station. In air photography however, this is not so. Owing to variation of air conditions, flying height may vary appreciably between exposures even when an automatic pilot is working.

The altimeter fitted in the instrument panel of most air survey cameras is a form of aneroid barometer which is calibrated to give heights. It has to be kept small to fit into the instrument panel and is usually capable of giving absolute heights to within about two hundred feet.

The aneroid barometer is affected by variations of atmospheric temperature and pressure, apart from the temperature and pressure gradient with altitude, and the height value recorded may be quite erroneous if these factors are not sufficiently considered. Until recently knowledge of the corrections required was scanty, but Aneroid Tables were published by the War Office in 1936 [93] giving the requirements of a standard aneroid,

and the corrections to be made to the readings. It is now possible by taking temperature readings and by comparison with an aneroid at a ground station to determine absolute heights much closer than before. An altimeter constructed on the principles laid down and of adequate size will give, after correction, a value for absolute height much more accurately than before. Such an instrument will usually be too large for the instrument panel, and arrangements will have to be made to synchronize photography of the altimeter and the ground.

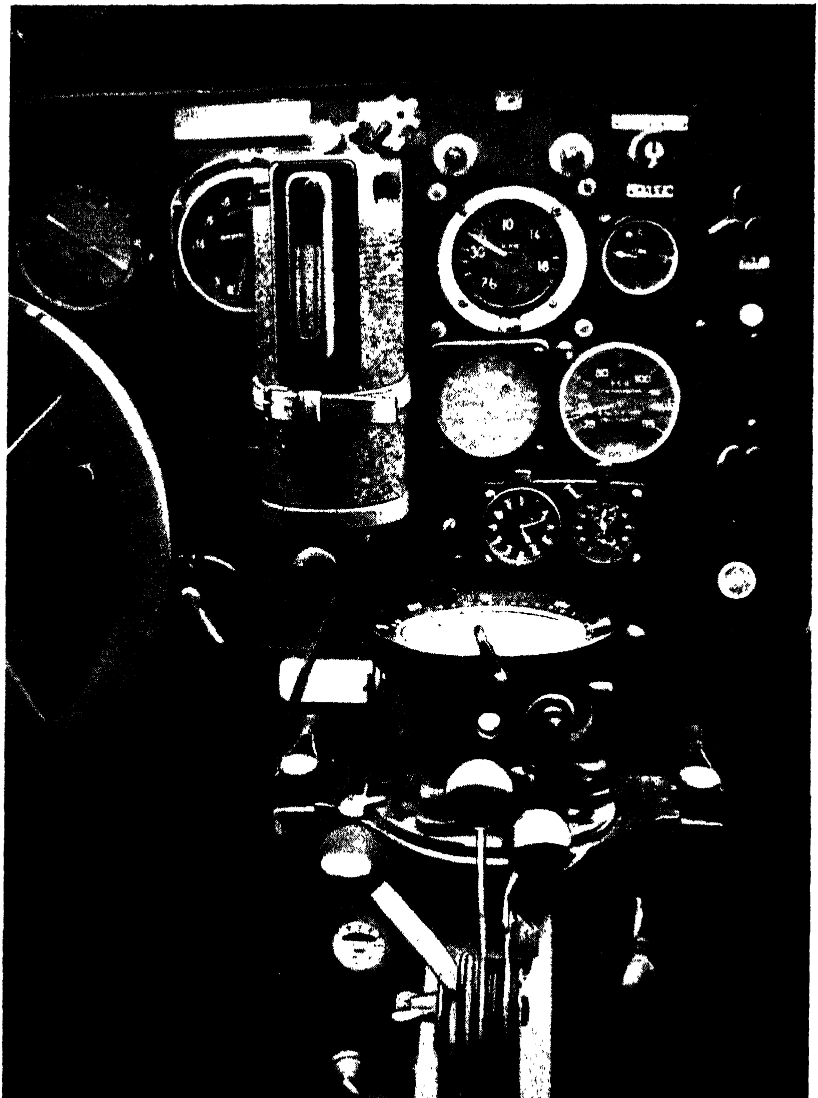
Usually, however, it is preferred to use in addition to an altimeter, a statoscope or differential aneroid, whereby variations of height between exposures can be much more accurately determined. Air survey firms have commonly used a type of statoscope which records height variations of a few feet from a starting height. The reading of the statoscope is synchronized with the photography. Various corrections have to be made [55, 93] and the resulting height should not differ by more than twenty feet or so from the real value. Even though the absolute height is not known with precision, known height variations from photograph to photograph enable scale variations to be determined, by direct measurement on the photograph.

Although in Britain an all-metal statoscope has been commonly used, during the last few years improved types of statoscope have been designed on the Continent which depend essentially upon the readings of a liquid manometer. The liquid commonly used is amyl alcohol, which will not freeze at high altitudes, and which is lighter than water so that small differences of pressure are recorded by an appreciable difference in the height of the two columns of liquid.

The statoscope of the French Aera Company will, it is claimed, record variations within $\pm 0{\cdot}5$ metres. The Väisälä statoscope gives a differential reading per 1 metre difference of altitude, which varies from 1·6 mm. at 5,000 feet altitude to 0·91 mm. at 16,000 feet altitude. [1] The statoscope itself consists of a glass tube, in communication with the open air by means of a capillary U-tube containing a small quantity of amyl alcohol and any variation in atmospheric pressure is recorded by displacement of the alcohol. In order to provide a reference reading, the bulb is placed in a vacuum flask containing an ice and water mixture. A three-way tap is provided so that the starting reading may be set and the liquid kept in the tube during ascent and descent.

Examples of statoscope equipment available are those of Zeiss and Wild. That supplied by Wild will be briefly described.

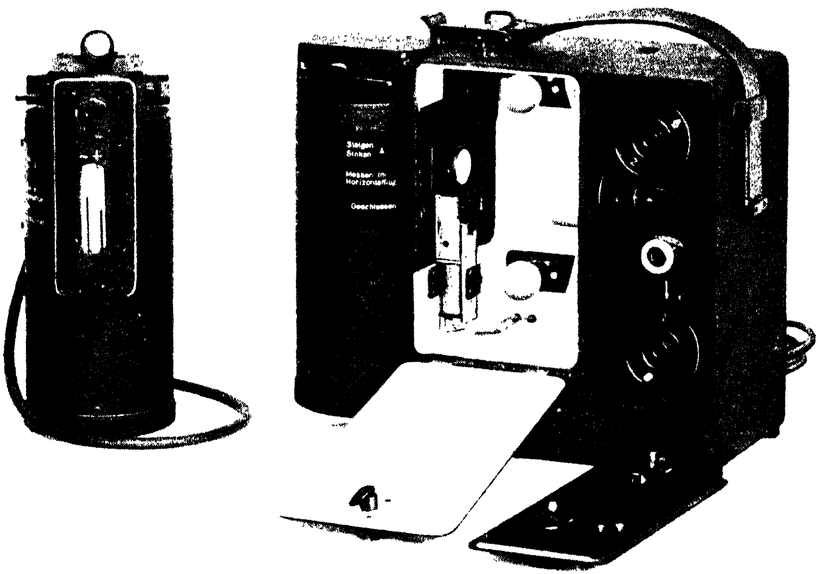
The Wild equipment consists of two statoscopes and a recording camera. One statoscope is fitted in the pilot's cockpit (Fig. 37) and



[*Courtesy of Wild Surveying Instrument Co., Ltd., Heerbrugg, Switzerland.*

FIG. 37—STATOSCOPE EQUIPMENT IN COCKPIT OF AIRCRAFT.

assists him in maintaining the required altitude, which it is said can be done with a variation of ±5 metres under good conditions. The second statoscope is photographed by a recording camera (Fig. 38) connected with the survey camera, so that its reading is photographed at the instant of the exposure of the former, on to a ciné-film, illumination being provided by the lighting of an electric lamp synchronized with the shutter of the survey camera. A clock and counter are also photographed, and, after one exposure, the film is automatically moved round into position for the next exposure.



[*Courtesy of Wild Surveying Instrument Co., Ltd., Heerbrugg, Switzerland.*

FIG. 38—WILD RECORDING STATOSCOPE EQUIPMENT.

From the differential reading of the statoscope, the variation of altitude from the starting altitude may be determined, after applying corrections for temperature and altitude. These corrections are obtained from tables derived from a theoretical formula.

It is stated that in general a relative accuracy of one to two metres may be expected, and it is being found that much longer strips may be photographed between ground control points when statoscopes are fitted. They are now being used extensively by Continental air photographers. Automatic pilots do not yet appear to have been employed much there,

although in this country it is generally agreed that the standard of flying is much improved thereby.

*Tilt of the Aircraft.*

It has been found impossible to take vertical photographs free from small tilts in unknown directions. A good survey pilot, unaided by gyroscopic stabilization, can limit the tilt to 2°, but when an "automatic pilot" is fitted it can be limited to $\frac{1}{4}$°, or so.

When a photograph is inclined from the vertical there is a scale distortion in the direction of tilt. This effect cannot be noticed by inspection when the tilt is as small as 2°. An exaggerated effect of tilt is given in Fig. 39; at the top being a low oblique with about 20° of tilt and below it a rectified print. In Fig. 2 the distortion in oblique photography of a rectangular grid on the ground is obvious. These tilt errors must either be eliminated in printing or during the plotting process.

*Inclination of the Air-base.*

Another difficulty is that due to actual variation of the aircraft height between exposures as a result of air currents or pockets. The air-base or line in space joining two adjacent camera positions will therefore be inclined to the horizontal, and this slope must be allowed for in any plotting process.

When an automatic pilot is fitted to the aircraft, the course is maintained much more nearly level, straight and at constant height than can be achieved by manual control.

## Flying for Air Survey Photography

Continental and British methods show considerable divergence in the technique of air survey. On the Continent more attention has been paid to producing accurate plots from photographs not usually taken in a manner enabling results to be obtained by the simplest method. Greater energy has been directed to the design of complicated and expensive machines to plot from photographs inclined a few degrees out of the vertical, than to ensuring approximate verticality of exposure.

In this country, largely on the initiative of the Air Survey Committee, it has been the object to produce as a matter of routine approximately vertical photographs, so that simple graphical plotting is adequate for many purposes. In special cases, or for greater accuracy, one may employ a measuring and plotting machine for producing the desired result from such photographs.

TILTED PHOTOGRAPH.

[*Courtesy of Carl Zeiss (London), Ltd.*

RECTIFIED PHOTOGRAPH.

FIG. 39.

It has been said that the difference between the British and Continental technique is that in this country the aim is comparable to that of the surveyor who adjusts his instrument carefully before making his observations to avoid elaborate methods of eliminating errors in the office. On the other hand Continental methods have been required for rather a different purpose than has been the case here. While most of the pioneer work here was in improving a simple method suitable for producing military maps on medium scales with minimum ground control, the Continental work was, to a considerable extent, for the preparation of large-scale contoured maps.

In the former case, improvement of photographic conditions made it easier to produce the required plans by a simple method, while, in the latter case, the work was related in most instances to ample ground control, and instead of rectifying the photographs this process was incorporated in the plotting machines.

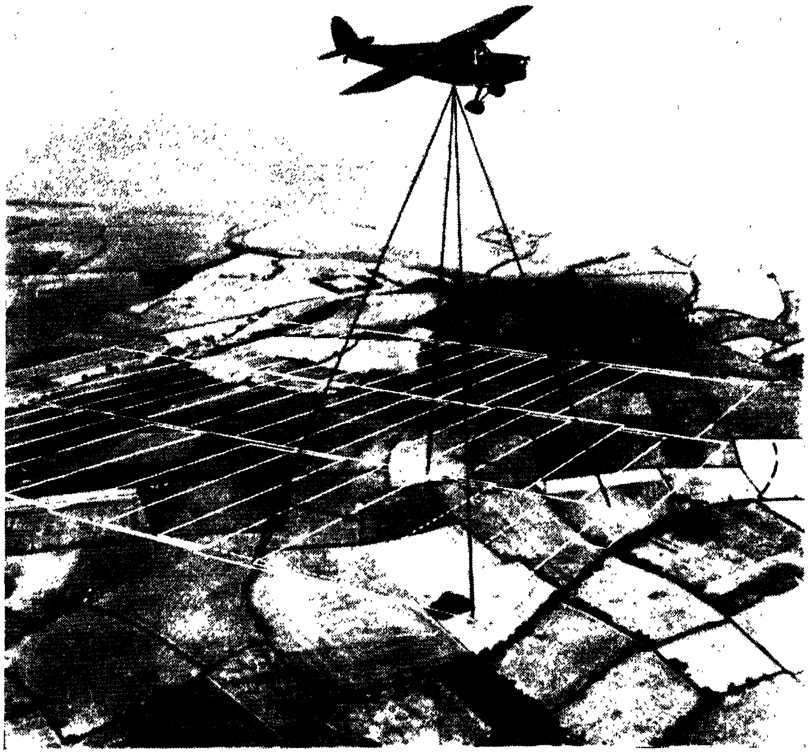
*Strip Photography.*

Most air survey photography involves taking the photographs in a series of strips. In some cases of oblique photography the photographs are taken in groups along the strip so as to cover as large an area as possible for small-scale mapping, and although it is necessary to ensure some overlapping of common detail, the plotting is carried out by non-stereoscopic methods. Some of these special methods will be discussed later, and here only flying for vertical and almost vertical photography and stereoscopic observations will be considered.

Survey photography is a very specialized task and it is usually arranged that the ground surveyor or engineer shall state his requirements of area and scale to the air party, together with any relevant information in reference to ground control.

A longitudinal overlap of 60 per cent is aimed at between photographs as shown in Fig. 40. This allows a margin in case of fore-and-aft tilt which might reduce the overlap to less than 50 per cent. Thus the common overlap of the photographs in series will be between 10 and 20 per cent, so that plotting may be carried out from the photographs without ground control points on each photograph. If there is only one case of a "short overlap" in a strip the chances of preparing an accurate plot with minimum ground control are appreciably reduced.

In order that adjacent strips may be connected by common detail attempts are made to maintain the lateral overlap at about 25 per cent. This allowance should be sufficient to preserve overlap in spite of lateral tilts and deviations from course.



[Courtesy of Aerofilms, Ltd., London and Wembley.

FIG. 40—DIAGRAMMATIC REPRESENTATION OF LINES OF FLIGHT COVERED BY OVERLAPPING PHOTOGRAPHS.

Some difficulties of navigation and photography are shown in Fig. 41. The upper diagram (Fig. 41a) shows the course taken by the aircraft, and area covered by photographs when the air is still and navigation perfect. If the aircraft is set on its course by compass without allowing for wind velocity the machine will "drift" from its course as shown in Fig. 41b. If the wind is correctly allowed for in calculating the course, the aircraft will "crab" (Fig. 41c) because, owing to the relative velocities of wind and aircraft the nose of the latter will not point in the direction of motion. To correct this the camera is turned in its mountings so that the photographs are taken square as in Fig. 41d. A device such as the Aldis Camera Aiming Sight is used to set the camera.

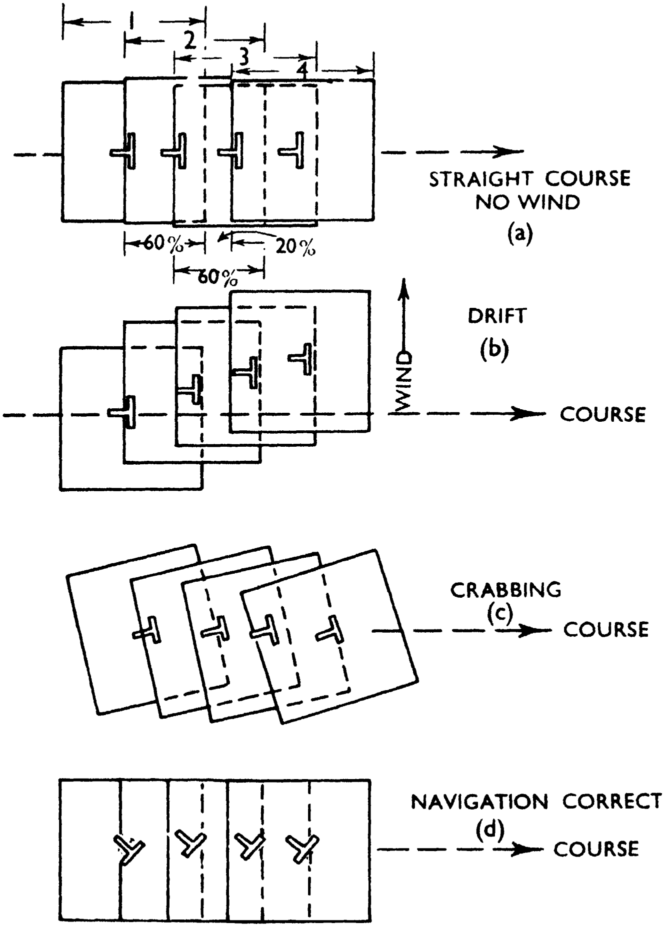



FIG. 41.

The effect of a tilt is, of course, to deflect the optical axis out of the vertical and displace the photographed area as shown in Fig. 42. ABCD is the area covered by a vertical photograph from O, and A′B′C′D′ is the area covered when there is a tilt of $\theta$. Tilt may be in any direction, and is not necessarily in the direction shown. Thus it may be seen that if there are equal opposing tilts between two adjacent photographs, either laterally or longitudinally, the overlap will be increased or reduced by twice the amount. Brown [10] has shown very clearly how a tilt affects the overlap and how this can be a very serious problem when working on large scales, by causing gaps or resulting in increased cost of photography.

He compares photography at a scale of 1/20,000 on a 7 × 7 inch plate with a lens of 7 inches focal length, with that at 1/5,000 on a plate $8\frac{3}{4} \times 6\frac{1}{2}$ inches with a 21 inch lens. These two cases are respectively at about the military or medium scale, and the scale of photography for the 1/2,500 Ordnance Survey experimental revision. Limiting tilts of 2° for manual control and $\frac{1}{4}$° for the automatic pilot are considered. Conditions for minimum amount of photography, are that the longitudinal overlap must be at least 50 per cent and the photographs just touch laterally.

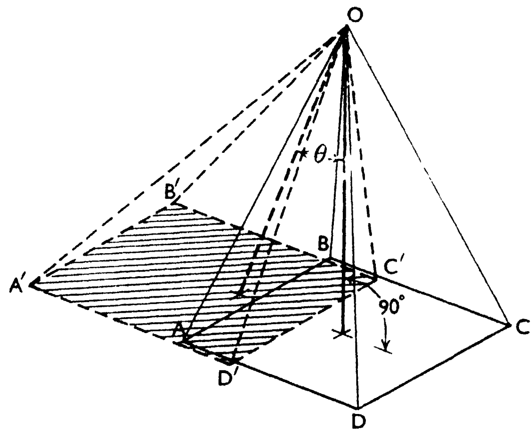



FIG. 42.

TABLE IV. 1

| *Scale.* | 2° *Opposing Tilts: Overlaps %* | | | | $\frac{1}{4}$° *Opposing Tilts: Overlaps %* | | | |
|---|---|---|---|---|---|---|---|---|
| | *Fore and Aft.* | | *Lateral.* | | *Fore and Aft.* | | *Lateral.* | |
| | *Actual.* | *Loss.* | *Actual.* | *Loss.* | *Actual.* | *Loss.* | *Actual.* | *Loss.* |
| 1/20,000 | 51 | 9 | 16 | 9 | 59 | 1 | 24 | 1 |
| 1/5,000 | 37 | 23 | 8 | 17 | 58 | 2 | 23 | 2 |

In table IV 1 are considered longitudinal and lateral overlaps of 60 and 25 per cent respectively under ideal conditions. It can be seen that the effect of a tilt on overlap is much greater for large scales than for small ones. If tilts are limited to $\frac{1}{4}$° it will be safe to work on a basis of longitudinal overlap of 55 per cent and lateral overlap of 10 per cent, thereby making an appreciable saving in photographic costs alone.

The course for strips is often established by magnetic compass. The effect of the magnetic dip, combined with tilting of the aircraft, makes the compass sluggish on North runs and very sensitive on South runs. Also

the dazzle and haze caused by the sun makes it preferable to aim at East and West courses. Many pilots prefer to set on a landmark or even a distant cloud. Owing to possible sluggishness of the compass, a landmark is preferred when setting a course to be flown by the Automatic Pilot.

For scales of about 1/25,000, it is usually uneconomic to fly a strip less than ten miles long, while for lengths greater than twenty miles it is almost impossible to fly accurately enough without intermediate control points. When some sort of a map is available it is usually possible to plot projected flight lines on it for the use of the pilot and navigator. These lines will be fixed in consultation with the ground party, and one or two strips will have control points at each end where this can be arranged.

It is possible to fly longer strips, but the lack of control must naturally result in reduced accuracy. Thus strips have been flown in Arabia, with distances up to a hundred miles between control points. It is becoming more usual to fly long strips between ground control points, when an aerial triangulation is to be made by means of a stereoscopic plotting instrument.

When flying for 1/5,000, or larger scale, photography, a much closer control and co-operation is necessary because the allowable limits of deviation are much smaller. Even with the automatic pilot limiting tilts to $\frac{1}{4}^\circ$, the allowable deviation from course is only about 140 yards at 1/5,000. When the survey covers an appreciable area, six miles is about the economic minimum length of strip. This takes three minutes to fly at 120 m.p.h., and an error of 1° in the course causes a deviation of 180 yards during this period. When the ground control is limited the use of the automatic pilot is most desirable in large-scale work in order to save a large percentage of wasted runs.

Brown [10] also points out that although fore-and-aft tilt can be kept down so as to keep the aircraft nearly level and ensure the required longi-

TABLE IV. 2

| | *Tilt of 2°* | | *Tilt of $\frac{1}{4}^\circ$* | |
|---|---|---|---|---|
| *Scale.* | *Departure from plumb point—yards.* | *Loss of overlap %* | *Departure from plumb-point—yards.* | *Loss of overlap %* |
| 1/20,000 | 135 | 4·3 | 17 | 0·5 |
| 1/5,000 | 102 | 8 | 13 | 1·1 |

tudinal overlap by means of the automatic pilot, the lateral overlap must be maintained by skill in navigation. This will entail setting the course for the automatic pilot to control. He gives a table which shows the deviation from track due to tilts, the information from which is given in Table IV.2. The *plumb point* is the point on the ground vertically below the camera lens at exposure. The departure figure given is the movement of the principal point away from this due to the tilt.

If the lateral overlap should be 25 per cent the deviation which will cause loss of overlap is 488 yards on the 1/20,000 scale and 152 yards on the 1/5,000 scale. Brown remarks, "It will be seen at once from these figures that whereas the small-scale photographer has nearly a quarter of a mile to play with on either side of his line for navigational purposes, the large-scale photographer has only 50 yards, when the tilt is 2°. Even if he could reduce his tilt to $\frac{1}{4}$° he has only the narrow margin of 140 yards."

This type of flying is compared to a child's efforts at "kerb-walking."

*Stabilized Flying.*

For a number of years experiments were made in various parts of the world in attempts to ensure that the camera axis was vertical at the moment of exposure, but little success was gained until the introduction of the Smith Three-axes Automatic Control which stabilizes the aircraft gyrostatically and enables it in good weather to be maintained on a course with a probable tilt not exceeding $\frac{1}{4}$°. Greater difficulty has, however, been experienced in obtaining an exactly straight course because of some unbalanced gyroscopic forces which give the course a slight bend in azimuth. Nevertheless the course is noticeably superior to that which can be flown by the most experienced and accurate pilot.

Extensive experiments have been made in the use of this automatic stabilization of controls and the established commercial air survey companies have now adopted it with enthusiasm. They have perhaps been a little slow in adopting the instrument because much of their work can be executed by manual control, since the vast amount of ground control available from the Ordnance Survey maps has made it easy to rectify the tilted photographs during the printing process. Also, many surveys abroad have been of a reconnaissance nature, in which such refinements have been unnecessary.

The teething troubles of these instruments have been largely eliminated, and recently additional research and experimental work in this connection has been carried out by Captain Charles Lloyd of C.L. Air Surveys.

In addition to the employment of the Pollock Brown Auto-hydraulic Pilot in these experiments, navigation has been controlled by wireless and

the camera level stabilized gyroscopically. The gyroscope used is a spinning wheel instead of a freely swung gyroscope as used formerly. An essential feature is that some provision is made for correction of the path of the pilot which is liable to show a bend in azimuth due to gyroscopic precession. A special type of gyroscope known as a "Deviator" is fitted which allows an external control to be applied. This control is by radio from a broadcasting station, small currents being generated which operate in turn relays and solenoids and so alter the setting of the automatic pilot in relation to the transmitting station. By tuning in to such a station lying at right angles to the required direction of flight, the path of the aircraft will be along the arc of a circle centred at the broadcasting station. Other strips are similarly centred and are therefore parallel. The apparatus does not appear yet to be beyond the experimental stage.

In the United States the automatic pilot is not yet considered satisfactory owing to tendencies to precessions.[1] It must be remembered however that in the United States where very large-scale maps are required, plotting machines such as the Zeiss Stereoplanigraph are employed and also that some operators are not always prepared to give the constant attention and maintenance required to keep an automatic pilot in first-class condition. Eliel [1] has described a Solar Navigator, which enables a pilot to use the sun for accurate navigation.

When a stereoscopic plotter is used, it is considered by some that statoscope readings are sufficient.

*Requirements of Aircraft and Personnel.*

The photographic work of the Royal Air Force has generally been carried out in ordinary machines, but these have certain disadvantages for regular service. Salt[85] in 1933 gave the following requirements. Endurance should be from six to eight hours to allow of operations at a distance from the base and for preparatory work before photography is commenced. The time required for the latter in making estimations for "drift," etc., will take nearly an hour. Since survey flying is often at a height where the temperature is low, the aircraft should be designed for comfort. There should be a good view ahead and to the sides with the instruments grouped near the normal line of sight. The machine should cruise at at least one hundred miles an hour and be able to climb rapidly. A multi-engined aircraft gives greater security, and the better the aerodynamic stability of the machine the easier it will be for the pilot to keep on his course. The automatic pilot is a valuable addition and is fitted to some types of official aircraft.

Lloyd (1938) also gave certain requirements for survey aircraft. The

aircraft must be capable of cruising at 90 to 110 miles per hour at 15,000 feet. Use of the automatic pilot has shown that multi-engined aircraft have the disadvantage that slight variations in propeller properties and slight differences in pulling power of the engines will cause vibration that cannot be avoided as synchronization is impossible. The high-winged monoplane is most suitable as the pilot has a better view of the ground; and it should be designed to be operated solely by the pilot.

Special aircraft have been constructed in a number of countries for air survey photography. In Germany both single and two-engined aircraft are used, the cruising speed being usually higher than above (up to 180 miles per hour) and the service ceiling higher (19,000 to 21,000 feet). In Switzerland, the Survey Authority uses a single-engined, high-winged monoplane. Photography is carried out at as slow a speed as possible and the aircraft employed will fly at 68 miles per hour at a height of 3,200 feet and 83 miles per hour at a height of 19,600 feet. In the United States some aircraft have been specially adapted for survey photography.

A very large proportion of survey photography has, however, been carried out with standard machines adapted for the purpose.

Notable improvements have been made in navigational instruments and methods and it is clear that for the best results, the aircraft must be specially designed or adapted for air survey photography.

The following comment on the present position in this country has been made in the final report of the Departmental Committee on the Ordnance Survey 1938 [73] (Davidson Committee).

"The standard of photography required for Ordnance Survey purposes demands aircraft fitted with automatic pilots and equipped with special and costly photographic apparatus, while the technique involved in the taking of good-quality photographs free from abnormal tilts necessitates the employment of personnel specially trained in air survey methods."

After remarking that the present resources of civil firms are not at present adequate to deal with Ordnance Survey photography owing to short-term contracts, the Committee recommends that a special Air Survey Unit be formed consisting of trained personnel and specially equipped aircraft, and that this is the only satisfactory method of fulfilling the requirements of the Ordnance Survey.

The Aircraft Operating Company is having built two special De Havilland Rapide Aircraft fitted with Smith Three-axes Control Automatic Pilots to form the nucleus of a commercial air survey unit, the formation of which has been accelerated by the above Report. The crew will be augmented and will consist of the pilot, navigator and photographer.

*Ground Control in Relation to Photography.*

Except in the roughest of reconnaissance surveys and in military surveys over enemy country, air surveys are always fitted to a ground control.

In air surveys, no less than in other types of surveys, it is desirable to "work from the whole to the part." Thus the basis of ground control should be a triangulation and the spacing of the trigonometrical stations may be anything up to twelve miles for a medium-scale survey when plotted graphically, to several times that distance when more elaborate methods and instruments are employed. The geodetic surveyor is sometimes accused of unnecessary accuracy in observation and excessive precision in computation and adjustment. Admittedly in some cases quite good results could be obtained with a triangulation of less precision but this would apply, however, only to a limited area and the tediousness of correcting out larger and larger errors would become very great.

If the ground control survey is made with care and with a knowledge of the lay-out of the various strips of photographs, it is easy to save many hours' work in plotting by putting in a few hours on the ground.

The actual amount of ground control must vary with the scale and requirements. For instance, in a rough reconnaissance survey it is possible to dispense with ground control altogether. The resulting plan is likely to be in error by at least one or two per cent, and levels can only be determined very approximately. Moreover, the plan will be distorted in shape because of the effect of tilts, and errors of scale and shape will accumulate rapidly.

At the other extreme, four points per photograph give complete control for rectification and plotting. This number is usually excessive in all but very large-scale surveys and a strip of large-scale photographs can be plotted accurately by the Arundel Method with a ground control point near each end of the strip, provided an automatic control is used in flying. A greater number of points is required for contouring. When stereoscopic plotters are used, the complete map may be produced from four points on a strip. Manual control of photography on large scales will involve a greater density of control. It is possible on medium scales to manage with fewer points than two per strip when plotting a block consisting of a series of strips overlapping laterally. The general method of compilation will be described later on.

*Organization of Photography.*

In the case of medium scales, the usual procedure is as follows:

If no maps are available, it is necessary first of all to fly navigating strips

enclosing rectangles about 60 miles by 20 miles. The tie strips divide up the area into squares of 20 mile side. (See Fig. 43.) The ground surveyor will mark up navigation points as required and make the Minor Control Plots for each strip. Such plots, which are described fully in Chapter VII,

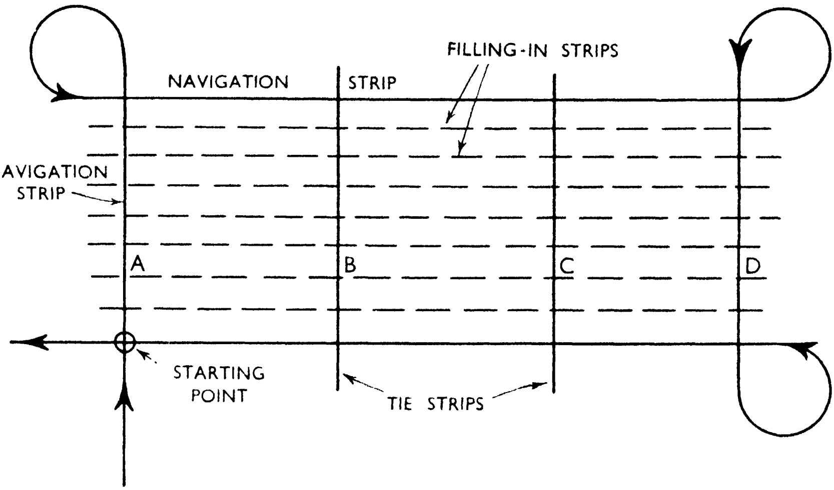
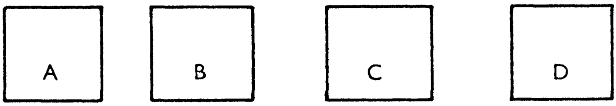



FIG. 43.

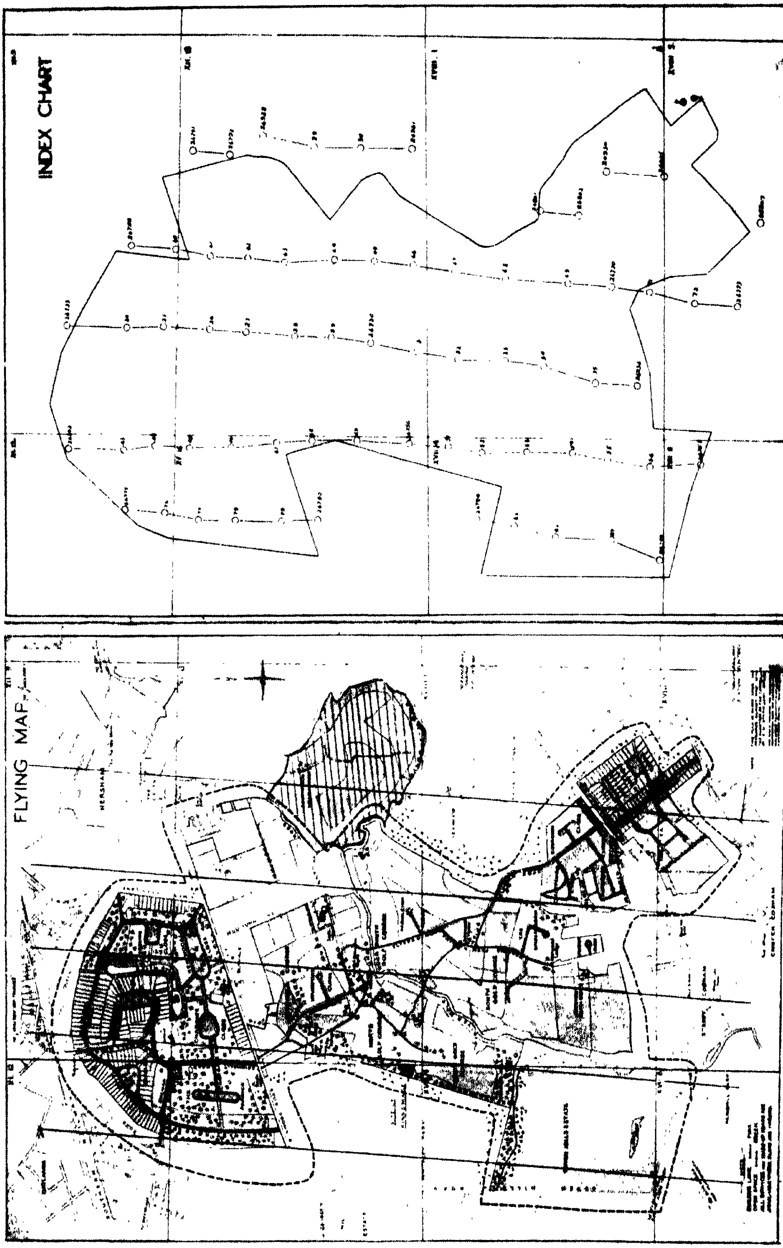
are actually "traverses" of the principal points of the photographs in a strip. The photographs are then placed over the minor control plot to make a mosaic framework.

The track lines and allowable deviations for the filling-in strips are marked. These strips are flown on an east and west course if convenient. The pilot and navigator are each supplied with photographs, divided up into sets of four for each strip, pasted on to card. (See Fig. 44.) It is




NAVIGATION PRINTS FOR FILLING-IN STRIP A. B. C. D.
FIG. 44.

necessary to make sure that there is some ground point shown on each photograph near the track line which can be identified from the air. While the strip is being flown the navigator can then always keep one point on his course in view. Great care has to be taken in working out the course,




[*Courtesy of Aerofilms, Ltd., London and Wembley.*

FIG. 45. PROJECTED AND ACTUAL FLIGHT LINES FOR AIR PHOTOGRAPHY.

owing to drifts caused by cross-winds. A deviation as small as 1° from the correct course will soon destroy the overlap. The addition of a third person to the crew when an automatic pilot is fitted should enable the accuracy of navigation to be considerably increased. An example of the air photographer's proposed lay-out of flight lines and how they appeared after plotting is given in Fig. 45.

In cases where the ground levels are changing fairly regularly, e.g., where the land rises from sea level to a range of hills, it is often desirable to fly the strips approximately parallel to the contours and at varying heights to preserve scale and overlap.

For small-scale maps also, some form of strip navigation is used.

The flying problem will frequently be much simpler in the case of large-scale surveys for engineering construction because the area or length is not very great. For example, when the best route for a proposed road has been decided from the preliminary survey, the construction survey will consist of one, or at most two, strips flown along the general direction of the centre line. In such cases, the ground control is also much easier to arrange. This is particularly useful where the survey is plotted from one strip taken along a centre line which is not straight. Here, it may not be economical to make use of an automatic pilot as the photographs are easily rectified with respect to ample ground control. Actually, however, the difficulty of keeping along a track which is not straight makes it often easier to fly the route in a series of straight strips, so as to avoid any "flat turns." Here again it may be considered unnecessary to use the automatic pilot for the short strips, because of the time taken to set the course by it for each short run.

CHAPTER V

# ELEMENTARY PERSPECTIVE

## Elementary Perspective as Applied to Air Photography

The air surveyor is faced with a problem exactly the reverse of that of the artist. The latter must be able to make a perspective drawing of any object from a particular viewpoint, while the former is required to reproduce actual dimensions to scale from a perspective picture as given by a photograph. The latter process is known as *iconometry*.

In order that the fundamental problems of air survey may be understood, it is necessary to be acquainted with certain definitions and rules of perspective projection and reconstruction. Projection from vertical, or nearly vertical, photographs, will be found to be a simplification of the general case of the tilted photograph.*

*Definitions.*

The essential definitions may be followed by reference to Fig. 46. The principal axis SP of a camera, perpendicular to the photographic plane ABCD, is tilted at an angle $\theta$ from the vertical at exposure, so that the plane of the photograph itself is inclined at $\theta$ to the horizontal plane CDEF, which represents a level piece of ground. S is the camera station as defined by the inner or rear node of the lens system, and is the *perspective centre*, the point through which all rays are assumed to pass straight from object to image. When referring to such a photograph, any plane which contains the perspective centre and a line in the plane of the photograph is called a *perspective plane*. For instance PVS is a perspective plane [similarly Z′SZ (Fig. 48)].

It is a fundamental principle of perspective that any straight line contained in a perspective plane will project as a straight line into any other plane. It will be seen later that this statement does not apply to a line along the ground surface which is of variable height above the datum plane.

* While it is desirable to understand the general case before certain aspects of air survey can be appreciated, the reader who is concerned primarily with plotting from verticals may prefer to return to this section later, or to refer to it as required.

The points V and $v$ vertically above and below the camera station, S, on the photograph and ground planes in Fig. 46 are known respectively as the *photo and ground plumb points.*

The *photo principal point* P is determined by dropping a perpendicular from S on to the plane of the plate, the principal distance SP being equal

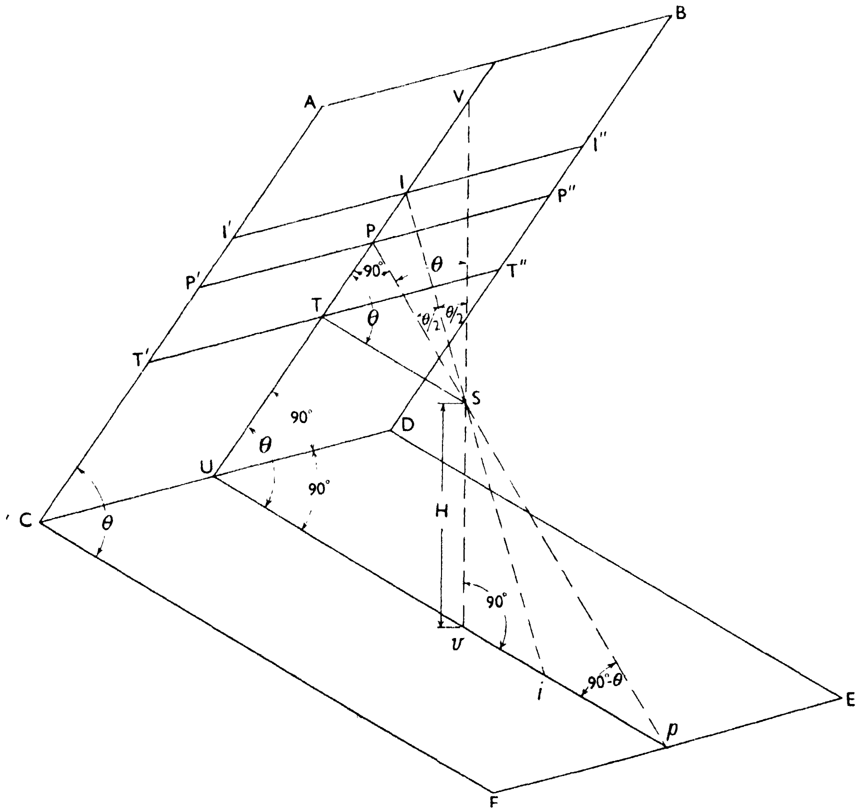



FIG. 46.

to the focal length $f$ of the lens when the camera is in adjustment. The ground principal point $p$ is found where the principal line. produced back from S meets the ground plane.

Pairs of corresponding points such as V, $v$; P, $p$; are known as *homologous points*, one point being the *homologue* of the other. The straight ray joining a pair of homologous points passes through the perspective centre S.

An inclined plane in space such as ABCD meets the horizontal ground

plane CDEF in a horizontal line called the *horizontal trace*, and the inclination $\theta$ of plane ABCD to the horizontal plane CDEF is equal to the inclination of a line normal to the horizontal trace CD along the inclined plane.

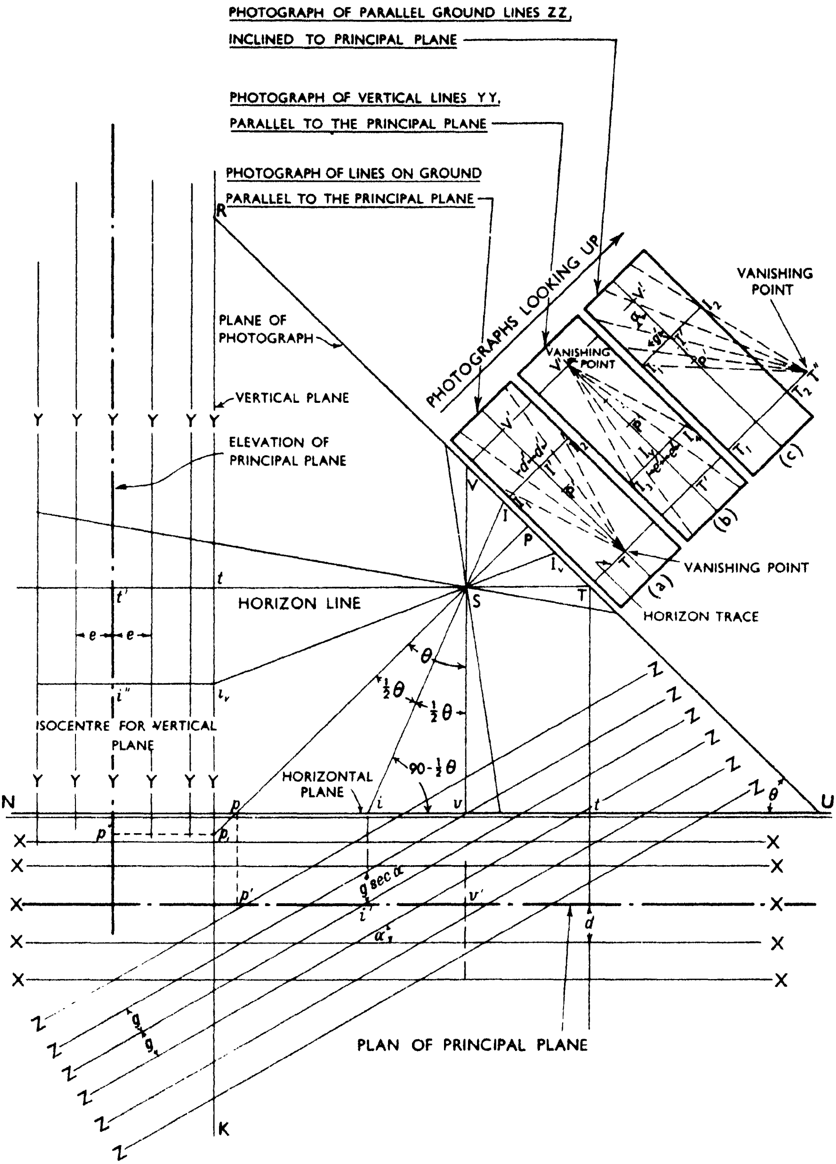
This horizontal trace CD, along which any point coincides with its homologue, is known as the *perspective axis*. A line UP is drawn perpendicular to the perspective axis along the photograph plane, and this projects as U*p* in the ground plane and is also perpendicular to CD. These are called *photo and ground principal lines* respectively, and are contained in the *principal plane* which also contains the perspective centre S and the plumb points V, *v*.

The horizontal plane which contains S is the *horizon plane* and meets the plane of the photograph in the *horizon trace* T′T″. Horizontal lines drawn in the photograph plane are *plate parallels*, or *plate horizontals*. These are, in effect, contour lines, and the direction of maximum slope perpendicular to this, i.e., along the principal line, is conveniently known as the direction of tilt. The *axis of tilt* as defined by the Air Survey Committee is that plate-parallel which passes through the principal point. The *isometric axis* is fixed by bisecting the angle PSV, giving the line IS*i*, which is inclined equally to both the photograph and ground planes; I and *i* being the *photo* and *ground isocentres* respectively. The plate parallel I′I″ passing through I is called the *isometric parallel*. It will be shown later that this is the only parallel along which there is no scale distortion resulting from tilt. It will also be shown that angles between points on the ground as measured from the ground isocentre *i* will be equal to the corresponding angles as measured in the photographic plane from I, the photo isocentre.

*Vanishing Points.*

From the definition of a system of parallel lines it is known that they will meet at infinity. If such a system is projected from an object plane through a converging lens system on to an image plane, the image of this meeting point will be found at the point where the ray, which passes through the perspective centre and is parallel to the system, meets the image plane.

In Fig. 47 a section is shown of the principal plane of Fig. 46, the same lettering being used. The elevation of the ground plane is NU and the plan of the ground principal line *pv* in it is represented in plan below by *p′v′*. A series of lines equally spaced at a distance *d*, and parallel to the principal line is shown in the plan. If a horizontal line be drawn through S, the perspective centre, and parallel to this system, it will meet the plane of



PHOTOGRAPH OF PARALLEL GROUND LINES ZZ, INCLINED TO PRINCIPAL PLANE
PHOTOGRAPH OF VERTICAL LINES YY, PARALLEL TO THE PRINCIPAL PLANE
PHOTOGRAPH OF LINES ON GROUND PARALLEL TO THE PRINCIPAL PLANE
PHOTOGRAPHS LOOKING UP
VANISHING POINT
PLANE OF PHOTOGRAPH
VERTICAL PLANE
ELEVATION OF PRINCIPAL PLANE
HORIZON LINE
VANISHING POINT
HORIZON TRACE
ISOCENTRE FOR VERTICAL PLANE
HORIZONTAL PLANE
PLAN OF PRINCIPAL PLANE


FIG. 47.

the photograph in T, which is the elevation of the *vanishing point* of the system, i.e., the point to which the images of all such parallel lines will converge. A view (*a*) of the photograph as seen looking up shows how these lines converge to T. The isometric parallel is represented by $I_1I_2$, normal to the principal line P′V′, and since there is no distortion of scale along this line due to tilt, the scale along it is $f/H$, where $f$ = SP = the focal length of lens, and $H$ = S$v$ = the flying height at exposure. Hence distances $d'$ may be set off on either side of I′ along this parallel so that $d' = d.f/H$. Lines are then drawn through these points to T giving the system of converging lines shown.

Now let the vertical plane RK be the object plane. The intersection of principal line and vertical plane is in the line $p_1t$, represented by $p''t'$ in elevation. A series of vertical lines YY is shown equally spaced at a distance $e$. The vanishing point for this system will be at the intersection of a vertical ray through the perspective centre S and the plane of the photograph, namely the plumb point V shown in view (*b*) as V′. The horizontal distance from perspective centre to picture plane is St, which may be called D. The isometric parallel is no longer the same as for the horizontal plane and will pass through the isocentre $I_v$. This isometric axis is fixed by the line $I_vi_v$, which is inclined equally to the photograph and vertical planes. The spacing of lines along the axis $I_3I_4$ is $e' = e.f/D$.

Finally, consider a second system of parallel lines ZZ equally spaced at a distance $g$ on the horizontal ground plane, which are skewed to the principal line $p'v'$ at an angle $\alpha$ as shown. For convenience one of these lines is drawn through $i'$. Since the ground angular ratio is preserved when lines are drawn from the isocentre, a line drawn through I′ (view *c*) inclined at $\alpha$ to the principal line V′P′, meets the horizon trace at T″, the vanishing point for the system. Distances $g = d.\sec \alpha.f/H$ are set off along $I_1I_2$. These points are joined to T″.

An appreciation of the determination of vanishing points is useful as an introduction to some of the problems of perspective and is of importance in plotting from oblique photographs. ✓

*Distortion of Image on a Vertical Photograph due to Height of Object.*

Owing to the added complication of tilt distortions when the photograph is not exposed vertically, the effect may first be considered for the simple case as in Fig. 48. When the exposure is a vertical one, principal point, plumb point, and isocentre are coincident, and reference to Fig. 46 will show how plumb point and isocentre move away from the principal point as tilt increases.

In Fig. 48, *abcd* is a horizontal plane and ABCD its image on a verti-

cally exposed photograph. The principal axis P$p$ is vertical, S being the perspective centre as before. The photograph is a true plan of the plane abcd at a scale of SP/S$p$ = $f/H$. Angles measured from P on the photograph will be equivalent to the corresponding horizontal angles measured with a theodolite from $p$ on the ground. This is true, not only for points such as $z$, the base of a chimney in the ground plane, but also for all other points such as $z'$, the top of the chimney at a height $h$ above the ground. The image of $z$ is at Z and that of $z'$ at Z′. The latter is in the same position as the image of a point $z''$ on the ground, fixed by the intersection of lines $pz$ and Z′$z'$ produced. The distortion of image due to the height $h$ of the object is ZZ′, PZZ′ being, of course, a straight line.

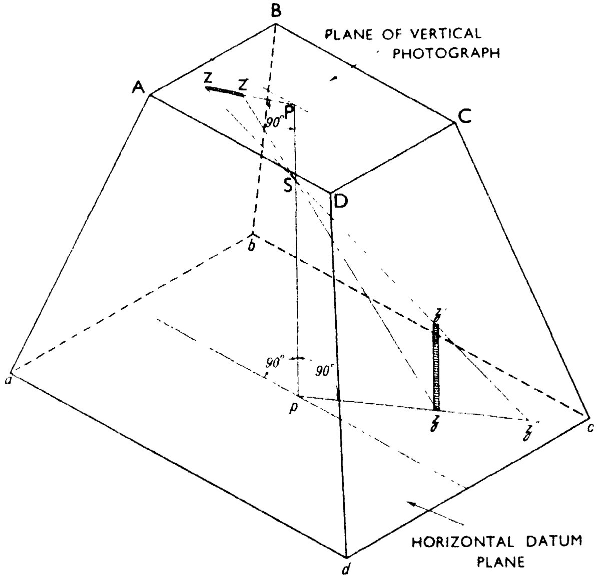



FIG. 48.

From the diagram it will be seen that triangles SZZ′ and S$zz''$ are similar, as also are triangles ZPS and $zp$S:

Hence, $ZZ' = zz''\cdot\dfrac{f}{H}$: also $zz'' = PZ'.\dfrac{h}{f}$.

so that $ZZ' = PZ'\cdot\dfrac{h}{H}$ . . . . . . . . (V.1.

The distortion of image is therefore proportional to the distance of the image from the centre of the photograph, and to the ratio of height of object to flying height when there is no tilt.

*Relation of Principal Point, Plumb Point, Isocentre, and Horizon Plane.*

In general terms, a perspective representation of an object is obtained from a particular viewpoint if a transparent sheet is placed between the object and this viewpoint, the picture being formed on it by the intersection with it of all rays from the viewpoint to the object. It will be seen that by moving the position of this sheet any one object can have an infinite number of perspective projections.

Much of the pioneer work in applying perspective theorems to the solution of problems of air survey has been due to Hotine, who investigated them very thoroughly while he was Research Officer to the Air Survey Committee. The selection of the more important theorems which follow is largely due to him,[55] although solutions of most of them have been furnished also by Von Gruber[46], Higgins[53], Gordon[45], and others.

The terms used are those employed by the Air Survey Committee.

Only those perspective properties of direct use in air survey are given here, and details of some of their applications are left until the appropriate section in later chapters.,

From Fig. 46 the following relations may be deduced for a tilt $\theta$ of the principal line from the vertical:

(*a*) Distance from principal point to plumb point,
$PV = SP \tan\theta = f.\tan\theta$ . . . . . . . (V.2a)

(*b*) Distance from the principal point to the isocentre,
$PI = f.\tan\frac{1}{2}\theta$ . . . . . . . . . . (V.2b)

(*c*) Distance along the principal line from principal point to horizon plane $=$ PT, and since VST is a right angle and ST is horizontal
$\therefore PT = f.\cot\theta$ . . . . . . . . . (V.2c)

The height of aircraft at exposure $= Sv = H$.

*Scale of Photograph.*

In Fig. 46 the principal line UV gives the direction of greatest slope and it is in this direction that the "fore-shortening", or scale variation, is a maximum. Along plate parallels, such as P′P″, there is no fore-shortening, i.e., the scale is constant along a particular parallel.

(*a*) Let P′P″ be the plate parallel through the principal point P. Scale along this parallel will be in the ratio $SP/Sp = \dfrac{f}{(H.\sec\theta)}$ . (V.3a)

(*b*) In the case of the plumb point, the plate parallel is V′V″, and the scale is $VS/Sv = \dfrac{(f.\sec\theta)}{H}$ . . . . . . . (V.3b)

(*c*) For the isocentre, Ii is inclined equally to both photographic plane

and the ground plane (see also Fig. 47), so that $IU = Ui$. The scale along the isometric parallel I'I'' is $SI/Si = \frac{f.\sec\frac{1}{2}\theta}{H.\sec\frac{1}{2}\theta} = \frac{f}{H}$ . . . (V.3c)

Hence the scale along the isometric parallel is the same as that over the area of a vertically exposed photograph taken of the same level ground, from the same height, and with the same camera.

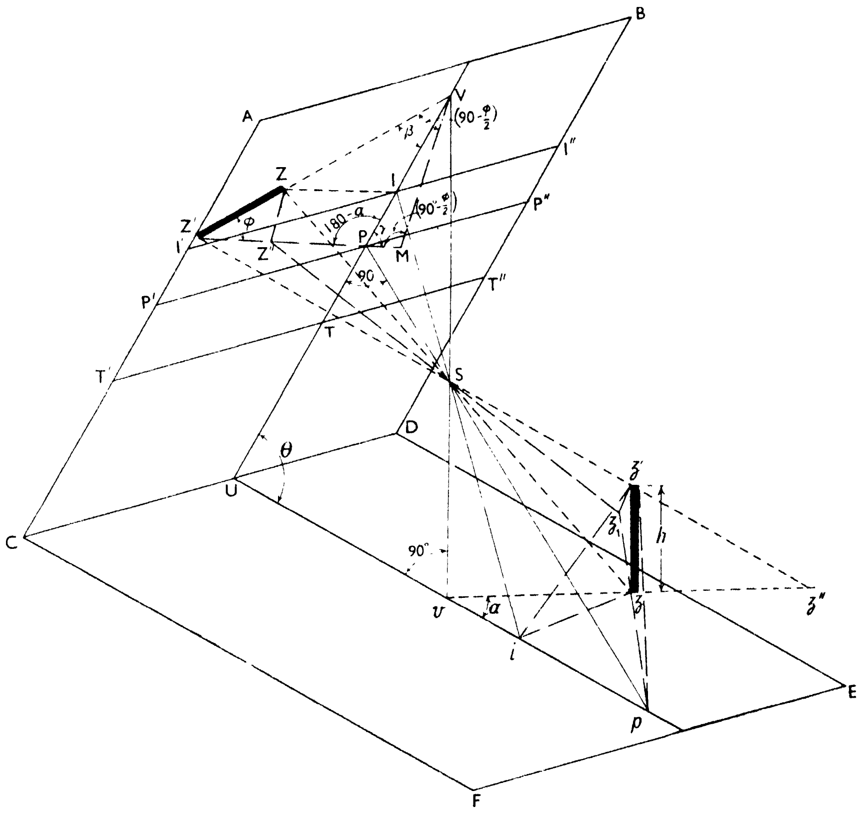


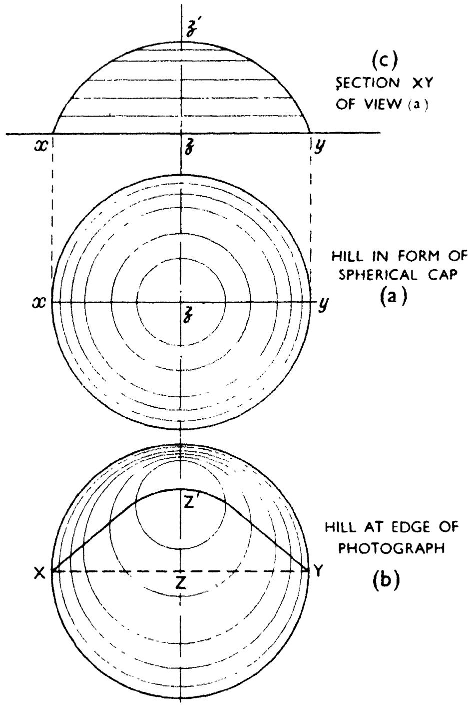

FIG. 49.

The scale of photograph is constant along any plate parallel, being too small on the principal point side of the isocentre and too large on the plumb point side.

Surveyors often define a line contained in a vertical plane along the surface of the earth as a straight line, i.e., even though there is height variation along it, the line is called "straight."

The rule that any "straight line" (in the surveyor's sense) will project

as a straight line into any other perspective plane must be modified for the production of plans from photographs where the true horizontal ground plane is never obtained, and the "straight" line is not in a horizontal plane. In Fig. 49, the chimney $zz'$ is vertical, but its image ZZ′ lies in a line radial from the plumb point V, all the points being contained in one perspective plane.




HEIGHT DISTORTION IN VERTICAL PHOTOGRAPH

FIG. 50.

Now consider the vertical plane containing the horizontal line $iz$, the vertical chimney $zz'$ and the isocentre $i$. The horizontal angle $zip$ is equal to the projection of angle $z'ip$ into the horizontal. From the property of the isocentre it follows that $\angle zip = \angle ZIP$.

It will be noticed that in the image plane, the height distortion ZZ′ is not a continuation of the line IZ. Such a condition will arise for all cases except for those lines which pass through the plumb point.*

Perhaps this will be seen more clearly if point $z$ is supposed to be the centre of the base of a spherical cap of circular base. The true plan as seen looking down vertically on to $z'z$ is as in Fig. 50a, xzy representing a section through the cap as in Fig. 50c. The contours are represented by concentric circles. The actual photographic view as obtained with $zz'$ as the vertical axis of the cap and ZZ′ its image in a case similar to that in Fig. 49, is as seen in Fig. 50b. The top is now distorted by an amount ZZ′ and while the vertical plane $xzyz'$ now projects as an inclined plane XZYZ′, as seen in Fig. 50b, so that the line measured along the surface

* The validity of the commonly used radial method of plotting from nearly vertical photographs depends upon the extent to which a line may be assumed straight although not passing through the plumb point. (See Chapter VII.)

of the ground between $x$ and $y$ is now represented by the curve XZ′Y, the height distortion of $z'$ being ZZ′ as in Fig. 49.

*Homologous Distances.*

In Fig. 51, z and Z are a pair of homologous points. The lettering corresponds with that in Figs. 49 and 46, but some separation of diagrams

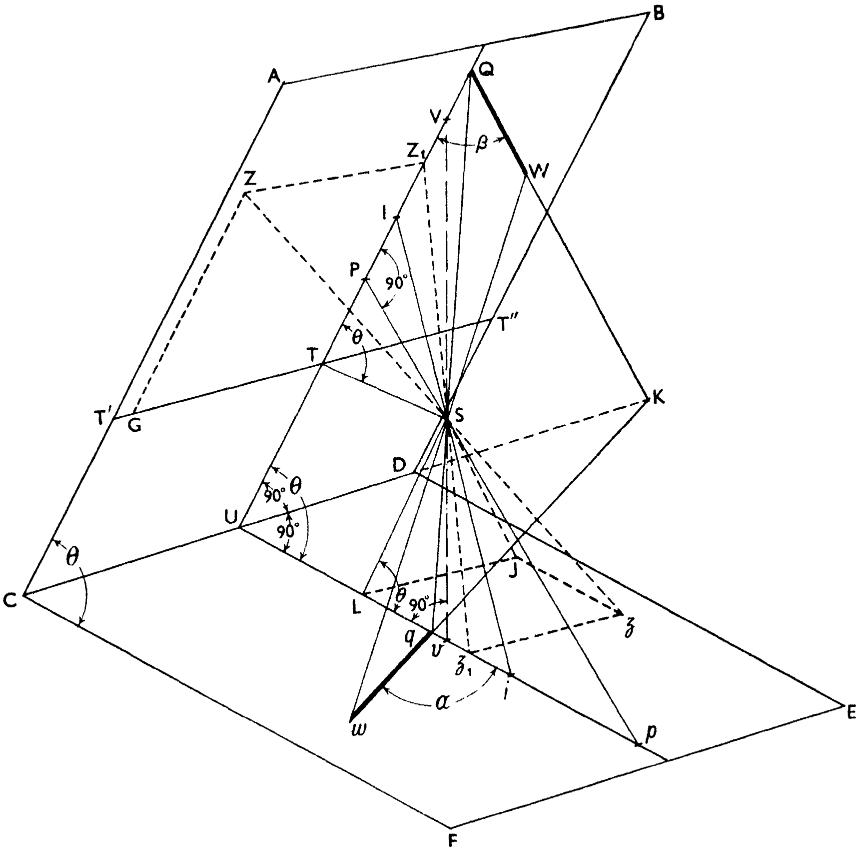



FIG. 51.

is necessary in order to avoid confusion. The plate parallel through Z cuts the principal line in $Z_1$, while a line drawn in the plane of the plate in the direction of tilt, i.e., parallel to the principal line, meets the horizon trace T′T″ in G. Through S a line is drawn in the principal plane and parallel to the principal line, meeting the ground principal line in L. Through L a line is drawn parallel to the horizon trace and another line through $z$

parallel to the principal line, intersecting at J. To complete the diagram a line is drawn from J to S.

Now $Jz = Lz_1$ and $ZG = Z_1T$, and since TS is parallel to LU and $Z_1Sz_1$ is a straight line, the triangles $TSZ_1$ and $Lz_1S$ are similar;

Hence $\frac{Z_1T}{TS} = \frac{SL}{Lz_1}$, and since $TS = f.\operatorname{cosec}\theta$ and $SL = H.\operatorname{cosec}\theta$,

therefore $ZG.Jz = Z_1T.Lz_1 = TS.SL = fH.\operatorname{cosec}^2\theta$ . . (V.4)

ZG and $Jz$ are known as homologous distances and the evaluation of the product in such a simple form is very useful in the design of rectifiers which enable prints to be made from tilted exposures as though the photographs had been taken vertically. It is also used in the construction of perspective grids for oblique photographs.

*Effect of Tilt on Angular Ratios.*

Consider Fig. 51 again, where Q and $q$ are homologous points in the principal plane, and W and $w$ are homologous points out of the principal plane. The angles made by lines $wq$ and WQ respectively with the principal line in their planes are $\alpha$ and $\beta$.

These four points will be contained in a single perspective plane. The line QW is in the intersection of this perspective plane and the plane of the photograph. It is the *trace* of the perspective plane in the plane of the photograph. Similarly, $wq$ is the trace of this perspective plane in the horizontal plane CDEF. When two planes in space intersect in a line (as in this case when ABCD and CDEF intersect in CD) and a third plane cuts each of them, it is a fundamental property of planes in space that the traces of this third plane on the two planes of projection must meet in a point on their line of intersection.

Hence in this case if QW and $wq$ be produced they will meet at point K on CD produced, CD being the perspective axis.

In a diagram such as this it may be difficult to distinguish all the right angles at first. These have all been marked and will be mentioned as met.

QU and $q$U are both perpendicular to the perspective axis CD so that in the triangle $q$UK, $KU/Uq = \tan\alpha$; and in the triangle QUK, $KU/UQ = \tan\beta$.

Hence $\frac{UQ}{Uq} = \frac{\tan\alpha}{\tan\beta}$ . . . . . . . . (V.5)

This enables the connection between angles in the picture plane and ground plane to be established.

Suppose that Q and $q$ coincide with the photo and ground principal points P and $p$; i.e., $\angle UPp = 90°$; then $\frac{\tan\alpha}{\tan\beta} = \frac{UP}{Up} = \cos\theta$ (V.6)

If Q and $q$ coincide with the photo and ground plumb points, then

since $\angle UvV = 90^\circ$, $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{UV}{Uv} = \sec\theta$ . . . . . (V.7)

Finally when $q$ and Q coincide with the ground and photo isocentres respectively, $\angle UiI = \angle iIU = (90^\circ - \frac{1}{2}\theta)$ so that $Ui = UI$:

Hence $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{Ui}{UI} = 1$, i.e., $\alpha = \beta$ . . . . . (V.8)

The essential property of the isocentre is now apparent, namely, that angles measured on a tilted photograph from the photo isocentre will be equal to corresponding angles measured on the ground from the ground isocentre. Any distortion of photographic images due to tilt must therefore be radial from the isocentre. In practical plotting from air photographs a difficulty arises from the fact that these tilt distortions are radial from the isocentre, while height distortions are radial from the plumb point.

*Height Distortions on a Tilted Photograph.*

Consider the homologous points Z, $z$ in Fig. 49, where $z$ is the base of a vertical chimney of height $h$, its top being $z'$. On looking towards the plumb point, the top will appear to be displaced outwards, and from S it will appear in the same position as the image of a point $z''$. If $z$ and $z'$ are projected into the picture plane, which is inclined at $\theta$ to the vertical, the displacement of the image is ZZ′, radial from the plumb-point V.

On a vertical photograph $ZZ' = PZ' \cdot \dfrac{h}{H}$, from Fig. 48 and equation (V.1).

On a tilted photograph, however, this distortion will be affected by the angle of tilt $\theta$, and the angle $\beta$ between the principal line, and the direction of Z from the plumb point.

In practice it is usually better to consider the displacement of individual points separately, and the somewhat complicated formula for the full solution of the above case is not required.

*Tilt Distortions.*

In Fig. 52a a sectional elevation is given through the principal plane of a photograph taken with a tilt of $\theta$, and in Fig. 52b is shown a view of the photographic plane looking up in the direction of the arrow. Symbols for principal point, plumb point and isocentre are as before. Since the principal axis SP is inclined at an angle $\theta$ to the vertical, then VT, the principal line of the photograph, will be tilted by an angle $\theta$ to the horizontal.

The image of a point $a$ on the ground, not in the principal plane

(Fig. 52c) projects to $a_1$ in the principal plane as shown in Figs. 52a and 52c. The image of point $a$ is at A in the photographic plane (Fig. 52b). Similarly the image of $a_1$ appears at $A_1$ as shown in both Fig. 52a and Fig. 52b.

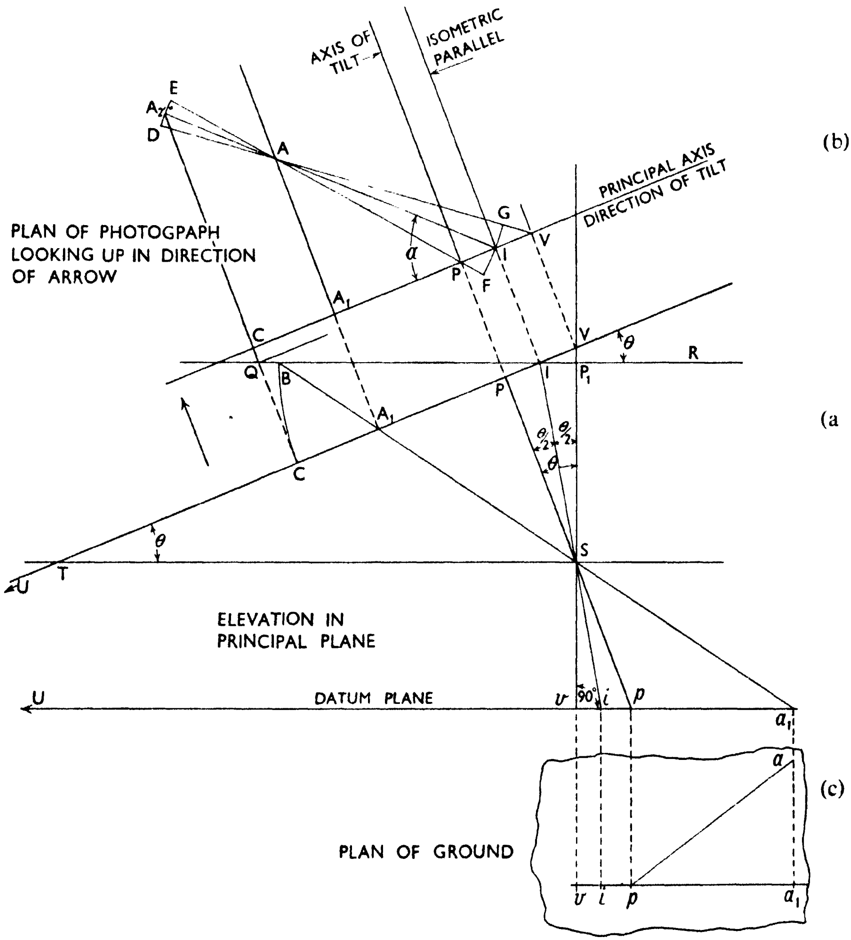



FIG. 52.

Had the photograph been taken truly vertically from the same camera station and with the same camera, then QR would represent the photographic plane which passes through I, the elevation of the isometric parallel, i.e., the scale along this parallel is $f/H$ on the tilted photograph corresponding to that on the vertical one.

The image of $a_1$ is at B in the untilted view. In order to show the relative positions in the tilted photograph, the correct distance of the image of $a_1$ from I, IB, is set off along IT, i.e., with centre I and radius IB describe an arc cutting the principal line of the tilted photograph in C. Thus C is the corresponding point to $a_1$ for the untilted photograph, and by drawing a line through C parallel to the axis of tilt, $A_2$ is fixed by the intersection of this line and IA produced. Then $A_2$ is the position that would be occupied by the image of $a$ on the untilted photograph. The tilt distortion is therefore $AA_2$.

Let $\alpha$ be the angle in the photographic plane between $IA_2$ and the principal line IP produced. $A_1$ and C are the projections of A and $A_2$ respectively on to the principal line.

Then $AA_2 \cos \alpha = IC - IA_1 = IB - IA_1$ . . . . (V.9)

Now the triangles $IA_1B$, $TA_1S$ are similar, and IT = TS since IS is inclined equally to the photographic plane IT and the horizon plane TS.

$$\text{Then } \frac{IB}{IA_1} = \frac{ST}{A_1T} = \frac{ST}{IT - IA_1} = \frac{f.\operatorname{cosec}\theta}{f.\operatorname{cosec}\theta - IA_1}$$

$$\therefore \frac{(IB - IA_1)}{IA_1} = \frac{f.\operatorname{cosec}\theta - f.\operatorname{cosec}\theta + IA_1}{f.\operatorname{cosec}\theta - IA_1} = \frac{IA_1}{(f.\operatorname{cosec}\theta - IA_1)}$$

Now from equation (V.9) it follows that

$$AA_2.\cos\alpha = IB - IA_1 = \frac{IA_1^2}{(f.\operatorname{cosec}\theta - IA_1)}$$

Also $IA_1 = IA.\cos\alpha$

$$\therefore AA_2.\cos\alpha = \frac{IA^2.\cos^2\alpha}{(f.\operatorname{cosec}\theta - IA.\cos\alpha)}$$

$$\text{and } AA_2 = \frac{IA^2.\cos\alpha.\sin\theta}{(f - IA.\cos\alpha.\sin\theta)} \qquad \text{(V.10)}$$

If the expression is written in the form

$$AA_2 = \frac{IA}{\dfrac{f}{IA.\cos\alpha.\sin.\theta} - 1}$$

it will be seen that the distortion has a maximum value when $\alpha = 0$; i.e., when the direction of the line from the isocentre is along the principal line.

When the tilt $\theta$ is not more than a few degrees, the term $IA.\cos\alpha.\sin\theta$ in equation (V.10) is small in comparison with $f$, and may be neglected, so that the expression becomes:

$$AA_2 = \frac{IA^2.\cos\alpha.\sin\theta}{f} \qquad \text{(V.11)}$$

The maximum value when $\alpha = 0$ is

$$AA_2 = \frac{IA^2.\sin\,\theta}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V.12)$$

*Effect of Assumption that Tilt Distortions are Radial from the Plumb point.*

Referring to Fig. 52b, A is the distorted position of the image on the tilted photograph of a ground point $a$, while $A_2$ is the position of the image when tilt is absent. Since height distortions are radial from the plumb point, it becomes desirable to find the amount of error in plotting by assuming that the tilt distortion is radial from the plumb point V.

Produce line VA to D, so that $AD = AA_2$. Then the error due to the assumption is $A_2D$.

When the tilt is not more than a few degrees an approximate solution may be found.

Draw IG parallel to $A_2D$; let the angle VAI between the rays from isocentre and plumb point to A be $\gamma$; while the angle PIA between the ray from the isocentre to A and the principal line IP is called $\alpha$.

From the similar triangles, $DA_2A$ and GIA; we have $A_2D = AA_2.\dfrac{IG}{IA}$

From equation (V.11) $AA_2 = IA^2.\dfrac{\cos\,\alpha.\sin\,\theta}{f}$

In the triangle IVG,

$$\frac{IG}{\sin\,(\alpha - \gamma)} = \frac{VI}{\sin\,(90 + \frac{1}{2}\theta)}, \text{ since } AG = IA$$

Again from Fig. 52a, $VI = f(\tan\,\theta - \tan\,\frac{1}{2}\,\theta)$

Hence by substitution

$$A_2D = \frac{(IA)^2.\cos\,\alpha.\sin\,\theta.f}{f} \times \frac{(\tan\,\theta - \tan\,\frac{1}{2}\,\theta).\sin\,(\alpha - \gamma)}{\sin\,(90 + \frac{1}{2}\,\gamma).IA}$$

$$= IA.\cos\,\alpha.\sin\,\theta.(\tan\,\theta - \tan\,\tfrac{1}{2}\,\theta).\sin\,(\alpha - \gamma)\sec\,\tfrac{1}{2}\,\gamma. \quad . \quad (V.13)$$

Generally $\theta$ may be taken as a small angle and I and V are so close that $\gamma$ can be neglected for points which are not very near I.

Then $A_2D = IA.\cos\,\alpha.\sin\,\alpha.\theta.(\theta - \frac{1}{2}\,\theta) = IA.\frac{1}{4}\,\theta^2.\sin\,2\,\alpha$ . (V.14)

IA is the measured distance of the image of the point from the isocentre.

This error is a maximum for $\alpha = 45°$ when $\sin\,2\,\alpha = 1$.

Hence the maximum error due to the assumption is $AI.\frac{1}{4}\,\theta^2$. (V.15)

## Special Features of Vertical and Almost Vertical Photographs

*Height and Scale.*

If a particular scale is required in a photograph of particular size, the area shown on the photograph *cannot be varied*, i.e., the ratio $f/H$ must remain constant.

Thus if a scale of 1/5,000 is required and $f = 7$ inches.

$$\frac{7}{12H} = \frac{1}{5{,}000} \text{ or } H = 3{,}000 \text{ feet approximately.}$$

If $f = 20$ inches, then $H = 8{,}000$ feet approximately.

There are certain advantages in flying high with a long-focus lens in such a case because, at the greater height, the air is less likely to cause unsteadiness, and, possibly more important, the incidence of rays at the edges of the photograph is more nearly vertical (see Fig. 53). The scale is the same in each case, i.e., $f/H = f_1/H_1$. By increasing $H_1$ to H and correspondingly increasing $f_1$ to $f$, the "dead ground" due to the height of the building shown is reduced from *ac* to *ab*. It will be shown later that a lens of shorter focus is better for contouring.

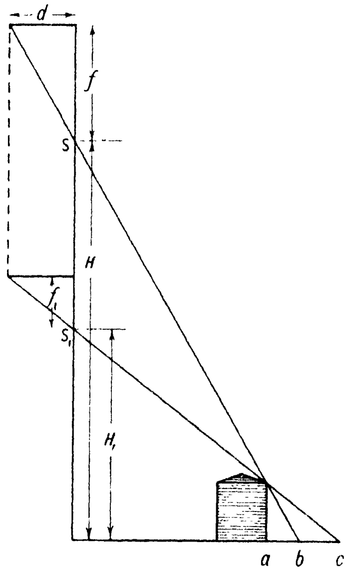


Fig. 53.

*The Radial Assumption.*

Since tilt distortions are radial from the isocentre and height distortions radial from the plumb point, a compromise must be made in deciding on a basis of plotting. If the plumb point is used for this purpose, it must be located either by referring to four points in the photograph which are known in position on the ground, or, if there is less control, by setting a pair of photographs in an elaborate stereoscopic measuring machine. Since the principal point is marked on the photograph, or can be easily plotted, it is convenient to use this point wherever possible. It will be shown that when the tilt is very small and variations of ground height limited, both tilt and height distortions may be assumed to be radial from the principal point.

*Effect of Assumption that Height Distortions are Radial from the Principal Point.*

Consider again the chimney $zz'$ in Fig. 49. In the photograph, $ZZ'$ is the height distortion, radial from the plumb point V. If this distortion is assumed radial from the principal point P, then $Z''$ will be plotted from the photograph instead of Z, by making $Z'Z'' = Z'Z$. If a line be drawn from p to z on the ground plane and produced until it meets $Z''S$ in a point $z_1$, then $z_1$ is the position of a ground point in the datum plane, whose image will appear at $Z''$ in the photograph.

The error of position of the image of $z'$ in the photograph due to height distortion will be $ZZ''$.

An approximate solution for small values of $\theta$ may be found as follows:

Produce $Z'Z''$ back through P to M, making $Z'V = Z'M$.

(*N.B.*—Although the diagram shows $Z'V > Z'P$, the reverse may also be true.)

$Z''$ is fixed very nearly by drawing a line through Z parallel to VM and cutting $PZ'$ in $Z''$.

Let $\angle VZ'M = \phi$; $\angle VPM = \gamma$ and $\angle PVZ' = \beta$.

i.e., $\gamma = \phi + \beta$.

Now $\dfrac{Z'Z''}{Z'M} = \dfrac{ZZ'}{VZ'}$.

From equation (V.7), $\dfrac{\tan\alpha}{\tan\beta} = \sec\theta$, so that if $\theta$ be small, then $\alpha = \beta$ approximately, from which it follows that

$$\frac{ZZ'}{VZ'} = \frac{zz''}{vz''} = \frac{zz'}{Sv} = \frac{h}{H}$$

Hence $Z'Z'' = Z'M\cdot\dfrac{h}{H}$ approximately.

Also $\dfrac{ZZ''}{Z'Z''} = \dfrac{VM}{Z'M}$, so that $ZZ'' = VM\cdot\dfrac{h}{H}$.

Now $PV = f.\tan\theta$, and in the triangle PVM

$$\frac{PV}{\sin(90 - \frac{1}{2}\phi)} = \frac{VM}{\sin\gamma}\text{, and by substitution}$$

$$ZZ'' = \frac{f.\tan\theta.\ \sin\gamma}{\sin(90 - \frac{1}{2}\phi)}\cdot\frac{h}{H} \qquad \text{(V.16)}$$

We have a maximum value of $\phi$ when $\angle Z'PV = \angle PVZ'$, i.e., when M coincides with P,

Then $\beta = (180 - \gamma)$ and $[2(180 - \gamma) + \phi] = 180°$, i.e., $\gamma = (90 + \frac{1}{2}\phi)$. Hence the expression in equation (V.16) will tend to a maximum when $\dfrac{\sin\gamma}{\sin(90 - \frac{1}{2}\phi)}$ approaches its maximum value which is given by $\dfrac{\sin(90 + \frac{1}{2}\phi)}{\sin(90 - \frac{1}{2}\phi)} = 1$.

Now PV is small, so that $\frac{1}{2}\phi$ will be small near the edges of the photograph and maximum distortion $= ZZ'' = f.\tan\theta\cdot\dfrac{h}{H}$ . . . (V.17)

*Effect of Assumption that Tilt Distortions are Radial from the Principal Point.*

The same diagram (Fig. 52) may be used as in the case of the plumb point.

In a similar manner $AA_2 = AE$, and by drawing IF parallel to $A_2E$ (the error to be found), it can be shown as in the case of the plumb point that

$$A_2E = I\,A\cos\alpha\cdot\frac{\sin\theta.\tan\frac{1}{2}\theta.\sin(\alpha + \beta)}{\sin(90 - \frac{1}{2}\beta)} \quad . \quad . \quad . \quad . \quad (V.18)$$

and when $\theta$ is small and $\beta$ is also small, the expression reduces to the same form as equation (V.14), namely $A_2E = IA.\frac{1}{4}\theta^2.\sin 2\alpha$ and the maximum value which occurs when $\alpha = 45° = AI.\frac{1}{4}\theta^2$, as before.

*Validity of the Radial Assumption.*

The effect of the radial assumption must now be considered in relation to the permissible errors in the plotting of points due to inherent error in the assumption, compared with the likely error which may arise during the plotting process.

The effect of tilt alone is much less than that of varying ground height.

From the formula for maximum tilt distortion given in equation (V.15) the maximum value for a 2° tilt on a photograph 7 inches square, is 0·0015 inches, taking the diagonal distance from centre to a corner as 5 inches. This is clearly negligible since it is smaller than the likely plotting error and is even smaller than this value when the tilt is less than 2°. At $\theta = \frac{1}{4}°$ the error is 0·000012 inches.

In vertical photography the flying height is generally between 3,000 and 15,000 feet and the focal length of camera lens may vary from about 3½ inches for an ultra wide-angle lens, to about 21 inches for large-scale photography.

Maximum error due to principal point assumption for height distortion from equation (V.17) is $f.\tan\theta.h/H$.

For manual control flying at the standard military scale of 1/25,000, the tilt is limited to 2° and the ratio $h/H$ to 1/10. The photographs are 7 × 7 inches and the focal length is 7 inches. The maximum error is then 0·025 inches, which is considered satisfactory for the simple Arundel Method of plotting. The standard flying height for this scale is 15,000 feet, so that a variation of ground height of 1,500 feet is allowed from the datum. If, however, the variation is only 500 feet, then the error is only 0·008 inches.

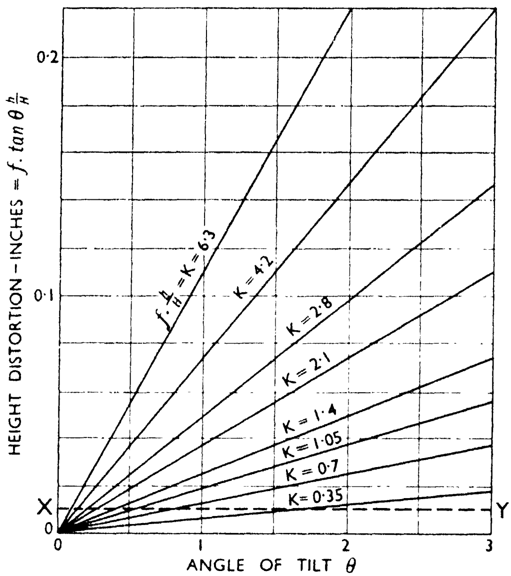



FIG. 54.

Focal length is not likely to be increased much above 7 inches for photography at this scale because of the discomfort of flying above 18,000 feet.

Reduction of tilt enables the limits of height variation and focal length to be varied.

Suppose an error of 0·01 inches is admissible. Fig. 54 gives curves for various values of $f\cdot\frac{h}{H}$ from equation (V.17). Taking the smallest values of $f$ and $h/H$ as $3\frac{1}{2}$ inches and 1/10 respectively where K = 0·35, and their largest values as 21 inches and 3/10 respectively where K = 6·3, the curves of error are plotted against tilt $\theta$, for $f = 3\frac{1}{2}$, 7, 14 and 21 inches, and for $h/H = 0{\cdot}1$, 0·2 and 0·3.

By reading off along the line XY (which represents an error of 0·01 inch), a tilt of about 1° 40′ is permissible for the focal length of 3½ inches when $h/H = 0{\cdot}1$ (K = 0·35) while a tilt of only 0° 05′ is permissible when using a 21-inch focal length, with a height variation of 3/10 (K = 6·3).

The choice of camera lens and other conditions must depend upon the requirements of the work. For instance in large-scale planimetry, rays at the edges of photographs may obscure much detail owing to obliquity, if a lens of short focal length is used.

If, in a particular case, the ratio of $h/H$ becomes too great, some means must be employed to determine the plumb point of the photograph. The process of rectifying photographs for tilt when printing is one such method. By using four known points on a photograph the rectification is done by projection, so that in the rectified print, the plumb point is coincident with the principal point, and the rectified photograph appears as though the camera axis had been vertical at the moment of exposure.

CHAPTER VI

# ELEMENTARY STEREOSCOPY

*Principles of Stereoscopy.*

STEREOSCOPY plays an important part in air survey, both in the interpretation of air photographs and in plotting from them.

A pair of human eyes possesses the power of binocular vision by which it is possible to obtain a conception of relief instead of seeing a flat panorama, as on a cinema screen. The lens of the human eye is a converging lens of variable focus, and alteration of focussing distance or *accommodation* is obtained by modifying, at will, the curvature of the lens. At the same time the optical axis of the eye can be changed in direction by rotating the eye in its socket. The relative direction of these axes is known as the *convergence*. When a near object is viewed the eye lens is focussed by altering the curvature, and the convergence is automatically suited to the distance of the object, so that the image impression is imprinted on to the screen or retina at the back of the eye. The physiology of the stereoscopic impression thus obtained is not yet perfectly understood.

The eye has front and rear nodes in the same way as an ordinary converging lens system, and similarly it is sufficient to consider only the rear node, as shown at point N in Fig. 55. Here A and B are two points in space which can be seen at the same time. Rays pass to the fovea of each eye via its nodal point and the images are seen at $a$, $a'$ and $b$, $b'$ respectively.

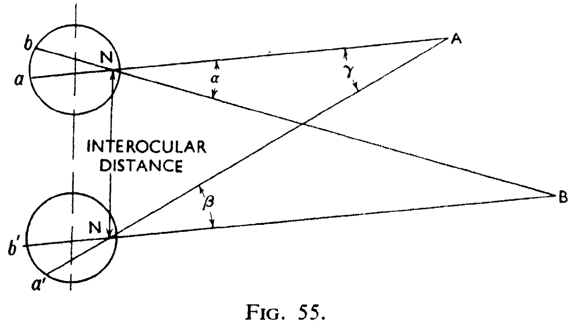



FIG. 55.

As a result of the variable focus of the eye, the nodal point moves in relation to the retina according to the distance of the focus. The focal length is the distance from the node to any image point on the spherical surface of the retina.

In the early part of the nineteenth century Wheatstone propounded

the fundamental principle of stereoscopy. He showed that if two perspective pictures of an object are placed before the eyes so that the same view can be seen by each eye as would be seen when looking at the solid object, then an impression of the solid object would be constructed to the eyes from the perspective pictures.

Although there are certain geometrical rules upon which the explanation of stereoscopic vision is based, these rules are not rigid as there are, apparently, physiological tolerances which allow some deviation from the stated conditions. This tolerance is of great convenience in stereoscopy as applied to air survey.

The line joining the nodes is called the *eye-base*, and their distance apart is the interocular distance, which is usually about 65 mm. or $2\frac{1}{2}$ inches.

The angle subtended at a point by the eye-base (such as $\gamma$ for A in Fig. 55) is called the *parallactic angle* or *angle of parallax*.

The eye-base NN is the *epipolar axis*, and the plane containing this axis, and a point (such as A) in space an *epipolar plane*.

In the introductory stages of the discussion, the point in space, its stereoscopic images and the nodal points of the eyes will be assumed to be in one plane.

Cover up the two black dots $b$ and $b'$ in Fig. 56 and look hard at a point midway between the dots $a$ and $a'$, keeping the direction of the eye-base approximately parallel to the line between the dots. Most people will find that these dots will appear to fuse together and three dots will be seen. A little practice may be necessary before the third dot is seen clearly and it is likely that it will first be seen with a yellowish ring round it. A post card held in between the two dots a and a′ may help. If fusion cannot be obtained by this method, the dots may be drawn on a piece of transparent material such as celluloid and fusion obtained by looking intently through the plane on which the dots appear and at an object such as a pencil point, held behind.

When $a$ and $a'$ can be fused easily, uncover $b$ and $b'$ and look at all four dots together. When both pairs have fused it should be noticed that the dot formed from $a$ and $a'$ appears to be floating in space above that formed from $b$ and $b'$.

The geometrical principle can be seen from Fig. 57. For these tests the spacing of the dots should be considerably less than the interocular distance. XY represents the plane of the paper on which the dots are drawn and which is parallel to the direction of the eye-base. The elevations of $a$ and $b$ correspond, i.e. $b$ is vertically below $a$. Consider the formation of a third dot when viewing $a$ and $a'$, which represent respec-

M


a
a′
b
b′

FIG. 56.

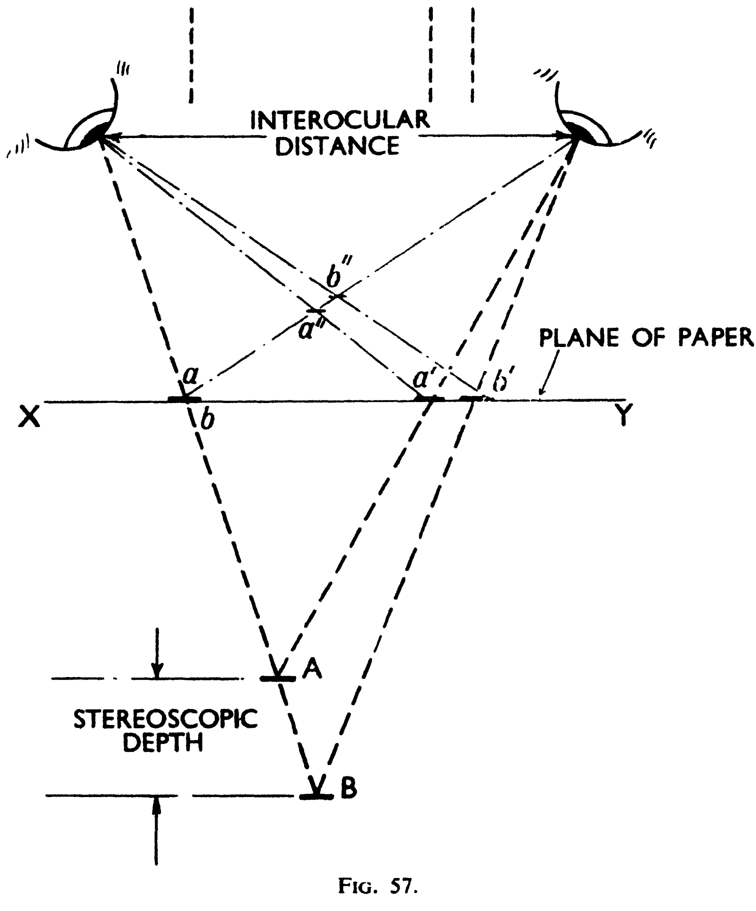

INTEROCULAR DISTANCE
b″
a″
PLANE OF PAPER
a
a′
b′
X
b
Y
A
STEREOSCOPIC DEPTH
B

FIG. 57.

tively the images as seen by the left and right eyes, of a point A in space. To reverse the procedure, if, when one is looking at an object such as A in space a transparent screen is interposed in a position corresponding to the plane of the paper, its images on that screen would be *a* and *a'*.

Similarly, *b* and *b'* will appear at B, the impression being gained that A is floating above B. The vertical depth between A and B is known as the *stereoscopic depth*, and, if the four dots on the photograph are supposed to be the stereoscopic images of two points in space, their relative heights may be obtained, given the values of certain characteristics of the photographs.

A stereo-pair of one of the pyramids has been given in Fig. 5 (Chapter II). In Fig. 58 sketches of a similar pair of truncated pyramids are given. It should be possible to fuse the two views at will to obtain the spacial impression. It is clear from Fig. 57 that the stereoscopic impression of depth can be varied by altering the spacing of the pictures. This may be tested by fusing two dots on *different* pieces of paper. If their separation is decreased the stereoscopic image approaches, and if increased the image recedes.

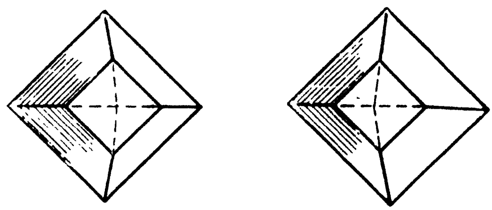


FIG. 58.

Another example is given in Fig. 59. After looking at the dots for a few seconds they will "click" together and the squares immediately surrounding each dot will also be fused, so that the third dot will appear to be floating well below the grid of squares. It is the principle of one type of measuring stereoscope to estimate differences of level by the relative movements of a pair of such grids over a pair of photographs.

The effect of stereoscopy in Fig. 5 may now be considered. By holding these views well away from the eyes the reader should be able to fuse them together and see the solid picture, and it will be noticed that the significance of the detail in the photographs can be much more easily appreciated than in the non-stereoscopic view.

Facility of stereoscopic observation requires some flexibility in the eye muscles and this usually requires practice. In Fig. 56 it will be noticed that if A is a point in space both the convergence of axes of the eye-lenses and the accommodation will be set in sympathy for the distance of A from the eyes. Actually, of course, when these points are fused stereoscopically, the curvature of the eye-lens is such that the focus is correct for points in the plane of the paper; but the convergence of the eye-

lenses are suited to the distance away of the fused point. There are certain physiological limits to the extent of variation of the accommodation distance from the eye to the plane of the paper and that at the distance from the eye of the fused stereoscopic image. If the spacing of the points is increased, the angle of convergence is increased, with the

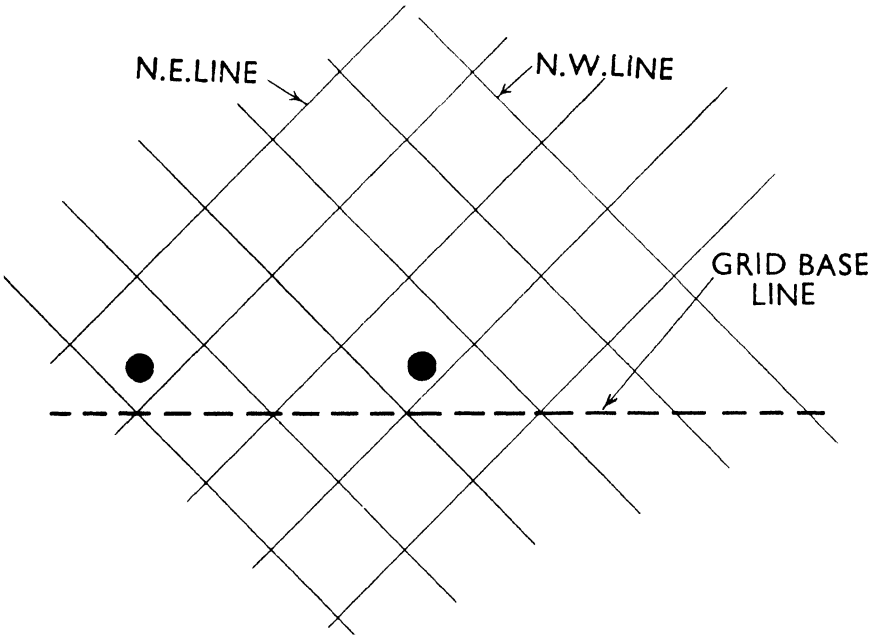



FIG. 59.

result that the impression of relief will be more pronounced as long as it is possible to retain stereoscopic fusion.

At first, some observers get a "pseudoscopic" effect in which the convergence of the eye axis is too much and the right eye picks up the view intended for the left eye and vice versa, so that everything is seen inside-out; i.e., hills will appear like valleys. In order to avoid this effect it is important that the photographs should be examined with the light coming from the same direction as in the original exposure. The chain dotted lines in Fig. 57 show how a pseudoscopic image may be obtained inadvertently. When it is desired to view a pair of photographs pseudoscopically to aid interpretation, the left and right views are interchanged.

The human eye can see distinct figures over only a small area. If one concentrates on a letter in a book held at a distance of, say, 20 inches,

only a very few letters round it can also be seen clearly. French [40] has stated limits for the stereoscopic range of sight. In Fig. 60, A and B are two points which subtend an angle of 1° at the node N of the left eye; while points C and D, in the same horizontal line as A and B, subtend an angle $\theta$ at the node N′ of the right eye. Assuming that the respective pairs are of shape and size similar enough for stereoscopic fusion, then B and D can be fused at the same time as A and C, provided that $\theta$ is approximately between about 30 and 90 minutes. Martin [65] has illustrated this relationship in a diagram as given in Fig. 61, in which the shaded area gives the range of values so that both pairs of points may be fused. Outside these limits the vision is separate.

The limits of accommodation of the human eye are given by two

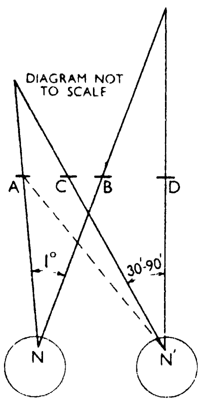



FIG. 60.

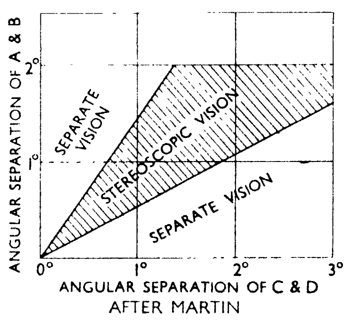



FIG. 61.

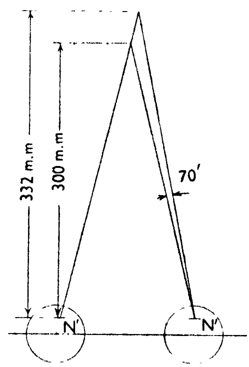



FIG. 62.

examples quoted by Hotine. [56] Investigators have indicated that objects between infinity and three metres can be fused clearly at the same time without any perceptible effort of accommodation, and Fourcade has shown that when objects at a distance of 300 mm. from the eyes are clearly focussed, then objects 332 mm. away can also be seen clearly. In each case the difference in convergence is about 70 minutes. It appears that within a field of about 2°, no greater range than this can be dealt with if all the stereoscopic picture is to be seen in fusion (Fig. 62). The range, however, can be extended somewhat, provided the focal limit is not exceeded. Thus, if there are two pairs of dots as in Fig. 63a, where the difference of convergence of the pair is somewhat greater than 70 minutes, it will not be possible to fuse them while they are in the same line or close together as in this case. If, however, they are separated as

in Fig. 63b, it will be possible, after fusing one pair, to get the other pair in fusion as well. The eyes are able to accommodate themselves provided that the *local* change is not greater than 70 minutes within a field of 2°.

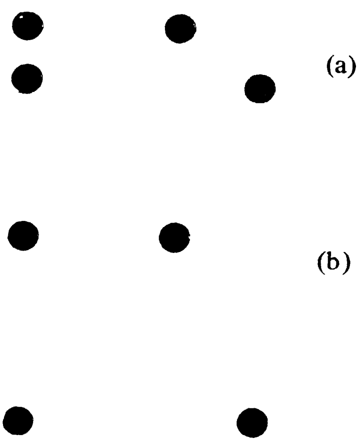


FIG. 63.

In order that stereoscopic fusion of two points may be effected, it is theoretically necessary that they should be in the same epipolar plane, i.e., the line joining them should be parallel to the epipolar axis, or eye-base. It is found in practice, however, that the human eye allows an appreciable margin of deviation from this condition, as may be seen if the pair of dots *a* and *a'* in Fig. 56 are turned slowly out of a common epipolar plane. It will be found possible to retain the fusion during a rotation of several degrees until the fused image becomes blurred and the points finally separate. This elasticity of stereoscopic vision is of value when orienting a pair of photographs by the correspondence method which will be described later.

It has already been shown how the impression of stereoscopic depth can be varied by altering the separation of a pair of points within the limit of convergence of the eyes.

The impression of stereoscopic relief obtained by a pair of human eyes from an aeroplane is negligible and the desired effect of relief is obtained by taking overlapping photographs at different positions of the aircraft. These photographs are placed in a stereoscope so that the common area may be fused stereoscopically in the same way as it would be by the eye of a giant whose eye-base is equal to the distance between the two positions of the aircraft.

The size of air photographs makes it impracticable to examine such common detail, except over very small portions, by direct fusion in the manner previously discussed, and it becomes necessary to provide some means of extending the human eye-base artificially.

The base line may be extended by parallel mirrors as shown in Fig. 64 giving an apparent reduction of separation of the photographs. Examination in a stereoscope of a stereo-pair of photographs taken with the eye-base increased by $\frac{B'}{b}=n$ (fig. 65) gives an appearance of accentuated relief.

Hotine[55] does not agree that the extension of the base increases the relief effect and remarks: "The relief effect is remarkable, but it is incorrect to describe it as exaggerated. If the length of the air base (usually about 3,000 feet*) is 15,000 times the length of the eye-base (usually about 2½ inches), we are really examining a true-to-scale relief model 15,000 times smaller than the original landscape and 15,000 times closer to the eyes than it was to the aeroplane." Actually, of course, the effect of presenting the view $n$ times nearer is to increase the perception of depth $n$ times.

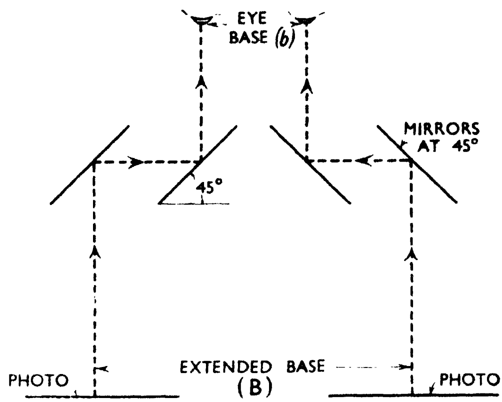



Fig. 64.

For field examination of pairs of air photographs some form of hand prism stereoscope extends the base length as in the case of prism binoculars. The principle is illustrated in Fig. 66, and is employed in such pocket stereoscopes as the Zeiss, or the Hilger. A simple stereoscope suitable for examining pairs of photographs can be made in a wooden frame using two convex lenses of the same focal length, and making use of the side curvature to obtain the stereoscopic base extension.

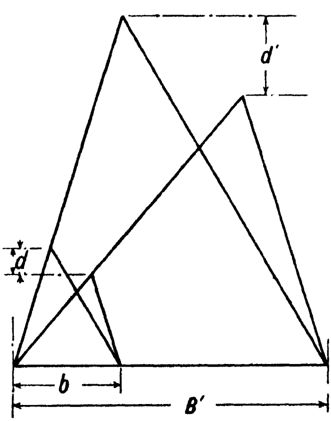



Fig. 65.

If magnification is applied to a pair of photographs from which a stereoscopic reconstruction is required, the angular ratios between pairs of points will be increased $m$ times in passing from the object space to the image space, where $m$ is the power of magnification. The result is that rays from the photographs to the observing point will be flattened, and the effect of stereoscopic relief will be reduced accordingly.

If a stereoscopic viewing apparatus has a magnification of $m$ times and the eye-base is extended $n$ times, the stereoscopic impression of depth

* For photography at 15,000 feet with a lens of focal length 7 inches.

will be changed from the natural value by $n/m$. Hence a suitable choice of ratio is very important in all instruments depending upon stereoscopic vision. These include field glasses and some types of range-finder as well as measuring stereoscopes.

*Stereoscopic Parallax.*

Everyone, no doubt, has noticed when travelling in a train how the telegraph poles along the track appear to rush backwards across the countryside. This effect is due to the varying rates of change of direction of the rays from the eyes to the poles and to points on the landscape. The effect is known as parallax.

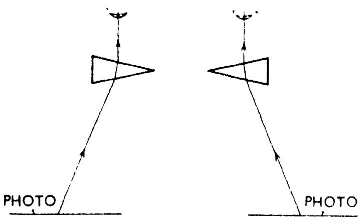


Fig. 66.

When points are sighted with both eyes, i.e., with binocular vision, their relative distances are estimated by the angle of convergence of the rays to them from the eyes. This is in addition to the help gained by previous knowledge of the approximate size of the objects. The effect of binocular parallax is seen if when looking through a window at a particular object, marks are made on the glass in line with each eye in turn, the other eye being closed. The relative distances of two objects from the observer's eyes will be indicated by the relative spacing of the dots of each pair. In the open this can be tested by standing with two points at considerably different distances in line with one eye. If this eye is shut and the other opened it will be found that the objects are no longer in line.

This effect of parallax has a very important bearing on the determination of the relative heights of points and in setting pairs of air photographs in their correct relative orientation.

At this stage it is convenient to consider the examination of a pair of air photographs as though each photograph has been taken exactly vertically. In the simpler methods, tilts are allowed for as far as possible in the plotting processes, while in some of the more exact methods photographs are set to their correct tilts in relation to the air-base before plotting. Rectification of photographs in printing with respect to ground control points is also frequently employed. In the simpler methods, the effect of inclination of the air-base with respect to a horizontal datum is also neglected.

Let S and S′ (Fig. 67) be two successive camera stations from which vertical photographs have been taken. These are spaced at a distance $B$ and each is at a height $H$ above a horizontal datum plane. The horizontal

planes of the photographic equivalent positives are at XX′ and YY′. When a viewing device is provided to reduce the length of the air-base *B* to that of the interocular distance *b*, a three-dimensional replica of the countryside in the common overlap will be seen to scale.

For convenience at this stage, the diagram is shown in the common principal plane of the two photographs. The photo principal points are

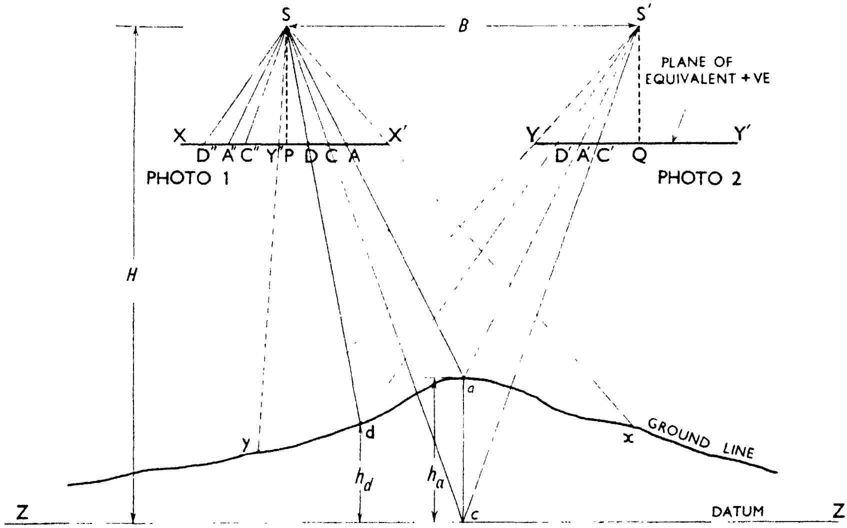



FIG. 67.

respectively P and Q. The datum line is ZZ′ and let it be assumed that the ground line is as shown. If S′Y is produced to meet the ground line in *y*, and S*y* is joined, cutting XX′ in Y″, then the extent of the left hand photograph (Photo 1) over which there is a stereoscopic overlap with the right-hand photograph is Y″X′, represented on the ground by *yx*. Similarly for the right-hand photograph (Photo 2).

Let it also be assumed, at the moment, that all points considered are in the plane of the diagram. A point *a* on the ground at height $h_a$ above the datum has images at A and A′, while *c* vertically below it on the datum has images at C and C′. From S draw lines parallel to S′A′ and S′C′ meeting the equivalent positive plane of Photo 1 at A″ and C″. Then CC″ and AA″ are measures of the separation of the pairs of images, and give the *absolute parallax* of *c* ($p_c$) and of *a* ($p_a$) respectively.

The actual spacing of the photographs or the reduction of the base in a stereoscope has no effect on the absolute parallax. For example, if the diagram is shown with half the spacing between the photographs,

the direction of the ray to any point is unaltered in direction from S and S′, so that when the parallel is drawn from S the amount of the absolute parallax remains unaltered. This fact is of great convenience in stereoscopic observation.

The focal length of camera lens ($f$) = SP = S′Q, and since the photographs were taken vertically, it may be seen from similar triangles that

$$\frac{CC''}{SP} = \frac{p_c}{f} = \frac{B}{H} \text{ so that } p_c = f \cdot \frac{B}{H}$$

$$\text{Similarly } p_a = \frac{f.B}{(H - h_a)}$$

This gives a general formula for the parallax of a point at height $h$ above the datum

$$\frac{f.B}{p} = \frac{f.B}{(H - h)} \qquad \text{(VI.1)}$$

Thus if the air-base $B$ and the flying height $H$ are known, measurement of the parallax of a point from a pair of photographs gives a measure of the height $h$ above the datum plane.

It is not possible to make routine measurements of height from air photographs without additional information. If the height of a is known as well as the values of $H$ and $B$, then the parallax $p_a$ can be calculated. Suppose that the height $h_d$ of point $d$ is required. Images of $d$ appear at D and D′, the absolute parallax being DD″. To determine $h_d$, the *difference of parallax* between $a$ and $d$ can be measured in a specially constructed stereoscope fitted with some form of floating mark.

If $\delta p$ = difference of parallax between $a$ and $d$ = DD″ − AA″, then

$$p_d = p_a + \delta p \text{ or } \delta p = p_d - p_a \qquad \text{(VI.2)}$$

Although this difference is usually measured stereoscopically, it can also be measured directly, provided the photographs are set suitably. It is considered, however, when the observer is experienced, that the stereoscopic method is more accurate.

Difference of parallax is measured in a direction parallel to the air-base. The principal point is taken as the basis of co-ordinates for a particular photograph, the $x$-ordinate representing the direction of measurement of parallax. In Fig. 68, P and Q are the principal points of two overlapping photographs as in Fig. 67. Corresponding images of these points on the other photographs are P′ and Q′. PQ′ and P′Q define the base-line direction, and when the photographs are set "in correspondence" for purposes of measurement, PQ′ and P′Q are in the same straight line. Let E, E′ and F, F′ be the corresponding images of two ground points $e$ and $f$.

Draw $jk$ through P normal to PQ′ and $lm$ through Q normal to QP′. Then $B'$, the spacing of $jk$ and $lm$, is the base-line for the stereoscopic model. This spacing is frequently a matter of convenience for stereoscopic setting. It will be seen that

$x_1 + x_2 = p_e$ and $x_3 + x_4 = p_f$.

If the points can be identified accurately on the photographs, these distances can be measured directly in a co-ordinate measuring machine with reference to the base line as the $x$-axis. The accuracy is much increased by stereoscopic identification of the points, and the *difference of parallax* is obtained by setting a floating mark to ground level at each point in turn, and noting the reading on the $x$-scale with the aid of a micrometer. If the height of one point is known, that of the other can be calculated.

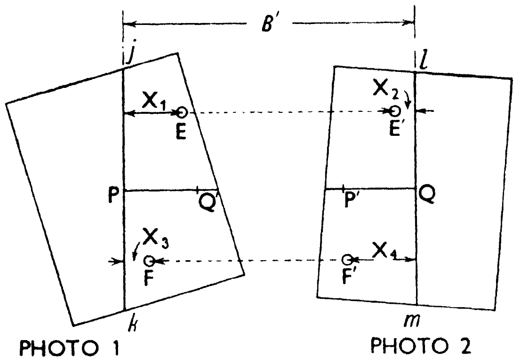



FIG. 68.

In some cases it is desirable to have a measure of the rate of change of parallax with respect to height; this is particularly useful when measurements are taken along a slope of considerable length.

From the parallax equation (VI.1), the rate of change of height with change of parallax is obtained by differentiation, when

$$\frac{dh}{dp} = \frac{-(H-h)^2}{f.B} \qquad \text{(VI.3)}$$

*General Principles of Stereoscopic Pairs and Stereoscopic Reconstruction.*

When two overlapping air photographs are taken, from which a stereoscopic picture is to be obtained, there are a number of variables which have an effect upon the reconstruction. It has been assumed up to the present that the photographs are taken truly vertically from a known height of aircraft, and that the air-base is of known length and truly horizontal. In the case of surveying from vertical photographs it is possible either to satisfy these conditions approximately or to make the necessary allowances within the limits of scale employed. This is called the normal case of stereoscopic pairs.

The general principles of stereoscopic reconstruction must now be briefly considered because the limitation of stereoscopy to vertical expo-

sures with a horizontal air-base cannot be maintained when precise measurements are required, and more elaborate stereoscopic instruments are then employed.

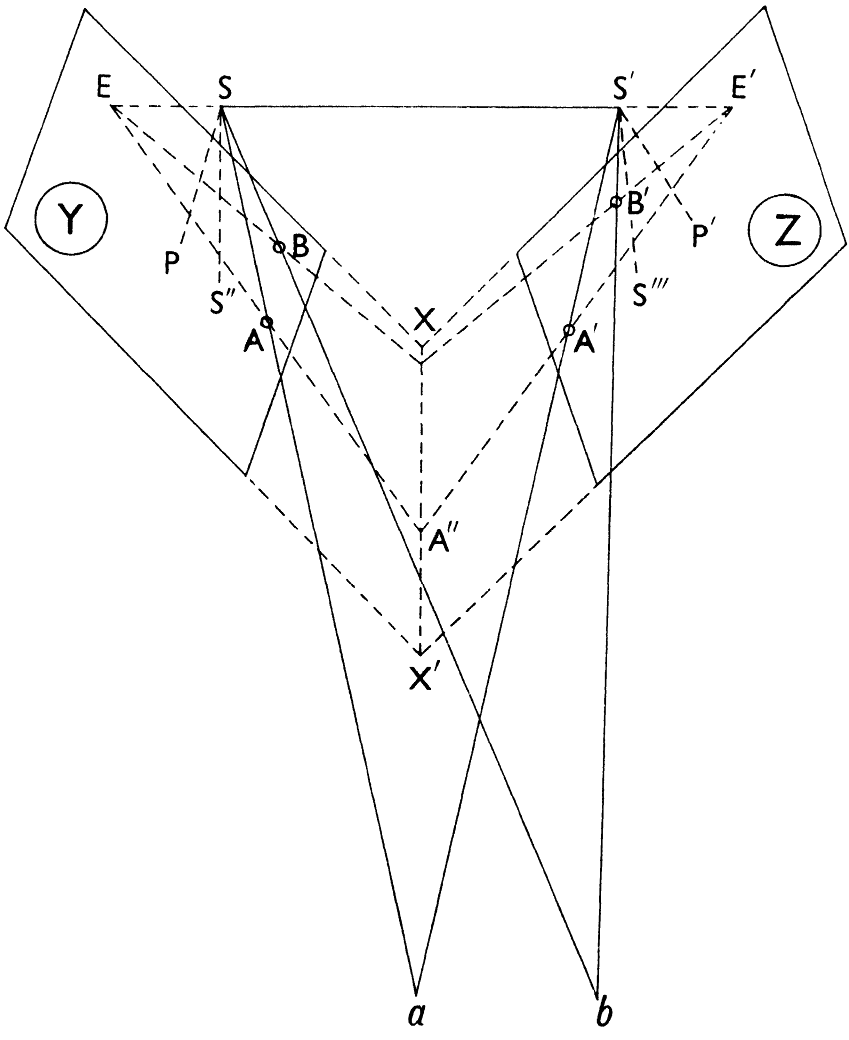



FIG. 69.

There are certain additional definitions which must be stated. In general, those laid down by the Air Survey Research Committee will be employed.

When a pair of photographs have both been exposed in the same plane which is parallel to the air-base, they are said to have been exposed in

*basal co-plane.* If, as in the cases previously considered, the air-base is considered horizontal, the exposure is said to be in *horizontal co-plane.*

Since epipolar planes contain the air-base, these planes are often called *basal planes.* Each point in a stereoscopic reconstruction has its own basal plane or epipolar plane, which contains the air-base, the ground point, and its two photographic images.

Theoretically, it is possible to take the photographs at any angular relationship, so that when they are placed in their correct relative positions as at exposure, a stereoscopic view can be seen. There are, of course, practical limits to this. For example, when the two views of an object are of very different size in two such photographs considerable difficulty may be experienced in fusing them.

In Fig. 69, S and S′ are the camera stations for two overlapping photographs represented by their equivalent positives Y and Z. The two views of a point *a* seen at A and A′ are called corresponding points, and it is clear that one condition of setting of a stereo-pair is that all pairs of corresponding points must be set into the appropriate epipolar, or basal plane. Thus, when photos Y and Z are set, A and A′ should be in the basal plane SS′*a*, and similarly B and B′ should be in the plane SS′*b*. All epipolar planes contain the base-line or epipolar axis, and this axis is produced to meet the picture planes in the *epipoles* E and E′.

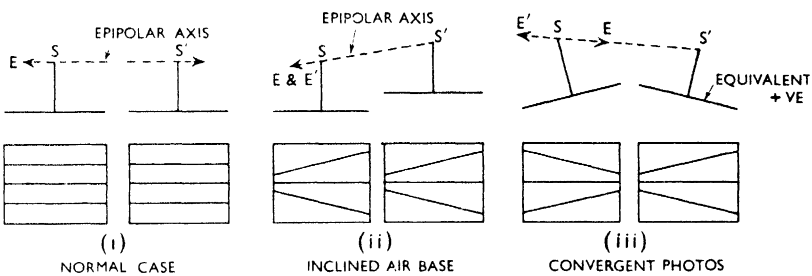



FIG. 70.

Let XX′ be the line of intersection of the two picture planes, then the *epipolar rays* EA and E′A′ when produced will meet XX′ at A″, and since E, E′, A and A′ are all in the epipolar plane of point *a*, A″ will also be in this plane; hence a relationship can be established between the epipolar rays of corresponding points. A condition of orientation of a pair of photographs is that the epipolar rays must be brought into the appropriate epipolar plane.

The relative directions of these rays will depend upon the relative positions of the photographs at exposure. Von Gruber [46] gives three main

cases for this relationship for epipolar rays as shown in Fig. 70. The ground plane is assumed horizontal. In case (i) where the exposure is in horizontal co-plane, the epipolar axis is parallel to the picture planes and the epipolar rays are lines parallel to the air-base. In (ii) where the air-base is inclined it is seen that the rays on each photograph are divergent in the same direction. In (iii) where the photographs are convergent, the epipolar rays also converge, while in the case of divergent photographs (as in Fig. 69), the rays would diverge to the common line XX′.

Until recently it was the custom to refer all stereo-pairs to the horizontal, derived, no doubt, from the procedure in setting pairs of ground photographs when the base can be precisely fixed and determined. Much complication was caused in the early days of air survey by the length of time required to set a pair of photographs in a stereoscope which allowed of tilt adjustments. These variations of air-base slope in unknown directions was the cause and it was left to Fourcade to enunciate his Correspondence Theory, [38] which has now been applied to observing and plotting machines other than the Fourcade Stereogoniometer.

If it be assumed that the photographs are set so that they are each in the correct position in relation to their perspective centres, there are movements about five different axes which can disturb the condition of "correspondence" of a pair of photographs.

These five movements are as follows: (i) and (ii), each photograph may be rotated in its plane about its principal axis (SP and S′P′ in Fig. 69).* (iii) One of the photographs may be rotated about the epipolar axis SS′; (iv) and (v). In order to establish a reference datum, photograph Y (Fig. 69) may be rotated about an axis in the epipolar plane SS″ perpendicular to SS′, and photograph Z about a similar axis S′S‴. Conversely, a pair of photographs may be set in their correct relationship by making these five adjustments. According to Hotine [55] this method of setting is more soundly based on theoretical principles than other methods and is comparatively simple to use practically. Other methods which involve setting with respect to the horizontal rather than to the air-base are, he considers, not so suitable because the setting movements cannot be so systematic and the operation takes longer.

Fourcade has proved that if a pair of photographs are set so that any five pairs of corresponding points are set "in correspondence," i.e., in their correct basal or epipolar planes, then all other pairs of corresponding points will be similarly set and the photographs are correctly set in relation to one another.

The principle is of great importance in the construction and operation

* It is assumed that the camera is in adjustment.

of the more elaborate stereoscopes and their attached plotting machines, and the whole matter is discussed more fully in Chapter X.

In the simpler type of stereoscopic reconstruction where the air-base and photographs are supposed to lie in horizontal planes, i.e., the "normal case" (see p. 169), there are only two degrees of freedom and the setting is correspondingly much simpler. It will, perhaps, be desirable to defer consideration of correspondence setting until methods of stereoscopic measurement have been described.

*Floating Marks.*

The principle of stereoscopic parallax and the appreciation of stereoscopic relief having been discussed, consideration can be given to the methods and applications of stereoscopic measurement.

The stereoscope, in which the stereo-pair under observation is brought to fusion, is fitted with a pair of glass plates which have a number of corresponding marks, or, alternatively, a single pair of marks which can be set anywhere in the overlapping area. Means are provided for measuring the $x$-ordinates of any point along the air-base and $y$-ordinates perpendicular to the air-base. The principal point is usually taken as the origin of co-ordinates.

The pairs of dots which have been previously used for illustration are a crude form of floating mark, and it has been seen how the change of spacing affects the stereoscopic depth. If two corresponding points are selected on a photograph, then a pair of marks used as a floating indicator, and with the same spacing, should appear at the same level. If they are more widely spaced they will appear below, and if less, above. Assuming that viewing is in a vertical direction, then if the floating marks are moved together sideways in a horizontal plane their fused images appear also to move in a horizontal plane, but when there is a relative movement there will be an apparent movement in depth.

Hotine remarks [56]:

"A good observer can place two entirely similar natural objects, possessing equal stereoscopic character to within an equality of depth represented by a difference of convergence as little as three or four seconds. Nothing like this result can be obtained between a floating mark and a photograph. . . . When the mark is so strong that it completely overpowers the photographic impression, no observations can be made with certainty."

There are a number of marks in use, the most common being the dot wedge or triangle, cross, and arrow as shown in Fig. 71. One or other of these types is used in the more elaborate machines, the most usual being a dot. Those with a point are particularly useful for contouring, since it is

easy to run the point round on the slope of a hill, so that it just touches the ground, with a constant parallax setting so that height above datum is constant. Three-dimensional marks such as cones and pyramids have been abandoned because they tend to upset the stereoscopic perception of the photographic overlap.

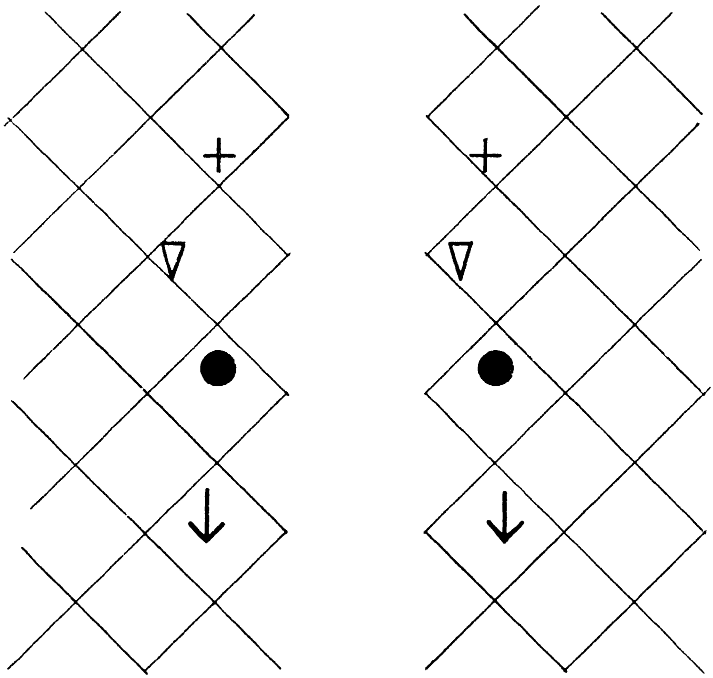


FIG. 71.

In all stereoscopes where it is required to set a pair of photographs in the correct relative orientation, it is desirable to have a floating mark which will enable this setting to be checked easily.

When a pair of photographs has been set in a stereoscope so that fusion is obtained it does not follow that the photographs are set in correspondence. The human eye has considerable tolerance once the stereoscopic view has been obtained and this can be tested by the angle through which a pair of photographs can be turned before fusion breaks down. Curiously enough, when a pair of photographs is set approximately and is in fusion, with a pair or pairs of floating marks above, it will be the floating marks which will appear out of correspondence. This is because the general form of a landscape is familiar to the eye while the floating mark is comparatively strange, and the eye (within limits, of

course) assumes that the landscape is right and the floating mark wrong. If, then, an adjustment can be made until the floating mark appears correct it will be found that the photograph, at least in that area, is in correct orientation. Consequently, instead of having floating marks which are boldly stereoscopic, they should be sufficiently weak so that the stereoscopic picture is not broken up. Suppose the two marks are as shown in Fig. 72a, the two vertical lines will fuse and the result should be as in Fig. 72b. If, however, the marks themselves are not in correspondence or the non-correspondence of the surrounding points throws it out apparently, as in Fig. 72c, the fusion will appear as in Fig. 72d. This may be tested by fusing the marks in Fig. 72c. Adjustment by turning one or both of the photographs in its own plane will put the floating mark back into correspondence. One vertical mark is made longer than the other to facilitate setting. This type of mark is very satisfactory and is being used to some extent for precision work in this country. An alternative mark is shown in Fig. 73.

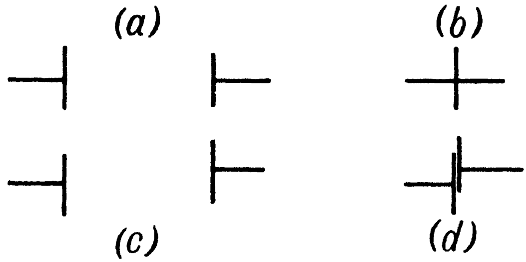



FIG. 72.

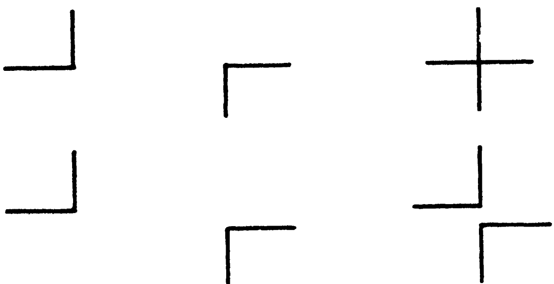


FIG. 73.

For work of a less precise nature, where speed is important, the "parallactic grid" has found considerable favour (Fig. 59).* This consists of a series of equally spaced lines ruled diagonally on a sheet of optically flat plate glass. A pair of these is set in the stereoscope, one over each photograph, and arranged with reference to a common base-line. This base-line is also used for setting the photographs, a condition being that the principal point of each photograph and the image of that on the next should be set in this base. When the stereoscopic view is seen, the effect is as in Fig. 59, the dots being considered as the landscape in this case. Here the parallactic grid fuses and appears to be considerably

* Parallax-bars (or stereometers as they were previously called)—see p. 178—are now generally used for simple parallax measurements.

above the dot. Where there are two grids on adjacent photographs these can be moved over the photographs so that when they fuse they appear to be at different levels. Hence if the spacing of the grids is measured it is possible to obtain a measure of difference of parallax between points. Here the dots and the grids are in correspondence. Suppose that the position is as before when the photographic detail is out of correspondence, while the grids, being fixed, should be in correspondence. The effect will be that the landscape will seem correct while the floating marks will appear out of correspondence.

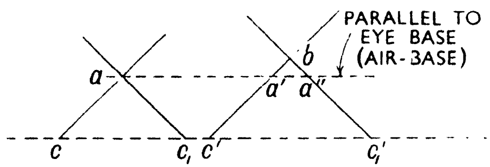



FIG. 74.

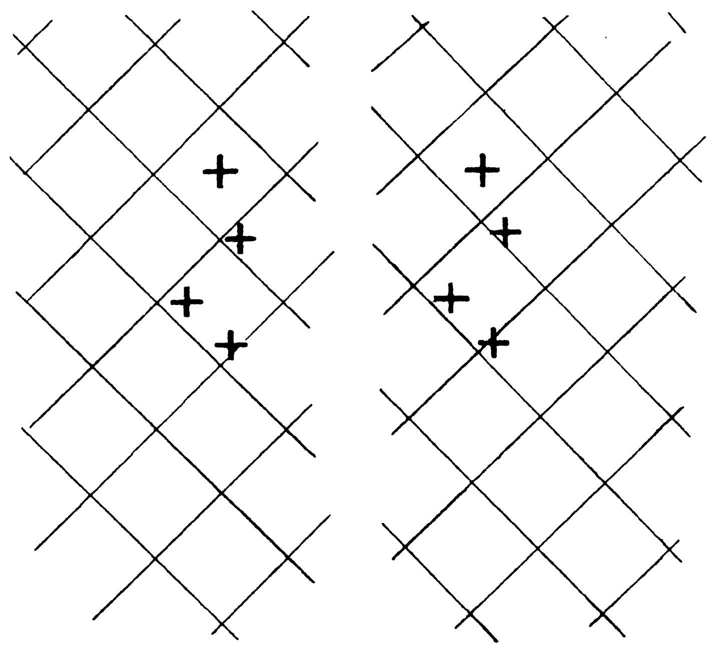


FIG. 75.

Suppose two grid crosses are out of correspondence (or appear to be) as in Fig. 74. The grid should, of course, appear to be horizontal, i.e., co-planar with the datum of the photographs, and this will be so if *ab*, the line joining the centres of the grid crosses, is parallel to the eye-base. In the case illustrated there will be *two* points on the other cross which

will fuse with $a$, namely $a'$ and $a''$, $aa''$ being parallel to the eye-base; and since $aa'$ is less than $aa''$, $a'$ appears to be above $a''$. Similarly $c$ will fuse with $c'$ and $c_1$ with $c_1'$, so that the point on the north-east line appears above that on the north-west line of the grid. Thus, although the lines of the grid crosses will fuse stereoscopically, they will appear to be at different


[Courtesy of Barr and Stroud, Ltd.

FIG. 76—TOPOGRAPHICAL STEREOSCOPE Z.D.4.

levels owing to the lack of correspondence. Suppose now that the right-hand cross is moved down without altering the direction of the lines of the grid so that $ab$ is now parallel to the eye-base. If this is done while observing stereoscopically, then the north-east and north-west lines will gradually become co-planar. If the process is continued so that $b$ is on the other side of the base-line, the north-east line will now appear to be below the other. This effect is illustrated in Fig. 75. After a little practice, the four pairs of crosses should be easily fused simultaneously. The absolute parallax of each pair is the same, so that they will appear on a horizontal plane. The grid is set a little out of correspondence and the north-east and north-west lines will appear as described above.

*Topographical Stereoscopes.*

Simple forms of measuring stereoscopes are used for surveying on medium scales from vertical, or nearly vertical, photographs. Fig. 76

shows a simple topographical stereoscope made by Messrs. Barr and Stroud, in which the air-base is reduced by means of parallel mirrors. The photographs, taken with about 60 per cent overlap are oriented so that fusion can be obtained. Each photograph is covered with a parallactic grid. The difference of level between two points can be estimated by adjusting the spacing of the grids so that they appear at ground level, first at one point, then at the other. Heights can only be determined approximately with this instrument owing to difficulties of exact measurement of grid movements and distortions due to tilts of the aircraft. Its chief use is for contouring when the levels of a number of points are known in the photograph.

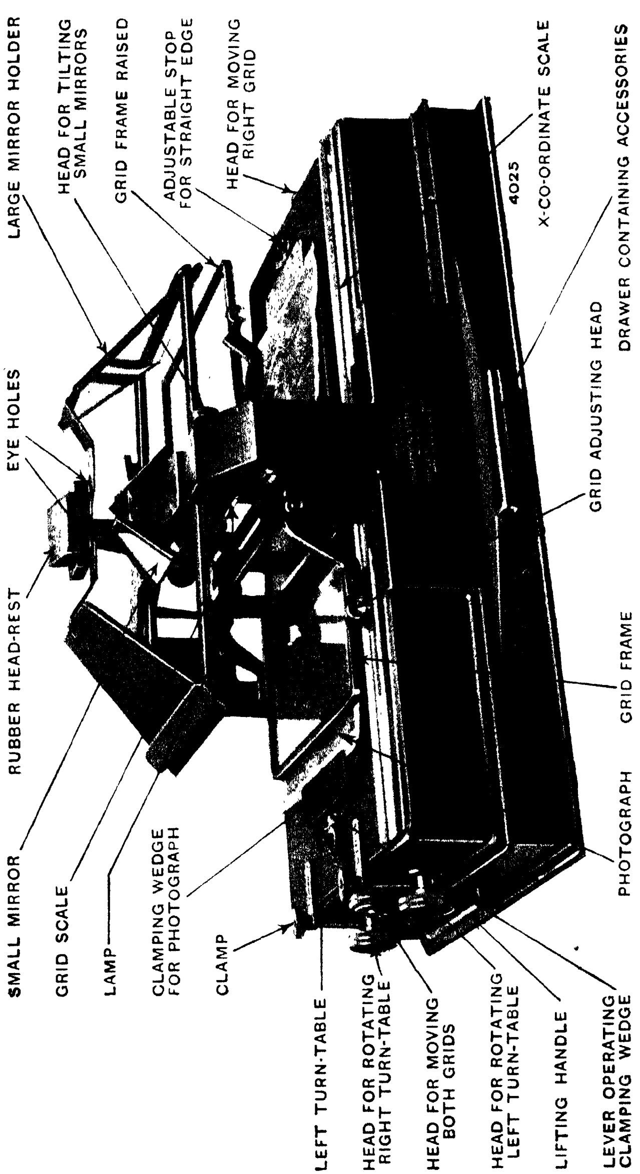
In the illustration one grid is shown lifted up so that contours can be pencilled in while observing stereoscopically.

It is possible to make approximate correspondence setting with the topographical stereoscope, although it is usual for the base-lines to be fixed on the photographs previously. The pair of photographs should be set so that the line joining the principal point to the image of the principal point of the adjacent photograph lies underneath the grid base-line, i.e., in Fig. 68, PQ and P′Q would coincide with the grid base. This can often be done by inspection where there is a lot of detail and the exact spot located. Sometimes, however, in country where detail is sparse, the only possible way is to orient by means of the correspondence method. This will be appreciably more accurate than inspection if the observer is skilled.

The general principle of setting in this way is to set the photographs approximately right by inspection with the principal points on the grid base-line. A grid cross is observed in the vicinity of the *left*-hand photo principal point and the *right-hand* photograph rotated until correspondence is obtained. The process is then repeated with a cross near the *right*-hand principal point and the *left-hand* photograph rotated until correspondence is obtained. Then the photographs will be set in correspondence.

A much better orientation of the photographs can be made with the Precision Topographical Stereoscope of Barr and Stroud (Fig. 77).* Over each photograph can be lowered a glass grid with rather finer etched diagonal lines than on the topographical stereoscope grid. The grid base-lines are in the same straight line and each photograph is mounted on a turn-table with its centre on the grid base-line. The photographs are set with their principal points exactly over the centre of the turn-table. Both grids can be moved together or the right-hand one can be moved in relation to the left-hand one. The photographs can be turned through small angles by means of the turn-tables, the grid directions remaining constant.

* Now superseded by the Cambridge Stereocomparator (p. 244), although a new type of grid stereoscope is under development.




[*Courtesy of Barr and Stroud, Ltd.*

FIG. 77—PRECISION TOPOGRAPHICAL STEREOSCOPE Z.D.15.

Whereas on the topographical stereoscope the parallax scale (as fixed by the movement of the grids) is graduated in millimetres with estimation to tenths of millimetres, on the precision topographical stereoscopes, the micrometer movement attached to the grid movements reads to one-hundredth of a millimetre. Hotine claims that a skilled stereoscopist can consistently obtain readings to 0·01 mm. under the best conditions, but others, including Crone of the Indian Survey, consider that 0·03 is a more likely figure.

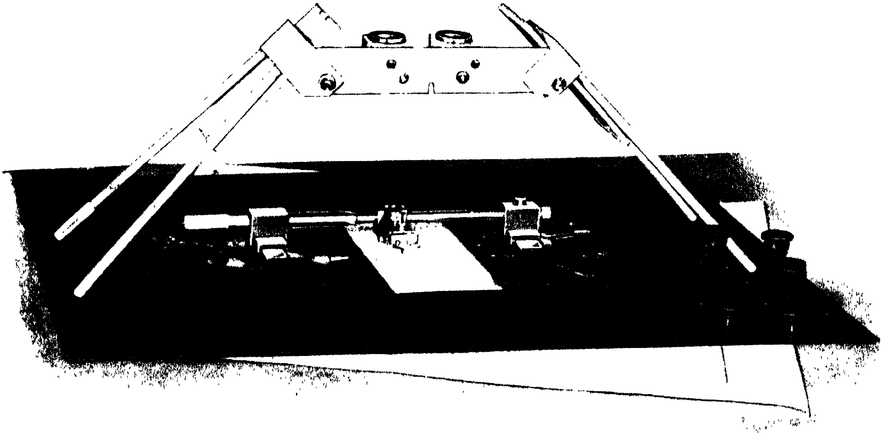
Several types of simple stereoscopes are available for the preliminary examination of air photographs.

The Zeiss Folding Mirror Stereoscope (Fig. 78) is an instrument of this type, and it is made to fold up for storage and transport. A similar instrument is the de Koningh Stereoscope (Fig. 79).

These instruments can be used for examination and interpretation of stereo-pairs of air photographs, with or without magnification. In each case a stereometer may be used for parallax measurement and during certain stages of plotting. In both cases a pair of photographs is placed on a flat surface under the stereoscope and moved about until an impression of stereoscopic relief is obtained. The principal points of the photographs will have been previously marked, and, by means of a straight edge, the photographs may be base-lined by inspection. The photographs are then fastened down, preferably with adhesive cellophane tape.

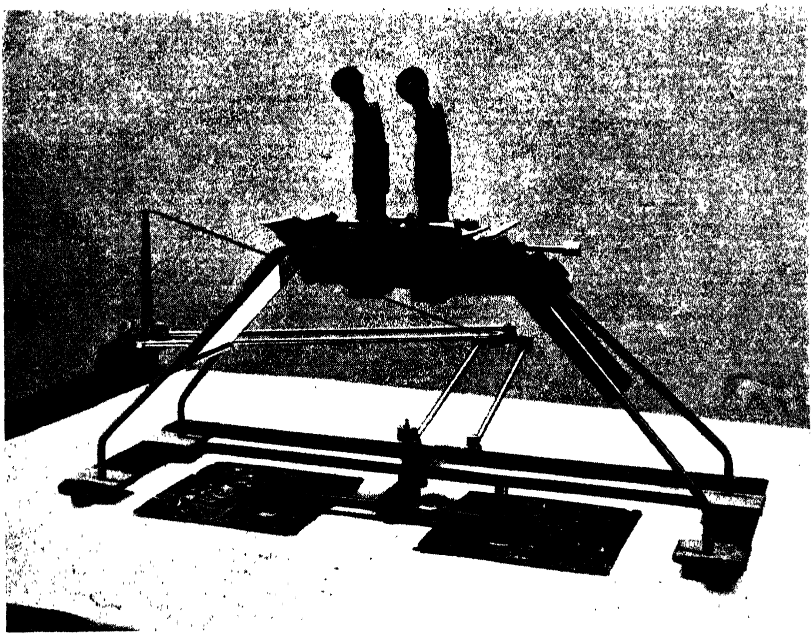
The stereometer may now be used for measurement of difference of parallax between points of detail. Floating marks are carried on small flat glass plates, one over each photograph, so that it is possible to obtain the effect of the mark floating in space. Two floating marks are provided, one a small dot and the other of different shape, and either of these may be used according to convenience or suitability. A micrometer adjustment is provided at one end so that the spacing of the marks may be altered, and when each half-floating mark covers an image of the point of detail the fused mark should appear at ground level at that point. If a reading is taken of the parallax micrometer and the stereometer is now moved and adjusted so that the mark appears at ground level at another point of detail, the difference of parallax may be read off from the micrometer and the difference of level between the two points computed. In order that the stereometer may be moved easily while maintaining a direction parallel to the base-line of the pair of photographs, a parallel-guidance mechanism is often fitted. In Fig. 79, such a mechanism is shown fitted to the de Koningh instrument and one of similar type is available for use with the Zeiss instrument.

It will be noticed that the mechanism is quite simple and merely



[Courtesy of Carl Zeiss (London), Ltd.

FIG. 78 FOLDING MIRROR STEREOSCOPE AND STEREOMETER.



[Courtesy of G. de Koningh, Arnhem, Holland.

FIG. 79—STEREOSCOPE AND STEREOMETER WITH PARALLEL GUIDANCE.

ensures that when the stereometer is moved its direction remains parallel to that in which it is set, i.e., parallel to the photographic base-lines. In this case the stereoscope is also connected to this parallel motion so that when magnification is used the eye-pieces are exactly over that part of the area which is being examined.

The use of these instruments in connection with methods of plotting and contouring will be described in later chapters.

There are also some of greater precision than are described here. In the precision instruments a plotting mechanism is frequently fitted and it is possible also to make measurements enabling aerial or three-dimensional co-ordinates of points to be calculated. The single type of floating mark is favoured in such cases.

The conditions of photography and of stereoscopic observation are likely to have an adverse effect on the accuracy of parallax measurements. For instance, the direction of lighting when observing should be the same as at exposure. The mental or physical state of an observer is likely to affect acuity of stereoscopic observation, and the probable accuracy of individual measurements is noticeably reduced if the observer has a slight cold in the head! A pair of photographs having uncorrected tilts will resolve stereoscopically with respect to an inclined plane, so that "false parallax" will appear in the measurements. Again, any variation in the intensity of sunlight between exposures may introduce "shadow parallax." Photographic blemishes, or imperfect materials, may cause errors, as also may incorrect development. Moreover, photographic materials may distort from the image dimensions at exposure and will also cause errors. Quantitative effects of errors are discussed in Chapter VIII.

*Practice in Stereoscopic Observation.*

To those making stereoscopic observations for the first time, it may appear that these impose a strain on the eyes, and headaches may be caused if the period of observation is too long. This is merely a result of exercising muscles which are not frequently called into play, because in ordinary observation we usually judge relative distances by our knowledge of size so that the stereoscopic powers are not so much used. Also, the conscious effort of divorcing accommodation from convergence contributes. It is quite a common practice among oculists, particularly in the United States, to prescribe stereoscopic observations in certain cases of eye defects.

The first stages are to practise with a variety of dots until they can be fused at will. Then a series of test cards are used with a number of objects having small parallax differences, and the observer must place them in

their right order of depth. Test cards of this type will be found at the back of Professional Papers No. 8 of the Air Survey Committee [86], and others may be obtained from Zeiss.

No really effective stereoscopic observations can be made with a measuring stereoscope until differences of the order of 0·1 mm. can be appreciated on these cards.

When stereoscopic observations are being made it is usual for the observer to get his eye in by playing "the dot game," as it is called, for ten minutes or so before starting measurement.

The reader may test his stereoscopic ability by referring to Fig. 71. Here right and left parallactic grids are represented, which, when fused, will give a horizontal plane of reference. First fuse each of the pairs of marks and note whether the fused image is above or below the grid. Repeat this for the other pairs of points, and finally decide the order of stereoscopic depth with respect to the grid. The correct order is given at the foot of the page. It should be noted that the stereoscopic differences here are considerable, and it is necessary to appreciate much smaller differences of parallax as given on the test cards before one can hope to make accurate measurements by stereoscopic observation.

Correct order of stereoscopic depth (Fig. 71): Nearest—cross—grid and dot at same level—wedge—arrow—Farthest.

CHAPTER VII

# THE RADIAL-LINE METHOD OF PLOTTING FROM VERTICAL PHOTOGRAPHS, AND ITS APPLICATION TO GRAPHICAL METHODS OF PLOTTING MEDIUM-SCALE MAPS

## The Fundamental Survey Problem

If mapping is attempted from single photographs it is necessary to know the position of the photograph in space and, unless the photograph is taken exactly vertically, the plan positions as plotted back into a horizontal plane will be distorted due to height variations. Thus mapping from single photographs has two possible applications. Firstly, preparation of plans from very nearly vertical large-scale photographs with little height variation, which can be rectified optically into a horizontal plane with reference to ground control points. Secondly, the preparation of small-scale maps from obliques in country where there are no variations of ground level sufficient to produce appreciable distortions at the plotting scale.

It is, however, generally agreed that the most satisfactory results in almost every case are obtained by making use of the principle of stereoscopic pairs. Not only does this make the interpretation of detail more sure, but it is also possible in many instances to determine and plot the topographical features adequately, i.e. heights as well as plan. There is little doubt that the advance of air survey as a scientific method has been largely due to the employment of stereoscopy.

Fundamentally, the problem is to find from the photographs, with the aid of a limited amount of ground measurement, that information which would be obtained on the ground with the theodolite, level, prismatic compass or clinometer.

Salt [85] states the general problem very clearly: "With a knowledge of the orientation of the camera axis at exposure, we have a pictorial record of angles in space to all objects in the field of view such as might have been obtained by an observer with a theodolite. Suppose now, that the exact position of the camera station is known and that another photograph is taken from a second known station covering much the same field of view from a different aspect. From the data provided by the two photographs the positions in space of all points in the common field of view can be

determined by two ray intersections. Various methods of applying this principle have been evolved ranging from actual determination of angles, followed by computation, to completely automatic plotting."

For medium and large-scale surveys there are two distinct paths open: improvement of flying and photography, so that comparatively simple methods can be used with photographs approximately vertical; or adoption of elaborate stereoscopic plotting machines. While both methods of approach have their applications, it may be as well to record the remarks of General McLeod (one of the pioneers of the subject) in a discussion on a paper by Cochran-Patrick [20] that the military problem approximated to the engineering problem:

"The alternatives when mapping contours boiled down to a choice between the Arundel Method and the continental plotting machine. These machines were extremely expensive and complicated; apart from this, one of the practical drawbacks was that engineering projects were always wanted in a hurry. The engineer always wanted his information at the last moment; he had to get on with his job and the time allowed to the surveyor was always short. The drawback to the continental plotting machine was that it formed a sort of bottle-neck through which one could not expedite the output in any way. The Arundel Method, on the other hand, required only cheap compact instruments which would stand on a table and once the preliminary lay-out was done and adjusted the work could be taken and divided up among a number of men, each with his own instrument. The plotting unit was not a single photograph but a stereoscopic pair, from which one man could plot all the details and do the contouring."

Also it was pointed out that the continental instruments "are three-dimensional optical instruments of precision, and, provided that sufficient ground control can be supplied on which to orient the photographs, and that the value of the survey is sufficient to cover the considerable costs and time required for the adjustment and final plotting, they can give accuracy suitable for even large-scale work."

It should also be noted that increased accuracy can be attained by the substitution of computation for certain of the graphical solutions.

In this chapter it is proposed to describe the simple method known as the Arundel Method and other forms of the radial-line method. The former method was developed by the Air Survey Research Committee primarily for military needs, the chosen scale being 1/25,000 and the contour interval ten metres. It has been shown to be suitable in many cases for larger scales than this, particularly for planimetry. With recent advances its scope has been enlarged, and further, by using ultra wide-angle lenses,

much smaller scales may be photographed directly than was formerly possible. The great merit of the Arundel Method is its simplicity.

Perhaps the most satisfactory approach is to describe first the method as applied to scales of the order of 1/25,000, and to deal with its other applications in later chapters.

### The Arundel Method

Consideration has been given in Chapter V to the scope of the radical assumption on vertical air photographs; namely, that the principal point of the photograph, within certain well-defined limits, may be taken as the "instrument station" for the measurement of angles to all points on the photograph.

Although the principle was originally enunciated by Scheimpflug in Austria, it was first put into a simple and workable practical form by Major J. W. Bagley of the United States Army.

Such a method where little ground is necessary and speed of production high was recognized to be of value from a military point of view, and its improvement and development in this country has been very largely due to the efforts of Major M. Hotine, R.E.

The standard of military map production from air photographs, both in plan and contours, has been brought to a high standard of efficiency on a scale of 1/25,000.

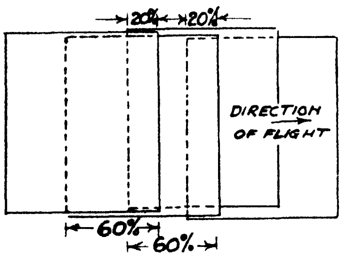



Fig. 80.

The principle of plotting is really the same as that of ordinary plane table surveying, except that rays are drawn on the photographs from the photo principal point to the image of the ground point considered, instead of being drawn from the plane-table station towards the ground point along the sight-line of the alidade.

Photographs taken in strips (Fig. 80) aim at 60 per cent overlap, allowing a margin for reduction of effective overlap due to opposing tilts to a minimum of 50 per cent, which gives 10 per cent common overlap to three neighbouring photographs. Unless this common overlap is maintained, the method breaks down, or at best its efficacy is much reduced.

A lateral overlap of approximately 30 per cent is often aimed at in order to connect neighbouring strips.

## Preparation of Plans

*Geometrical Basis.*

Let 1, 2, 3, 4, 5, 6 (Fig. 81) represent the principal points of a strip of photographs. The numbered arrows indicate the directions from these points to the principal points of adjacent photographs. A series of points A, B, C, D, E, is chosen so that each appears in the common overlap of three photographs, and the lettered rays indicate the direction of these points in relation to the base-lines. These angles are equal to the corresponding angles on the ground, and provided that one side of one triangle

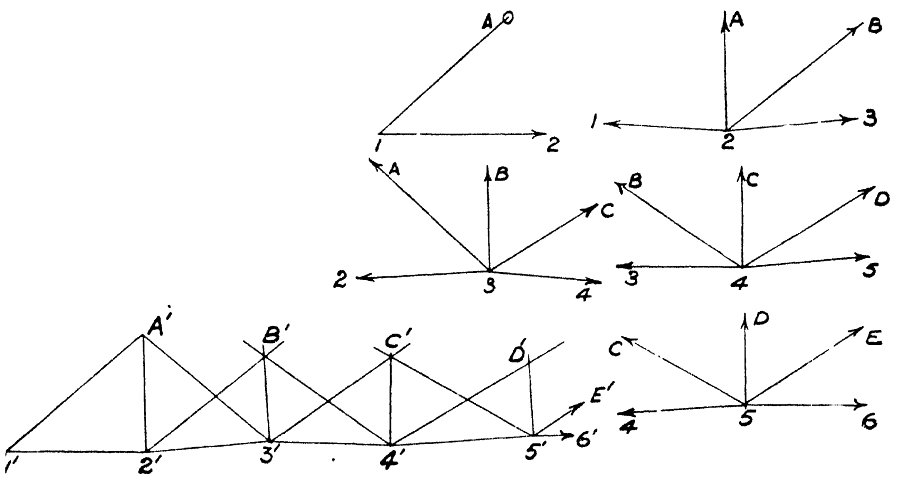



Fig. 81—Geometrical Basis of Arundel Method.

is known, all the points can be plotted in their correct relationship to the scale of the known line. Thus if the distance 1A is known, the lines can be traced off from each photograph in turn, making sure in each case that base-lines are correctly oriented and that the lettered rays pass accurately through previously intersected points. To obtain the exact position of the tracing over any photograph, it is necessary to slide the tracing along the base-line. The accuracy can be checked by the precision with which the three rays to points such as B and C (B′ and C′) all pass through the same point. Alternatively the triangulation could be drawn by setting off 1A and plotting the angles which have been measured on the photographs.

*Base-lining the Photographs.*

Frequently the collimating marks are short lines at the edges of the photograph, and the principal point must be drawn in on the print. This is done by setting a straight-edge along the marks and ruling in very carefully a finely drawn cross about a quarter of an inch long.

Until recently a matt photographic paper had to be used because ordinary glossy paper will not take ink or pencil. Matt paper loses much of the clarity of detail of the glossy, and there is now available a special paper of fairly glossy surface and remarkably free from distortions upon which fine lines may be drawn in pencil. It is customary in the best work to draw this cross in red ink, but some plotters prefer to use pencil even though it is difficult to see pencil lines at later stages of the plotting.

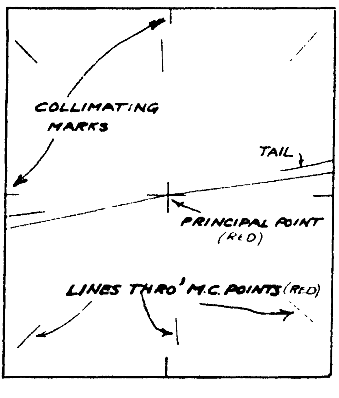



FIG. 82.—BASE-LINED PHOTOGRAPH.

All the principal points along the strip are similarly marked. Next, the base-lines are drawn on each photograph through the images of principal points of adjacent photographs (see Fig. 82). It is essential for this process that each principal point should appear in three photographs. Otherwise the evils of the "short overlap" can only be overcome at great trouble and loss of accuracy. Similar difficulties arise if a principal point is obscured by cloud.

There are two methods of "base-lining":

(*a*) *By inspection of detail.*—When the forward and back principal points have been identified *very carefully and accurately* on these photographs, the two lines representing the common base-line, when drawn, should pass through the same points of detail. Accuracy at this stage is of paramount importance and the lines are drawn as finely as possible in *red ink.* This method is satisfactory where there is sufficient detail to identify the principal points precisely. If, however, the topography of the district is of the type sometimes called "miles and miles of damn-all!" such identification cannot be used and a stereoscopic method becomes necessary.

(*b*) *By stereoscopic correspondence setting.*—As has been stated stereoscopic setting of two photographs in their correct relationship is more precise than the inspection method, provided that the observer is sufficiently skilled. The Precision Topographical Stereoscope has been shown

in Fig. 77 and briefly described. The parallactic grids which cover each photograph have been shown in Fig. 71.

If the observer now looks through the stereoscope, he will see the relief of the ground, but the north-east and north-west lines will probably appear to be at different levels.

The reason for this is that the photographs are not properly oriented and although the grids are correctly set, the human eye knows what to expect in the ground view and suits itself to the photograph positions, thus causing a *want of correspondence* in the grid lines. By rotating the turn-tables so that the grid lines appear co-planar, a very precise setting can be made.

The right-hand grid can be moved longitudinally in relation to the left-hand one and the two can be moved together and by altering the spacing and position of the grids, their stereoscopic image can be made to "touch the ground" at any point. First a "grid cross" is set over the centre of each turn-table, and a photograph placed under each grid, with the principal point exactly under the central grid cross. The photographs are set by inspection so that the line joining the principal points passes approximately through the same points of detail. The base-line direction is established accurately by making an adjustment to each photograph as follows: A convenient point of detail is selected in the vicinity of the principal point of the left-hand photograph, and, on looking through the stereoscope, it is probable that the view of the ground will be stereoscopic, but the grid lines will appear to be at different levels. If the right-hand turn-table is adjusted slightly, carrying the photograph with it, it will be possible to make the grid lines appear co-planar in the neighbourhood of the selected point. Hence the right-hand photograph is set. Next, in order to set the left-hand photograph, a suitable point of detail is selected in the vicinity of the right-hand principal point and the left-hand photograph is adjusted in a similar manner. It is important to realize that by selecting a point *very near* the principal point on one photograph, all the base-line adjustment for that point, for practical purposes, is on the other photograph. The slightest of movements of the turn-table will change over the relative heights of the lines and after some practice great precision can be reached. The base-lines are drawn in fine red lines, with the straight-edge provided, the side principal points not being marked, as it is only their direction which is required.

*The Principal Point Traverse.*

(*a*) *Minor Control Points.*—In order that a graphical triangulation with sufficient checks can be carried forward as shown in Fig. 81, it is neces-

sary to choose two accurately identifiable *minor control points* in the common overlap of each three photographs. These are marked on each photograph with a fine pricker and ringed with coloured chalk. Fig. 83 shows diagrams of a strip of photographs starting with No. 1. Short rays about half an inch long are drawn in fine red lines radial from the principal point through the control points (see also Fig. 82). It is desirable to identify such points by stereoscopic examination of the pairs, because the precision at this stage influences the accuracy of all the subsequent work.

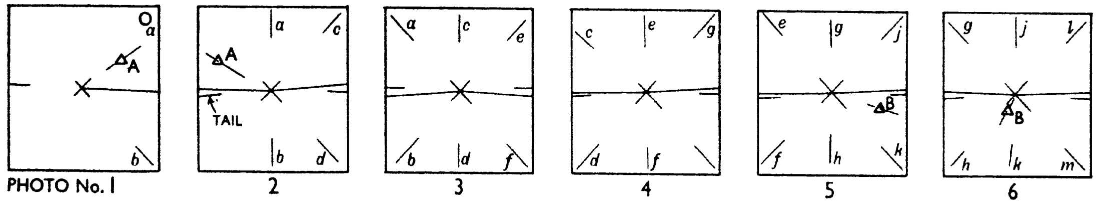
Short rays are also drawn through the images of A and B, two ground-control points which each appear on two photographs.

It is not possible to plot the relative position of the principal points to any known scale since the height of any exposure will not be known precisely, and the heights along a strip may vary by one hundred feet or so. Fortunately a variation of height means only a variation of scale, which can be eliminated during the process of plotting. The photographs will, however, have been taken approximately to the right scale and by stereoscopic examination a *scale point* such as *a*, preferably at one end of the strip, will be chosen in the common overlap and at about the average height along the strip.

(*b*) *Minor Control Plot* (M.C.P.).—The graphical triangulation is now plotted on tracing paper, or preferably upon the non-stretchable tracing material "kodatrace" which is much more transparent.

The kodatrace is placed over Photo 1, and the scale point *a* and principal point are pricked through. The base-line and short ray through *b* are drawn either with hard pencil or blue ink in the fine lines (Fig. 84a). The tracing is now placed over Photo 2 with the traced base-line exactly over that on the photograph. By sliding the tracing along in this direction the ray through *a* on the photograph may be made to pass through its plotted position on the kodatrace. The tracing is fastened down, the principal point and base-line 2-3 marked, and rays drawn through *b*, *c* and *d* (Fig. 84b). *Blue* lines drawn on the kodatrace give a precise setting by showing a *mauve* line when they are exactly over the *red* lines on the photographs. Actually a drawing-pen needs special setting for drawing on kodatrace, and many (including the writer) prefer to do this part of the work in hard fine pencil lines. The base-lines are drawn somewhat longer than necessary so that by using "tails" drawn on the photographs the precision is improved.

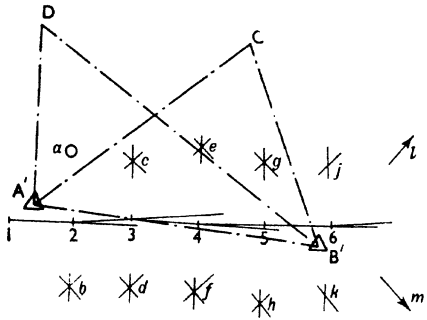
The process is repeated with Photo 3, and a test of the accuracy will be the precision with which the third ray passes through the intersected position of *b* fixed from Photos 1 and 2 (Fig. 84*c*). Again when the rays from Photo 4 are traced, there should be three rays accurately passing




STRIP OF PHOTOGRAPHS - BASE-LINED AND MINOR CONTROL POINTS FIXED

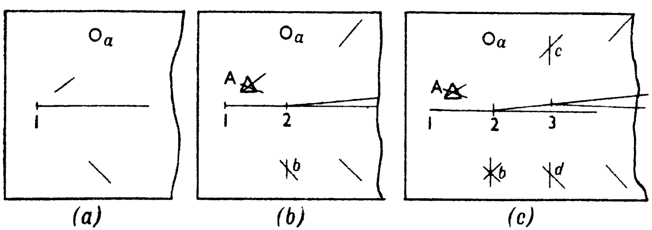
FIG. 83.

FIG. 85.

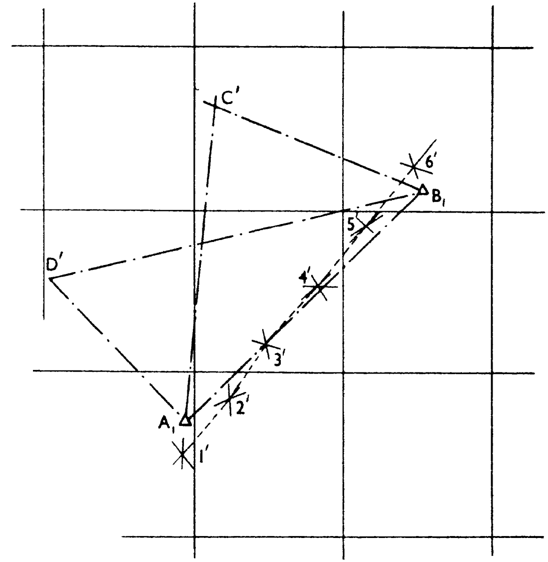


MINOR CONTROL PLOT

FIG. 84.




COMMENCEMENT OF MINOR CONTROL PLOT



RECTIFIED PRINCIPAL POINT TRAVERSE

FIG. 86.

through the previously plotted positions of *c* and *d* and so on. Unless these intersections are accurate considerable error may be carried forward. Random tilts in the aircraft may cause small "triangles of error" which will have to be adjusted. This difficulty should not arise if tilts are small.

The minor control plot is produced as shown in Fig. 85, the principal points 1, 2, 3, 4, 5, 6 being plotted in their correct relative positions the scale being unknown as yet. The amount of height distortion of any point may be seen by positioning the plot over the corresponding photograph.

*The Rectified Principal Point Traverse.*

Points A and B (Fig. 83) are ground control points, i.e., their relative positions are known on the ground usually from co-ordinates. Fig. 86 shows the grid upon which these points (indicated by $A_1$ and $B_1$) are plotted to the required scale. These points are each intersected from two photographs while making the minor control plot, giving $A'$ and $B'$, Fig. 85. Then the scale of the M.C.P. is given by the ratio of $A'B'$ to $A_1B_1$. The rectified principal points may be plotted on the grid by using another simple triangulation. Choose any convenient point C on the M.C.P. and join $A'$ and $B'$ to C. Place the M.C.P. over $A_1B_1$ so that $A'$ is on $A_1$; $A'B'$ along $A_1B_1$ and prick through C. Repeat with $B'$ on $B_1$ and $B'A'$ along $B_1A_1$. Join up the two rays and the intersection $C'$ determines triangle $A_1B_1C'$, which is similar to $A'B'C$. The process is nothing more than a method of transferring equal triangles.

A similar process is now used for plotting the principal points. Place $A'$ on $A_1$ and $A'B'$ with $A'C$ along $A_1B_1$ and $A_1C'$ respectively, and prick through the points 1, 2, 3, 4, 5, 6—repeat with $B'$ on $B_1$, C on $C'$ and draw the rays from $A_1$, $B_1$, and $C'$. Each principal point should be fixed by the precise intersection of three rays. If the rays to any point are badly conditioned any other convenient point such as D (Fig. 85) may be used instead of C. The principal point traverse $1'$ $2'$ $3'$ $4'$ $5'$ $6'$ is now plotted in its correct position on the master grid.

In plotting to engineering scales there will not usually be a shortage of ground control. In fact the amount available may prove embarrassing. If, however, it should happen that there are not two ground-control points in a strip, the rectification must be completed by a process such as that described by Salt, [86] in the Professional Papers of the Air Survey Committee, No. 8. Points of detail common to two adjacent strips, when used as minor control points, afford a useful check when plotting the second strip.

*Adjustment of Plotting to two or more Strips.*

Suppose a case arises in which two triangulation stations are not available on each strip. Fig. 87 shows two adjacent overlapping strips where trig. station X appears on strip 1 and trig. station Y on strip 2. Before making the minor control plots, points $p$ and $q$ near the ends of the strips and in the common lateral overlap are chosen. Point $s$ is also selected on strip 2 in approximately the position shown. When the minor control plots are made these points and the trig. points are very carefully intersected.

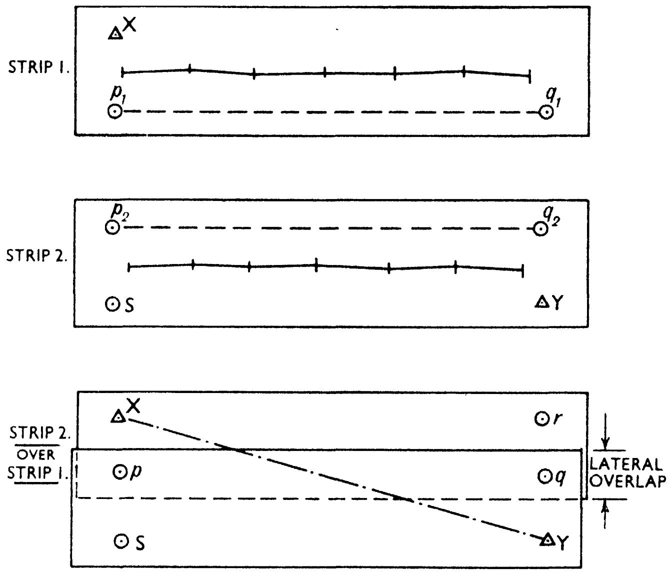



FIG. 87.

It is not possible yet to adjust these plots to any known scale and strip 2 must first be brought to the unknown scale of strip 1. Points X, $p_1$, $q_1$ are traced off on to a piece of kodatrace large enough to cover also the area of strip 2. The kodatrace is then placed over strip 2 with $p_1$ over $p_2$ and $p_1q_1$ along $p_2q_2$, and rays drawn through $s$ and Y from $p_1$. The process is repeated from $q_1$, the intersections fixing $s$ and Y to the scale of strip 1. It is now possible by placing this tracing sheet over the master grid on which X and Y are plotted to a known scale, to intersect on it the correct positions of $p$, $q$ and $s$. This is done as before by placing the kodatrace X over the master grid X with a common direction for XY and drawing short rays through $p$, $q$, and $s$; and repeating from Y. Each

minor control plot can now be adjusted independently to the master grid scale and the principal points plotted on it.

It might happen that no such point as Y appeared on the second strip. In such a case another auxiliary point in about the same position as Y would have to be chosen and the process continued until a known point is reached on another strip. The extension of plotting may introduce certain accumulated plotting errors, and some proportional adjustment may be necessary as described by Salt. [86]

Similarly any distribution of trig. points may be dealt with.

It is, however, preferable, when using graphical methods to limit to about two, the number of strips which must be plotted in this manner. Two strips form a suitable block, and unless it is unavoidable, this number should not be exceeded.

*Detail Plotting.*

Next follows the detail plotting. About ten distributed points, which can be identified on three photographs, are selected per photograph, and these are marked with red dots and numbered. The minor control points are frequently among them. Where there is an adjacent strip, points in the lateral overlap appear in six photographs. A tracing is made in blue on kodatrace of the rectified principal point traverse and the surrounding grid lines. The selected points of detail are now intersected on the kodatrace by three rays as before, their exact positions being marked in blue. Points in the overlap between strips will be intersected from six principal points. This plotting process is very similar to the original M.C.P. except that the scale is known.

If these intersections prove accurate, additional points required for completing the detail need be intersected from only two photographs. These additional points are spaced at three-quarters to one inch apart on the photograph and are marked and intersected as before. The final stage is quite simple, provided that the photo and plan scales are approximately the same, and consists of tracing the detail from the photographs, using the intersected points as local control. Only very slight adjustments will usually be necessary, and where an appreciable discrepancy occurs it will generally be worth while to intersect one or more extra points. Careful stereoscopic interpretation of prints is essential during this process. The detail is finally linked up in black, and the plan produced either by direct printing or by reproduction, when the blue lines will not show up. Sometimes the plan is transferred by direct tracing with the aid of transmitted light.

Where the scale is small and certain factors tending to inaccuracy

may be ignored, the tracing attachments provided with such instruments as the Zeiss and de Koningh Stereometers (Figs. 78 and 79) may be profitably employed for plotting details and contours (see Chapters VIII and XI).

*Effect of Hilly Country.*

If the ground height is sufficiently variable (when considered in relation to the tilt and focal length of the lens) to cause an error which cannot be neglected, some means of correcting or estimating the tilt must be provided. It is possible in some cases to get over the difficulty by flying adjacent strips at different heights.

The effect of height variations upon the accuracy of the radial assumption has been discussed in Chapter V, so that the limitations in any case can be readily determined. Obviously since the height distortions depend upon the ratio $h/H$, flying height may be varied to improve conditions.

Attempts have been made to determine the plumb point on a tilted photograph instrumentally at the moment of exposure. The spirit-level which is seen in the instrument panel of the air survey camera can only be a very rough guide, because random accelerations of the aircraft will cause the bubble to move off centre.

For surveys on engineering scales it will probably be desirable on those photographs where height distortions are appreciable, to provide sufficient ground control, i.e., four points per photograph, so that rectified prints can be produced.

Alternatively, the plumb point can be established on the photographs by a tilt-finder; or graphically, with reference to four ground points. The error introduced by assuming that tilt distortions are radial from either the principal point or the plump point is about the same (see pp. 150 and 152).

The radial method may then be used in plotting a plumb point traverse. The tendency, however, has been to improve flying so that it is unnecessary to use the plumb point for medium scales, while for large scales it is common to rectify the prints to ground control during the printing process.

## Ground Co-operation

*General Requirements.*

The difficulties of establishing the exact height and direction of air photographs make it impossible to produce accurate plans without control provided by a number of points fixed precisely on the ground, and which can also be identified on the photographs.

Close co-operation between the ground surveyor and the air photo-

grapher is essential if the work is to be carried out efficiently. The ground work may be divided into five stages:

(i) Preliminary work in consultation with the air personnel regarding proposed lines of flight in relation to existing and proposed control.

(ii) Identification and marking of trigonometrical stations and provision of additional stations as required. The photographs may be taken first on a skeleton control and the additional points chosen afterwards.

(iii) Interpretation of the photographs with regard to types such as vegetation, geological formation, fence lines. This involves inspection on the ground with the photographs and examination of overlaps with a hand stereoscope.

(iv) Collecting information and making ground measurements which cannot be fixed from the photographs either because the detail is hidden or does not show on the photographs.

This work may be grouped as follows:

(*a*) Classification of roads and tracks, adding and deleting where necessary. The extent of this will depend upon the map scale.

(*b*) Marking boundaries, such as parish and county boundaries and any others required which are not shown upon the photographs. Here again the extent of work will depend upon the scale.

(*c*) River and place names, etc., must be ascertained and the spelling checked.

(*d*) Additional measurements to obscured detail which it is necessary to show. Where individual buildings are to be shown on large scales it is often found that the overhang of eaves will cover the image of one or two sides. Also shadows, trees, clouds, or photographic defects may hide necessary information.

(v) The determination of heights, by either of two methods:

(*a*) For reconnaissance and preliminary surveys stereoscopic measurements on air photographs for difference of parallax may be made between ground height control points fixed by aneroid barometer and clinometer rays;

(*b*) For construction surveys, ordinary levelling methods, possibly combined with tacheometry and stereoscopic observation of photographs.

The ground work will include the location of bench marks, which are established on the ground, but which do not show on the photographs.

Levelling and contouring is discussed in Chapter VIII.

It will be gathered from the above list of requirements that the ground

surveyor has many duties to perform before the completed plan based on air photographs can be produced.

Hotine [55] remarks that there are those "who consider that geodetic survey and control are a form of useless hair splitting for purely scientific purposes. Actually the ultimate object is no less practical than the prevention of gaps or overlaps in subsequent detail surveys, leading to the same area being mapped twice over on different sheets or being omitted altogether."

Also when a traverse is made between triangulation stations the adjustment is much easier and more straightforward when such stations are fixed precisely.

Large-scale ground surveys include measurements to all main points of detail which are plotted. On medium and small scales a number of points are fixed accurately and the detail is usually plotted by plane-tabling, which involves a certain amount of sketching. When, however, one is plotting from air photographs there are sufficient ground points to control scale, and the detail plotting becomes, in effect, plane-tabling in the office where any convenient number of rays may be drawn. By giving a "bird's-eye view" of the area, the air photograph makes it easier to plot fuller and more accurate detail, provided that the photography is of a sufficiently high standard.

Although ground control is necessary for accurate air surveys a much smaller number of control points is required than for a ground topographical survey.

*Identification of Ground Control Points.*

It is usually preferable to use some easily identifiable point as a ground station rather than to mark them specially.

In order to show a square of 0·01 inch side in a photograph at a scale of 1/25,000, the station must be marked on the ground by a square of 20 feet side. It is quite likely that the image of such a beacon will not be identified and the normal variations of tone due to changes of vegetation and soil disturbances will make it necessary to have a much larger mark. On the same scale, a cross marking a station would have to be 100 feet long to show a length 0·05 inches in the photograph. Beacons of this size can be identified positively from the air and could be used for navigation marks.

The station positions are marked by whitewash, piles of stones or trenches. Such artificial markings are used sparingly. A war-time photographic pilot, in recounting his experiences on the Western Front, tells how on one occasion the existing detail had become so disturbed that no

ground control points could be conveniently selected. The remedy was to drop a few bombs in suitable places and locate them on the ground!

Road crossings, railway crossings, bridges, isolated trees, change of direction in wood boundaries, isolated buildings, hedge junctions, foot-path crossings are examples of points which are easily identified in photographs. The reader can verify this by referring to the examples of vertical photographs given in earlier chapters, but it should be remembered that much of the clarity and definition of the original photograph is lost in reproduction. It is somewhat risky to use buildings as control points, because the overhang of the eaves may obscure the exact ground point if this should happen to be near the edge of the photograph.

Any error in identification or in marking the ground control point in a photograph is equivalent to an error in fixing the point on the ground. At a scale of 1/5,000, an error of 0·01 inch corresponds to a displacement in the station of about 4 feet. Stereoscopic observation is of great assistance in fixing the exact positions.

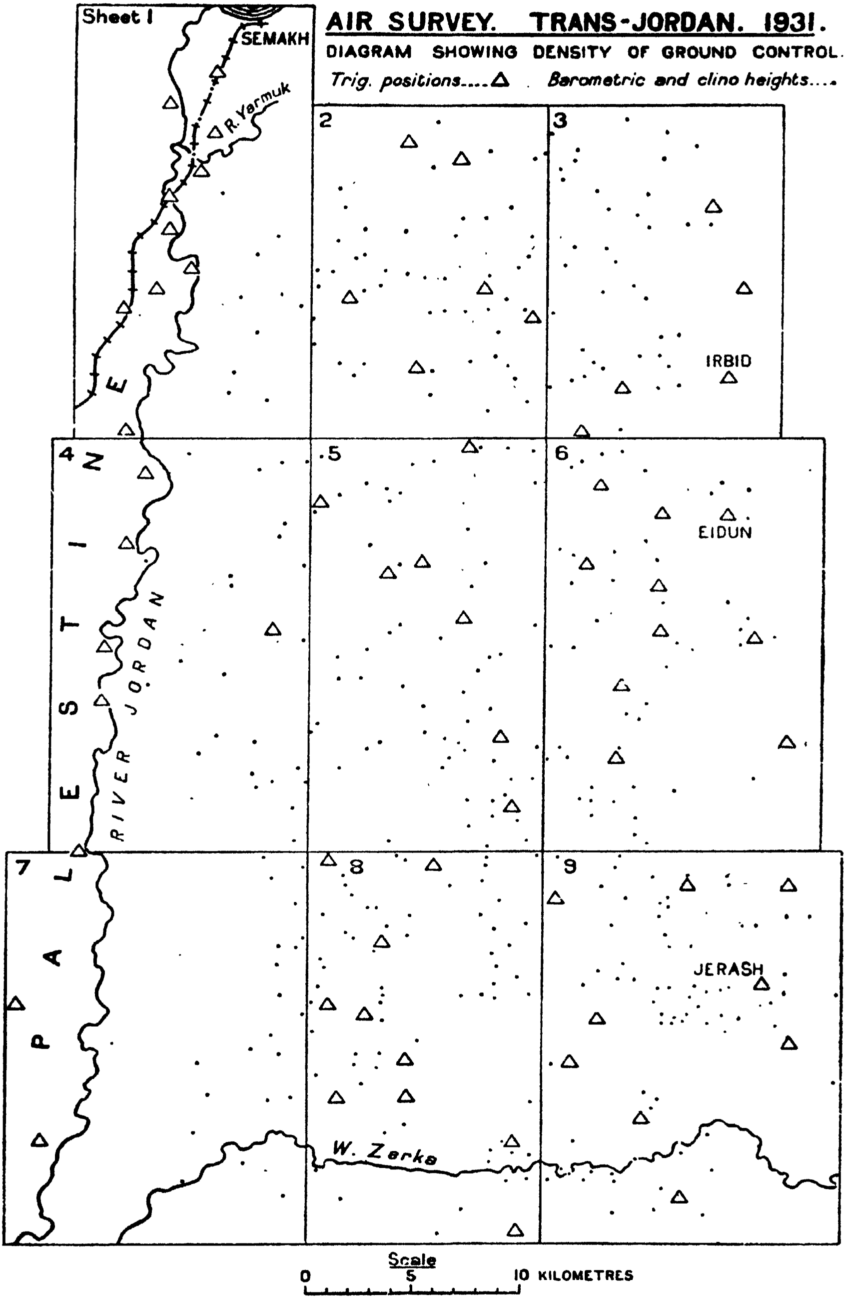
It is often found convenient in practice to select most of the ground control points after the survey photographs have been taken, although opinions differ as to which is the better procedure. There are obvious advantages in selecting good points after the photographs are taken but if the marks are fixed first there is a permanent record. Some form of compromise is probably the most satisfactory solution.

In the case of large-scale work (dealt with later), traversing between minor triangulation stations provides suitable ground control.

According to the results obtained by the Air Survey Committee of the War Office, a plan to a scale of 1/25,000 can be made as accurately as any ground survey, with ground control provided by a triangulation with sides averaging ten to twelve miles in length. Crone [32] points out that the ideal for principal point plotting is one point every five miles although it is possible to use less.

A diagram showing the density of ground control in the Trans-Jordan Air Survey of 1931 is shown in Fig. 88. This survey was made in connection with the Baghdad–Haifa Railway, and the final scale was 1/25,000. During part of this survey an automatic pilot was used and gave ample proof of the desirability of its employment for such work by producing much straighter runs with small tilts, thereby reducing the number of wasted runs and facilitating plotting and contouring.

The trigonometrical points were stations of the Survey of Palestine and in order to fix them on the photographs where they were not visible, marks were made on the ground and special photographs taken so that their positions could be plotted back on to the survey photographs,




[Reproduced from the Report of the Air Survey Committee, No. 2, 1935, by permission of the Controller of H.M. Stationery Office.

FIG. 88.

When some form of co-ordinate measuring stereoscope such as a radial triangulator (for polar co-ordinates) or a stereo-comparator (for rectangular co-ordinates) is used, it is possible to calculate positions of control points selected on the photographs and not known on the ground. Thus greater precision can be obtained than by graphical methods and the ground control points can be more widely spaced.

When using those stereoscopes and plotting machines in which the photographs are not assumed to have been taken vertically, it is possible to carry forward the air co-ordinates (three-dimensional co-ordinates) of control points for both plan and height, from overlap to overlap, with minimum ground control. This work is much more specialized.

*Cartographic Detail.*

While surveys on medium scales can be completed without any assistance from ground measurements for filling in detail, it is obvious that no air survey can possibly supply the names of places, rivers, or hills and a ground party must fill these in from information obtained on the ground and by reference to old plans and maps. It is always desirable to check names supplied by the "oldest inhabitant" or there may be frequent errors of nomenclature.

## Radial-Line Plotting direct to Scale

If there is sufficient ground control the plan may be plotted without the intermediate stage of a minor control plot as used in the Arundel Method. The Arundel Method aims at a minimum amount of ground control, such as that provided by a tertiary triangulation, and it becomes necessary to make the initial plot of a strip at an unknown scale.

Quite frequently, when maps at medium scales are required, ground control points much more closely spaced than points in a tertiary triangulation become readily available.

The approximate photographic scale can easily be found by reference to the flying height and focal length of lens, or by comparing photographic distances with known distances on the ground. The master grid is then prepared at a scale as close as convenient to the mean photographic scale and the known points plotted thereon. This master grid is either drawn on kodatrace or a tracing is made from it before plotting the detail. Adjustment to the map scale can be made during the stage of reproduction. Each photograph is base-lined and minor control points selected as before, while the ground control points are also marked.

Starting with a photograph on which there are three known points a

three-point resection can be made to fix the principal point of one photograph; this will also establish the direction of base-line for this photograph and plotting may be continued both ways along the strip. This corresponds to the method of fixing a plane-table station by tracing paper resection to three known points. Other ground control points when intersected should agree with their previously plotted positions.

If the photographs are comparatively free from tilt, triangles of error do not appear at the intersections of the three minor control point rays and the ground control points are accurately intersected. Discrepancies are likely to arise due to tilt and plotting errors, so that adjustments may have to be made to the ground control points. If it appears that a particular photograph has a definite tilt, it may be necessary to provide sufficient control to determine this. Much of the difficulty disappears if the automatic pilot is used, but when mapping on the smaller scales of the medium-scale range a decision must be made as to the relative importance of extra ground control and the automatic pilot. The special problems involved in large-scale planimetry are discussed in Chapter IX.

In the United States the radial-line method has been developed on rather different lines from the Arundel Method. In general the method has been that described above, but an additional assumption is commonly made, by selecting an identifiable point known as an azimuth point on the ground very near the principal point and using this as the photographic centre. The reliability of such an assumption must depend upon the degree of tilt of photograph and on the amount of height variations over its area. Limits of such an assumption based upon an allowable limit of error may be calculated by reference to the formulæ given in Chapter V.

A technique known as the template method has been developed, the templates being either transparent or slotted.*

*Transparent Templates.*

First the azimuth points and the minor control (or radial control) points are selected. Short radial lines are drawn through them for about half an inch on each side of the image of the point, and through any ground control (picture control) points. The template material is cellulose acetate which is of similar nature to kodatrace. Each photograph is laid face down on to a sheet of tracing material and fastened to it by adhesive tape. The radial lines are traced on to the template or they may be drawn direct through the images of the points. Base-lines are

* The writer is indebted to Mr. Leon T. Eliel of Fairchild Aerial Surveys, Los Angeles, and to Mr. Joseph M. Snyder of the United States Department of Agriculture, Soil Conservation Service, for the information on template methods of plotting.

drawn through the azimuth point to those of neighbouring photographs which appear on the photograph, the tails being drawn as in the Arundel Method, and each azimuth point is marked with a black circle. The lines may be drawn in ink or etched and filled in with opaque material. This is repeated so that finally there is a separate template for each photograph.

The block to be plotted will usually comprise a series of strips. A suitable strip is selected for starting plotting, and on this strip is chosen a pair of photographs having at least three ground control points in the common overlap forming in plan a well-conditioned triangle. Each of these templates is resected in turn upon the master grid which has plotted on it all the ground control points. It will be found that the two azimuth lines in the common overlap are coincident and the templates are taped down. The remaining templates are now fitted to these in turn both for the strip upon which the key pair appears and for the neighbouring strips. This involves a continuous process of checking with the ground control, and triangles of error may also need adjusting along the strip. This adjustment is considered quite legitimate and the result is good provided that excessive tilts are absent.

Finally each point is marked up on the back of the main grid sheet, and if any triangle of error persists, an average position is selected. When a particular photograph is badly tilted, it may become necessary to choose another azimuth point which is nearer the isocentre or plumb point.

A variation of the transparent template method sometimes used is as follows:

The principal points are marked in the usual manner, while base-lines and short lines through minor control points are traced off on to separate sheets of kodatrace each of the same size as a photograph. Any ground control points which appear on the photographs have also short radial lines drawn through them. The templates are then fitted together carefully, and the length between control points is checked. Where, as is usual, there is some disagreement, the templates are adjusted until the rays on the templates intersect at the position of the point as plotted on the master plan. This method may also be extended to the compilation of sheets.

*Slotted Templates.* *

An ingenious method of making the radial-line plot is that recently patented by C. W. Collier and controlled by Fairchild Aerial Surveys of Los Angeles. In this case cardboard is used instead of transparent templates. Each photograph is placed face upwards on a sheet of white cardboard and azimuth, minor control and ground control points pricked

* Use of these templates has now largely superseded graphical radial-line plots of the Arundel type.

through. A hole about 0·15 inches in diameter is punched through the template with the azimuth point as centre of rotation, and a special slot-cutting machine cuts slots of exactly the same width as the diameter of the central hole and about one inch long through the picture positions of these other points, as seen in Fig. 89.

In order to set the templates in the correct relationship, template pins are used, which have exactly the same diameter as the central hole and which will slide along the slots. Assembly is often started from a photograph where there are at least two ground control points and which is in

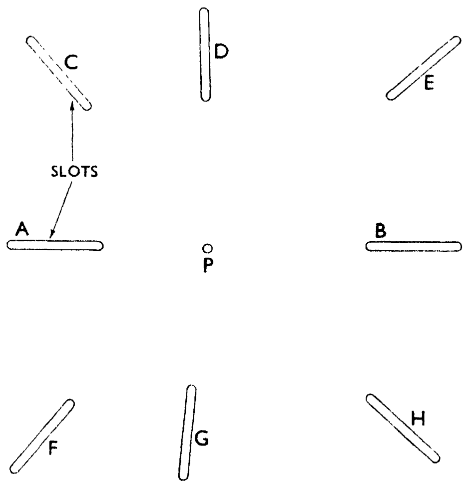



FIG. 89—SLOTTED TEMPLATE.

a good flight, although actually it is not necessary to have two starting points in one picture. A conventional triangulation system makes an excellent control for the method. Once the ground control points are set in their exact relationship as on the master grid, and the templates are set over it, the pins will automatically set themselves in the correct positions if tilt is absent. In the latter case the tilted photograph is soon discovered because its template will tend to buckle up during setting owing to the incidence of a triangle of error. Such a template can be left out of the original setting and adjusted to the established positions afterwards.

When the templates are adjusted, the system is free from play, and the exact points may be pricked through on to the grid through a needle hole which is drilled exactly in the centre of the template pin.

The slotted-template method is being used in the United States Soil Conservation Service. It is possible for one man to slot accurately several

hundred templates a day with a special punch, and the Soil Conservation Service have recently introduced a new slot cutter which is stated to be quicker and give improved accuracy.

Fairchild's have used the slotted-template method in mapping nearly 300,000 square miles. Many thousands of miles have been mapped with a ground control density of one point per 250 square miles. The maps have been assembled at a scale of two inches to one mile (1/31,680) and a usual specification is that all points should be within 0·05 inch of their true geographical position. The resultant maps have been repeatedly tested in the field and have been found to satisfy this condition.

Fairchild's have found that up to the limits of accuracy stated above, the slotted-template method is about four times as quick as the radial-line method (i.e. Arundel Method), while the ground control need only be 40 per cent of that required by the latter method.

*A Simple Approximate Method of Plotting Direct to Scale.*

A very simple method of plotting direct to scale depends upon the taking of photographs from approximately the same height, and upon the limitation of ground height variations to a small amount. Flying height is determined from statoscope readings to within about twenty feet of the correct height.

After the principal points have been marked on the photographs and the base-lines have been drawn in, the latter may be measured and the picture scale determined.

The principal-point traverse is traced direct on to kodatrace from the photographs by assuming that the length of base-line between two principal points is the same on each photograph of the pair.

Having obtained the principal-point traverse, the detail is plotted direct from the photographs. A sheet of kodatrace is set so that it is over one photograph of the stereo-pair and under the other. The photographs are set in their correct relative base-line direction by adjusting them until the two base-lines lie along a straight-edge. Then by viewing the overlap in a simple stereoscope and tracing direct on to that part of the kodatrace which covers one of the photographs, the detail may be plotted in approximately correct relationship to the photographic base-line.

If the stereometer is used it is possible also to plot form lines or contours as described in Chapter VIII.

This method of plotting direct to scale is of value in the approximate plots required for certain economic and engineering reconnaissance surveys. It does not produce results as precise as may be obtained by methods described previously in this chapter.

CHAPTER VIII

# SIMPLE METHODS OF LEVELLING AND CONTOURING FROM AIR PHOTOGRAPHS

## INTRODUCTION

THE employment of stereoscopic observation in the preparation of plans from air photographs and the resultant "solid" picture of the overlap, at once suggests the possibility of height determination. The problems involved are, however, very different from those which arise in planimetry, where accuracy of length and angular measurement is not appreciably affected if the plane of measurement is slightly dislevelled. Thus a slope of a degree or so will have little effect and even one of three or four degrees will not introduce uncontrollable errors. If, however, such a plane were used as a basis of measurement of difference of height, the results would be of little value unless the amount of correction to be applied for deviation of this plane from the horizontal could be determined easily. For instance, a slope of one degree from the horizontal corresponds to a gradient of 1·75 per cent, i.e., a difference of level of 1·75 feet per hundred feet of horizontal distance.

The assumptions which are valid in the preparation of plans from nearly vertical photographs have already been studied in detail in Chapters V and VII, where it has been shown that in most cases a small deviation from the horizontal plane of exposure can be overcome quite simply by making assumptions which introduce negligible errors, or by using a technique which will eliminate most of the error during the plotting process. One of the major problems arising in determination of differences of level is due to the impossibility of making the exposure in a truly vertical direction. This introduces inaccuracy in measurement of difference of level due to parallax errors or "false parallax" across an overlap when the stereo-pair of photographs is examined as though tilt were absent from each photograph.

Greater accuracy in relative height determination can be obtained by eliminating tilt distortion, either by rectification in printing so that the stereoscopic overlap may be examined in a common plane as in an ordinary stereoscope, or by setting the photographs of a stereo-pair in their correct

relationship at exposure. Instruments and technique for the latter are described in Chapter X.

Apart from plotting machines, most of the work done in this country in determining heights has been based on stereoscopic measurements of vertical photographs on the lines developed by Hotine.

Other simple methods of determining difference of level from air photographs have been evolved, but perhaps that of Crone [26] from high oblique photographs, which has been used with success in India, is most worthy of note. His method is described later in this chapter.

## Determination of Difference of Level

In order that absolute heights of points on the ground may be determined with useful accuracy from vertical air photographs, there are two principal methods available: (i) by observation with an elaborate stereoscopic instrument, with reference to fairly widely spaced ground control, or (ii) by a simple method of stereoscopic observation on approximately vertical photographs with extensive reference to ground control.

Inaccuracies arise from a number of factors, of which the most important is tilt, and these must be eliminated as far as possible by routine observation and allocation of corrections.

The usual method is to have a sufficient number of known spot-levels spaced over the stereoscopic overlap of a pair of photographs, and then to interpolate contours by eye as in a ground survey. An advantage of this method is that the contours are drawn on to the photographs while the overlap is being viewed stereoscopically, thus enabling the plotter to allow for variations of ground slope in spacing the contours.

## Contouring at Medium Scales from Vertical Photographs

It is proposed to describe first a simple method of contouring which has been found adequate when plotting at a scale of 1/25,000 from photographs taken with manual control at approximately the same scale. This process has been evolved by the Air Survey Committee of the War Office, and is described in Professional Paper No. 8 of the Committee. [86]

### *Selection of Control Points and Measurement of Parallax.*

Well-flown manually controlled photography may result in 2° of tilt in any direction, and reliable contouring necessitates the fixation of nine points per photograph, i.e., six per overlap. In addition, in an average case, heights for another twelve or so must be found from stereoscopic

measurement and computation. Occasionally, where the ground is very regular, fewer points may be used, but, where the country is irregular, additional points are required so that each important hill and valley has a control point somewhere near the highest or lowest level respectively.

The simple method does not lend itself to determination of levels over areas where there is no ground control, except in cases of military necessity where there is no alternative method available. The military aspect is beyond the scope of this book, and where sufficient control cannot be readily supplied, some form of aerial triangulation is preferably used, and this requires more elaborate instruments.

In some countries, such instruments are used almost universally, to the exclusion of the simple method.

In the Arundel Method, after the minor control plot has been made and rectified to the scale of the master grid, the spacing of the principal points may be measured, and if there is not much variation along a strip the average length of air-base $B$ may thereby be found. The average height of exposure can be found from the altimeter readings, and even if this is somewhat inaccurate, the method used will enable the effects to be eliminated.

The following example has been taken from a survey at a scale of approximately 1/25,000.

Focal length of camera = 177·8 mm.; average length of air-base along strip = 1,765 metres; average height of exposure along strip = 15,300 feet.

Fig. 90 gives a diagram of one of the photographs in the strip, with nine numbered ground heights marked upon it. These points are as evenly distributed over the area as the topography will allow. In this example the stereoscopic overlap of sixty per cent on the right-hand side is considered.

The absolute parallax of each of these points may be computed from the parallax equation, $p = \dfrac{f.B}{(H - h)}$ (p. 166).

Thus for point 20:

$$p = \frac{177{\cdot}8 \times 1{,}765 \times 3{\cdot}28}{(15{,}300 - 600)} = 70{\cdot}04 \text{ mm.}$$

In British practice heights are given in feet, and the necessity for conversion arises from the fact that instruments have in general been designed by Continental firms, or by the military authorities who are also concerned with contours at metric intervals.

Stereo-height control points are chosen in important positions on the overlap, so that adjustments can be made in circuits. In Fig. 90, points $E_1$, $E_2$, $E_3$, $E_4$, are shown approximately in line with ground control

points. These are selected at important points after the overlap has been examined carefully under the stereoscope.

The next step is to measure differences of parallax between known and required points. The Barr and Stroud Precision Topographical Stereoscope (Fig. 77) has been designed for this purpose as well as for fixing base-lines. The photographs are set carefully in correspondence in the

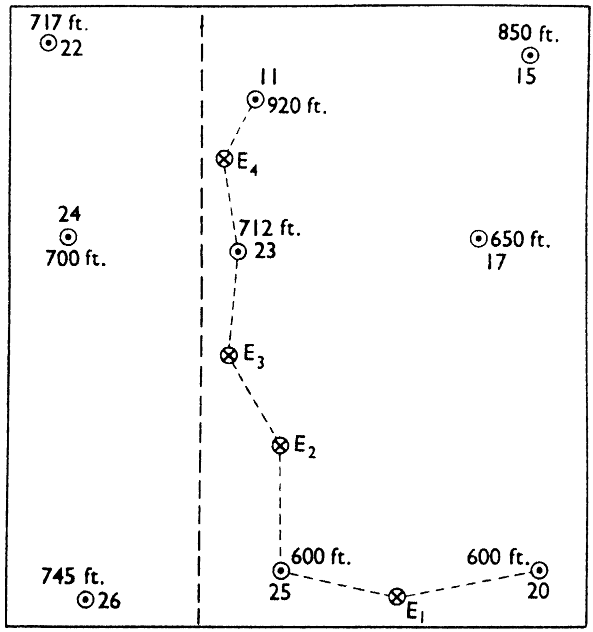



FIG. 90.

stereoscope, each photograph being covered with the parallactic grid which has lines running north-west and north-east. In addition there is a vertical or north line. The mode of operation of this stereoscope has been described in Chapter VI, the principle briefly being that the photographs are first set in correspondence in order that the base-line may correspond with the direction of lateral movement of the grids. These grids may be moved together in the direction of the base-line so that the spacing is maintained, and the right one may also be moved in the same direction in relation to the left grid in order that a difference of parallax can be measured between two points.

Differences of parallax may also be measured by means of such instruments as the Zeiss or de Koningh Stereometers (Figs. 78 and 79).

When the stereometer is set parallel to the base-line, differences of parallax are measured by setting the floating marks to ground level at various points, and noting the readings on the micrometer. The right-hand floating mark may be adjusted in the $y$-direction (at right angles to the base-line in order to allow for any "want of correspondence" due to tilts). The floating marks are engraved on glass plates which are superimposed on the photographs and which enable a much finer setting to be made than the parallactic grid. When the telescopes are fitted to the stereoscope, a much more accurate reading is obtainable and it is possible to measure differences of parallax with an accuracy which approaches 0·01 mm., provided that the mean of three or more readings is taken.

The process of determination of levels for control is similar for all stereoscopes of this type. The examples of parallax measurements which follow were made with a Barr and Stroud Z.D. 15 Stereoscope.

Parallax readings as taken are given in Table VIII. 1. In each case the reading on the micrometer drum was taken when one of the lines was brought to ground level at the point considered and to do this with precision needs considerable practice. During stereoscopic observation it will be noticed when the line goes below ground that it tends to become blurred or may appear to split up. By successive operations of the micrometer screw backwards and forwards, the exact setting is found where the line touches ground. If the north-east line is observed first, the point A on the photograph may be under point $a$ (Fig. 91). Since it is unlikely that the point will be exactly on the path of a grid cross, the reading for the north-west line at ground level will be different,

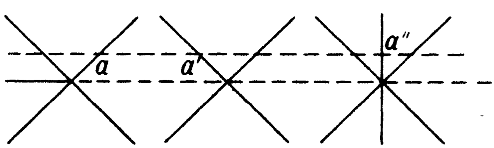


FIG. 91.

its position being now $a'$ over A. Again the north line will give a position at $a''$, and if the images are in correspondence this reading will give the mean value of the three, and will thereby check the accuracy of the observations. If there is a slight difference which is not eliminated after a check reading, the reason is a small local lack of correspondence, probably arising from a tilt error.

Occasionally some modification of reading has to be made where one of the lines is inaccessible or unreadable. Thus, for point 20, both north-east and north-west lines have been read a second time instead of one reading of the north line.

The mean values are given in the table and these considered with those along several strips appear to indicate that the figure of 0·01 mm. as the

limit of parallax error for a skilled observer may be of the right order, as the observer in this case was comparatively inexperienced.

Having determined the differences of parallax to the required picture control points, their absolute parallax is now found from the known values at the ground height control points. Thence the levels of the points

TABLE VIII.1

MEASUREMENT OF PARALLAX DIFFERENCE WITH BARR AND STROUD Z.D.15 STEREOSCOPE

| Point. | *Micrometer Readings of Grid.* (1) *N.E.* | (2) *N.W.* | (3) *N.E. or N.* | (4) *N.W. or N.* | *Mean Reading.* |
|---|---|---|---|---|---|
| 20 | 4·92 | 6·42 | 4·95 | 6·46 | 5·69 |
| $E_1$ | 5·72 | 6·92 | 6·30 | | 6·31 |
| 25 | 6·28 | 7·08 | 6·66 | | 6·67 |
| $E_2$ | 6·94 | 7·52 | 7·25 | | 7·24 |
| $E_3$ | 6·90 | 6·92 | 6·90 | | 6·91 |
| 23 | 7·26 | 6·81 | 7·06 | | 7·04 |
| $E_4$ | 7·42 | 7·12 | 7·26 | | 7·27 |
| 21 | 8·00 | 7·24 | 7·64 | | 7·63 |

may be computed, either from tables published by the Air Survey Committee as a supplement to their Professional Papers No. 8, [86] or from a graph.

*Heights from Parallax Tables.* [86]

The tables enable values to be interpolated for the likely range of the parallax equation, $p = \frac{f.B}{(H - h)}$

They are drawn up in ascending values of $f.B$ in square metres from 150 to 450, at intervals of five square metres, for values of $p$ from 30 to 130 mm. at intervals of 0·1 mm. and give the value of $(H - h)$ in metres. Intermediate values may be interpolated according to a straight line law.

In Table VIII. 2, the computed values of absolute parallax are tabulated for ground reference points of Fig. 90. Along each line is given a point measured stereoscopically and referred to the ground reference point. Thus for point $E_1$, the difference of parallax 20 to $E_1 = + 0{\cdot}62$ mm., so that the absolute parallax of $E_1 = 70{\cdot}04 + 0{\cdot}62 = 70{\cdot}66$ mm.

TABLE VIII.2

| Ground Reference Point (R). | | | | | Point Referred To (Q). | | | | | | Adjust-ment to $h_Q$. | Final Accepted Height $h_Q$. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | Height $h_R$ ft. | Height $h_R$ m. | $H-h$ m. | $p$ mm. | No. | $\delta p$ R to Q. | $p$ | $H-h_Q$ m. | $h_Q$ m. | $h_Q$ ft. | | (ft.) |
| 20 | 600 | 182·5 | 4,481 0 | 70·04 | $E_1$ | + 0·62 | 70·66 | 4,443 | 221 | 725 | — 107 | 618 |
| 20 | 600 | 182·5 | 4,481 0 | 70·04 | 25 | + 0·98 | 71·02 | 4,421 | 243 | 798 | — 198 | **600** |
| 25 | 600 | 182·5 | 4,481·0 | 70·04 | $E_2$ | + 0·57 | 70·61 | 4,448 | 216 | 709 | + 12 | 721 |
| 25 | 600 | 182·5 | 4,481·0 | 70·04 | $E_3$ | + 0·24 | 70·28 | 4,467 | 197 | 647 | + 27 | 674 |
| 25 | 600 | 182·5 | 4,481·0 | 70·04 | 23 | + 0·37 | 70·41 | 4,659 | 205 | 673 | + 39 | **712** |
| 23 | 712 | 217 | 4,446·4 | 70·58 | $E_4$ | + 0·23 | 70·81 | 4,432 | 232 | 762 | + 42 | 804 |
| 23 | 712 | 217 | 4,446·4 | 70·58 | 11 | + 0·59 | 71·17 | 4,409 | 255 | 837 | + 83 | **920** |

Since $f = 177{\cdot}8$ mm. and $B = 1{,}765$ m. therefore $f.B = 313{\cdot}8$ square metres. From the Parallax Tables [85] the values in Table VIII.3 are extracted:

TABLE VIII.3

| $p$ | $f.B$ 310 | $f.B$ 315 |
|---|---|---|
| 70·6 | 4,390·9 | 4,461·8 |
| 70 7 | 4,384·7 | 4,455·4 |

By slide-rule it is found that when $p = 70{\cdot}66$ mm. and $f.B = 313{\cdot}8$ square metres, then $(H - h) = 4{,}443$ metres, and since $H = 15{,}300$ feet ($= 4{,}664$ metres) therefore $h = 4{,}664 - 4{,}443 = 221$ m. Similarly all other points.

These levels have all been converted into feet and are given in the table.

It will be noticed that the heights found for those points of previously known level are considerably different from the true values as entered in heavy type in the last column under the heading of *Final Accepted Height*. Differences are largely due to tilt, but inaccuracies in length of air-base and height of aircraft contribute, as well as a number of errors due to instruments, observers and materials.

Since the points have been arranged in a circuit with approximately straight lines between ground reference points, it may be assumed that corrections to height are made proportionately to the photographic distance between them. For instance, the worst case is between points 20 and 25, where the deduced value of point 25 with reference to 20 is 198 feet too big. The distance 20 to $E_1$ is 0·55 of the distance between points 20 and 25, so that the correction is $198 \times 0{\cdot}55 = 107$ feet, thus making the final accepted value of $E_1 = 725 - 107 = 618$ feet. Similarly for the other points, the adjustments again being made in proportion to the distances.

In this instance an indication of accuracy attained is obtained as follows: First consider the circuit from point 25 to point 11, taking in point 23. Table VIII.4 shows that this gives an error of + 17 feet to the height of point 23. Secondly, consider the circuit from point 25 to 23, produced to point 11, giving an error of −23 feet to the height of 11. This is the order of accuracy which may be expected.

TABLE VIII.4

| *Point.* | *Proportion of Distance* 25—11. | *h from parallax readings (Table VIII.* 2). | *Circuit* 25—11. | | *Circuit* 25—23 *continued to* 11. | | *Accepted h from (Table VIII.* 2). |
|---|---|---|---|---|---|---|---|
| | | | *Adjustment.* | *Accepted h.* | *Adjustment.* | *Accepted h.* | |
| 25 | 0 | 600 | 0 | 600 | 0 | 600 | **600** |
| $E_2$ | 0·21 | 709 | + 17 | 726 | + 14 | 723 | 721 |
| $E_3$ | 0·43 | 647 | + 36 | 683 | + 27 | 674 | 674 |
| 23 | 0·67 | 673 | + 56 | 729 | + **39** | 712 | **712** |
| $E_4$ | 0·91 | 762 | + 76 | 838 | + 55 | 817 | 804 |
| 11 | 1·00 | 837 | + **83** | 920 | + 60 | 897 | **920** |

It follows that reduction of tilt by automatic pilot facilitates this work of adjustment, since the maximum amount of the height adjustment to be applied in the case given would probably be reduced to 20 or 30 feet.

Use of the parallax tables involves a considerable amount of computation in interpolation unless approximations are considered adequate.

*Heights from Parallax Graphs.*

Some surveyors prefer to draw a parallax graph, so that when the absolute parallax of point is known its height can be found direct from the curve. In most cases, where photographic heights and lengths of airbase are approximately constant along a strip one graph may be used for the whole strip, which results in much saving of time over tables. If the graphs are drawn to a large enough scale the accuracy compares satisfactorily with that obtained from tables, and heights may be read off directly in feet.

The graph may be drawn in two convenient ways: (i) By plotting absolute parallax against ground height, i.e., $p$ against $h$. This is very quick and simple. (ii) By plotting rate of change of height with parallax against absolute parallax, i.e., $\delta h/\delta p$ against $p$. This is the method described in a Professional Paper of the Air Survey Committee. [86]

(i) *Graph connecting Absolute Parallax and Ground Height.* When considering the same example detailed in Table VIII.2, it is found by inspection of the photographs and ground height control, that, along the length of the strip, heights are all between 500 and 1,000 feet above the Datum. From the values given above, namely, $f$ = 177·8 mm.; $B$ = 1,765 m.; $H$ = 15,300 feet, the parallax equation has been solved for $p$ between $h$ = 500 and $h$ = 1,000 (Table VIII.5).

TABLE VIII.5

| $h$ (*feet*). | $p$ (*mm.*). |
|---|---|
| 500 | 69 574 |
| 600 | 70 046 |
| 700 | 70 529 |
| 800 | 71·015 |
| 900 | 71 508 |
| 1,000 | 72·007 |

The graph of this is very nearly a straight line, and has been drawn on a small scale as a series of short straight lines in Fig. 92. The original was drawn on squared paper divided at intervals of 2 mm.; Height scale 5 cm. to 100 feet; Parallax scale 1 cm. to 0·1 mm. difference of parallax. The overall size of the graph was not unduly large, being about 9 × 10 inches.

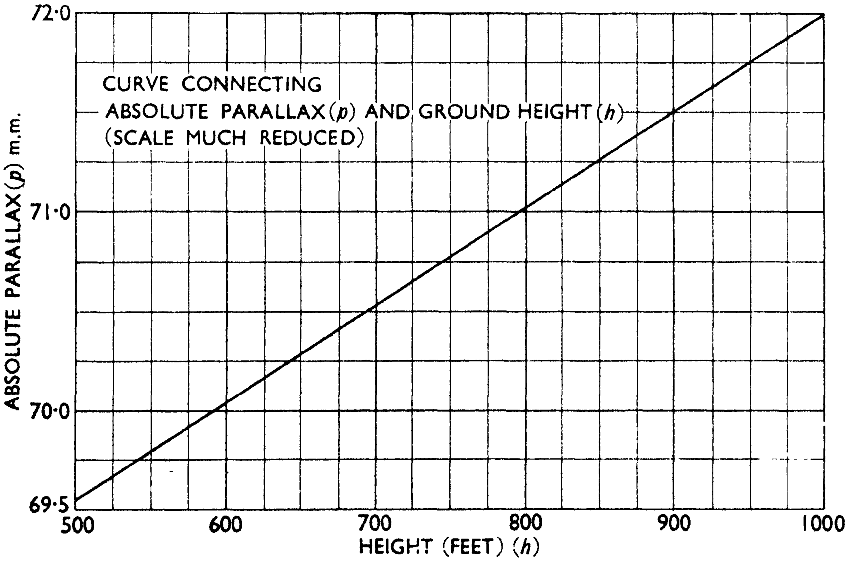



FIG. 92.

A table is drawn up as in Table VIII.6. The columns for the ground reference point are as in Table VIII.2, except that height are in feet. For the "point referred to," absolute parallax is found as before. From the curve, the height corresponding to its absolute parallax is read off for each

TABLE VIII.6

| *Ground Reference Point* (R). | | | | *Point Referred To* (Q). | | | | *Adjustment to* $h_Q$. | *Final Accepted Height.* $h_Q$. |
|---|---|---|---|---|---|---|---|---|---|
| *No.* | *Height* $h_R$ *ft.* | $H - h$. *ft.* | $p$ *mm.* | *No.* | $\delta p$ R *to* Q. | $p$ | $h_Q$ (*from graph*). | | |
| 20 | 600 | 14,700 | 70·04 | $E_1$ | + 0·62 | 70·66 | 726 | − 110 | 616 |
| 20 | 600 | 14,700 | 70·04 | 25 | + 0·98 | 71·02 | 800 | − 200 | **600** |
| 25 | 600 | 14,700 | 70·04 | $E_2$ | + 0·57 | 70·61 | 715 | + 13 | 728 |
| 25 | 600 | 14,700 | 70·04 | $E_3$ | + 0·24 | 70·28 | 648 | + 26 | 674 |
| 25 | 600 | 14,700 | 70·04 | 23 | + 0·37 | 70·41 | 674 | + 38 | **712** |
| 23 | 712 | 14,588 | 70·58 | $E_4$ | + 0·23 | 70·81 | 757 | + 45 | 802 |
| 23 | 712 | 14,588 | 70·58 | 21 | + 0·59 | 71·17 | 832 | + 88 | **920** |

point. The value can easily be interpolated to the nearest foot. Adjustments are made as before so that the final accepted heights appear in the last column. There is close agreement between heights found by this method and by that employing parallax tables. The values deduced give the general order of closeness of agreement.

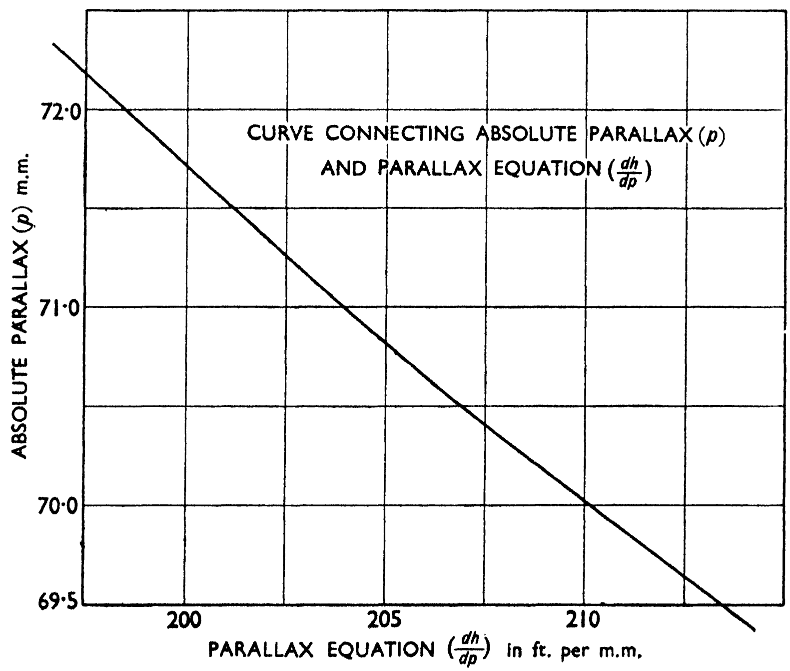



FIG. 93.

(ii) *Graph connecting Absolute Parallax and $\delta h/\delta p$.* For each point the values of $h$, $p$ and $\delta h/\delta p$ are computed. Those for $p$ are given above, while $\delta h/\delta p = \dfrac{-(H-h)^2}{f.B}$ from equation VI.3. Thus for $h = 600$ feet, $\delta h/\delta p =$ 208·5 feet difference of height per millimetre difference of parallax.

The values of $p$ are plotted against $\dfrac{\delta h}{\delta p}$ as illustrated in Fig. 93 which is much reduced in size from the original drawing.

Consider the point $E_4$, referred from point 23 (Fig. 90), the mean parallax between them being (70·58 + 70·81) = 70·69 mm. From the graph, $\delta h/\delta p$ for 70·69 = 205·7 feet per mm. over the range, so that the difference of level from 23 to $E_4$ = difference of parallax (23 to $E_4$) × $\delta h/\delta p$; = 0·23 × 205·7 = +47 feet, and the level of point $E_4$ is therefore 712 + 47 = 759 feet (compared with 762 and 757 by the other methods). Final accepted levels deduced as before are, in order, 616, 732, 676, 802 feet for $E_1$, $E_2$, $E_3$, $E_4$, and these are in quite close agreement when the possible sources of error are considered.

This method appears to offer advantages over the graphical method just described when there is considerable variation of height along a strip in which case the size of the graph connecting $p$ and $h$ would need to be very large to retain accuracy of interpolation.

Standard forms are used by the Air Survey Committee and the War Office for tabulating these heights. [86]

### *Interpolation of Contours.*

When the ground and stereoscopic spot heights are established contouring may be commenced. The stereoscopic impression is easily disturbed when marks are made on the photographs, and it is therefore preferable to mark the control points, together with number and height on to an index photograph which the stereoscoper keeps at his side for comparison with those under the stereoscope. Only actual positions of control points, and sometimes not even these, are marked on the photographs to be contoured.

Since the overlap is about sixty per cent, alternate photographs are contoured, each photograph to be contoured being used in turn with its left-hand and right-hand photographs.

For this purpose a simple stereoscope, such as the Barr and Stroud, Zeiss, or de Koningh may be used (Chapter VI). The stereo-pair is placed in the stereoscope and set in correspondence with respect to the base-lines which have been drawn previously during the plotting of the plan. Then

with the index photograph corresponding to that which is about to be contoured placed at one side, the stereoscopic picture of the overlap may be examined. The drainage system of the area can usually be identified easily, and this is marked with blue chalk as a guide to the contouring. Next it is ascertained if the slopes are fairly uniform between spot heights, as this will decide the spacing of the contours; for instance, on a convex slope, contours will be closer together at the bottom than at the top. It is possible to make approximate stereoscopic measurements with the stereoscope where ground slopes change noticeably, but generally, if sufficient control points have been chosen, this procedure should not be necessary.

The contours are sketched in by eye as for a ground survey from spot levels, but more accurately, because the "solid" view is seen. The grid over the photograph which is to be contoured is raised (Fig. 76) so that this can be done. The contours are generally drawn in red, and when completed alternate photographs will be completely covered with contours. These can then be transferred to the master plot in exactly the same way as the detail.

It is possible to delineate contours at a scale of 1/25,000 from photographs taken with manual control as accurately as they can be fixed by ground survey provided that an adequate number of ground heights are fixed as mentioned above, and that the stereoscopers are skilled in measurement and interpretation. In order to achieve these conditions it is necessary that the photographs should be of a high technical standard.

The tracing stereometers of Zeiss and de Koningh cannot be used for tracing contours accurately unless the photographs are free from tilt. If the floating mark is set to ground level at a particular point of known level, then by moving the stereometer so that the floating mark is constantly in contact with the ground, a contour is traced out if the photographs are free from tilt. Otherwise the lines traced out by the tracing pencil are only form lines, which may be of use on a very small-scale map, but which are not adequate on a map of medium scale.

*Ground Control.*

Tertiary triangulation stations used for plotting the plan will usually be of known height, and there may be a number of bench marks on the area. Height control for contouring is fixed on the photographs so that the number and spacing is adequate to the area and the number of ground-spot heights will, in most cases, be between six and twelve per overlap.

For maps at a scale of 1/25,000, or even somewhat larger, the trigonometrical heights, supplemented by barometric heights or from clinometer rays, will provide the required control. The bench marks, if available, will also be of use. Points are fixed on the photographs by inspection on

the ground, and are chosen, where possible, at ruling points such as hill-tops, changes of slope, and bottoms of valleys.

It must be remembered that where contours are required at spacings such as those on the official 1 25,000 map, namely 10 metres, it is necessary only to fix these levels to the nearest foot or two. This can be done fairly easily with a clinometer such as the abney level. Aneroid barometers are also extensively employed.

The vagaries of the household aneroid barometer may give rise to some misgivings among those who are used to working to 0·01 foot as in ordinary levelling, but surprisingly good results are possible with care, and values can be found accurately enough to produce the desired contours. The aneroid barometer records atmospheric pressure, which varies with altitude above sea level, and this variation will enable differences of level to be found if all other conditions remain constant. Changes of atmospheric pressure and even humidity affect the readings, as also do the diurnal changes of atmospheric pressure.*

A single aneroid may develop a tendency to give false readings, and the instruments should be mounted in batteries of three, so that if one barometer develops this tendency its readings can be discarded. Two batteries are used, one at a station of known height where barometer and temperature readings are taken at intervals, while the other is placed in turn at the points of which the height is required, aneroid and temperature readings taken, and the time noted. This enables corrections to be made to the movable battery for temperature and diurnal wave variations. The readings are worked out in the form of a traverse, starting and finishing at known points, so that the closure adjustment may be applied.

A graph connecting barometric pressure with the change of height per difference of one inch in barometric reading is given for various temperatures in the Professional Papers No. 8 of the Air Survey Committee. [86]

When the mean temperature between field and stationary battery is known and the difference of barometric pressure ascertained, (by averaging the readings of the instruments in each battery), the graph enables the rate of change of height with respect to barometric pressure to be read off. From this reading can be found the difference of level between stationary and field batteries.

In order to compare readings, values are interpolated for the stationary battery to correspond with the times of those taken in the field.

An example is given in Table VIII.7. Readings of the station battery were taken at intervals of fifteen minutes, together with the temperature. The field battery was taken round to the various points placed on the

* These changes are often considerable in tropical countries and may adversely affect determinations of level by this method.

ground and mean reading and temperature were recorded. In this case the field battery was read at stations at the commencement and end of the series of readings. The points were distributed over about 4 square miles of the South Downs.

Each aneroid was checked against a standard barometer calibrated at the National Physical Laboratory, and the necessary corrections were applied to the readings before the mean value entered in the table was determined.

Differences of pressure reading are likely to occur in different batteries due to variation in rate of response to pressure changes, but these can be eliminated. The field battery shows mean readings 0·018 and 0·022 inches less than the stationary battery at the beginning and end respectively, and these readings are adjusted to read from the same zero as the stationary battery, and are given here as "corrected readings." This enables the difference of pressure between station and field battery to be tabulated. Allowance has been made for slight changes in station battery readings with time. The adjusted values of station battery readings for times corresponding to field readings are given in brackets. From the mean temperature at station and field point, the difference of level per inch difference of barometric pressure can be read off from the curve in the Air Survey Committee's Professional Paper No. 8.

Differences of level between station and point may then be worked out. Consider point 1.

TABLE VIII.7

*Date, 21-6-39. Time, Evening. Place, Burpham, Nr. Arundel.*
*Weather: High Wind, Threatening Storms, Sky Overcast.*

| | | *At Station (S.B.).* | | | *Field Battery (F.B.).* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Time.* | *Point.* | *Mean Reading (ins.).* | *Temp. °F.* | *Height (h) ft.* | *Point.* | *Mean Reading (ins.).* | *Adjustment (ins.).* | *Corrected Reading (ins.).* | *Difference of Pressure F.B. to S.C.* | *Temp. °F.* | *δh (ft.). Per 1 in. Difference of Pressure.* | *Difference of Level (ft.).* | *Absolute Height (ft.) of F.B.* | *Height of Point above O.D.* |
| p.m. | Sta | | | | | | | | | | | | | |
| 5.40 | S | 29 979 | 70 | 36·7 | | | | | | | | | | |
| 5.43 | | (·979) | (70) | | S | 29·961 | +·018 | 29·979 | 0 | 70·4 | | | 36·7 | 36·7 |
| 5.52 | | (·979) | (70) | | 1 | 29 903 | + 019 | 29·922 | —·057 | 70·7 | 949 | + 54 | 90·7 | 95·28 |
| 5.59 | | (·979) | (70) | | 2 | 29·907 | +·019 | 29·926 | —·053 | 71·4 | 950 | + 50·3 | 87·0 | 88·85 |
| 6.00 | S | 29·979 | 70 | 36·7 | | | | | | | | | | |
| 6.13 | | (·985) | (69·9) | | 3 | 29·794 | +·020 | 29·814 | —·171 | 69 0 | 948 | + 162·1 | 198·8 | 195·47 |
| 6.15 | S | 29·986 | 69·7 | 36·7 | | | | | | | | | | |
| 6.21 | | (·986) | (69·3) | | 4 | 29·933 | +·020 | 29·953 | —·033 | 67·5 | 944 | + 31·2 | 67·9 | 72·38 |
| 6.30 | S | 29·986 | 68·9 | 36·7 | 5 | 29·924 | +·021 | 29·945 | —·041 | 67·1 | 943 | + 38·7 | 75·4 | 77·07 |
| 6.37 | | (·985) | (68·6) | | 6 | 29·966 | + 021 | 29·987 | +·002 | 67·6 | 943 | —1·9 | 34·8 | 41·96 |
| 6.43 | | (·984) | (68 3) | | 7 | 29·964 | +·022 | 29·986 | +·002 | 68·4 | 944 | —1·9 | 34·8 | 41·30 |
| 6.50 | S | 29·984 | 68·0 | 36·7 | S | 29·962 | +·022 | 29·984 | 0 | 68·1 | | | 36·7 | 36·70 |

Difference of level S to 1 $= 0{\cdot}057 \times 949 = 54$ feet. This is a rise, since pressure rises as altitude decreases. Hence absolute level of point 1 is $30{\cdot}7 + 54 = 90{\cdot}7$ feet.

The following approximate empirical formula gives fairly reliable values for $\delta h/\delta p$ in feet per inch of barometric height between pressures $p = 28$ and $p = 31$ inches of mercury and between temperature $t = 20°$ F. and $t = 80°$ F. $\delta h/\delta p = 830 + 1{\cdot}733\,(t - 20) + (31 - p)\,[29{\cdot}2 + 0{\cdot}06\,(t - 20)]$

$$\delta h/\delta p = 830 + 1{\cdot}733\,(t - 20) + (31 - p)\,[29{\cdot}2 + 0{\cdot}06\,(t - 20)] \qquad \text{(VIII.1)}$$

Thus for point 1, $\delta h/\delta p = 951$ feet per barometric inch, and the difference of level $0{\cdot}057 \times 951 = 54$ feet as before.

In order to test the accuracy of heights determined in this way, the reduced levels of all the points were referred to Ordnance Datum by ordinary levelling, and these levels are given in the last column of Table VIII.7. The atmospheric conditions were not particularly favourable and the results obtained were indicative of the general order of accuracy obtained.

If the barometric traverse is closed to some known point other than the station at which there is a battery, it is possible to adjust field-battery readings only by the difference of battery readings at the beginning, and adjustments are made proportionally to the distances between the points in the traverse.

Until recently, surveying aneroids made in Britain were provided with a height scale graduated according to Airey's Table of Altitudes published in the Meteorological Society's Journal in 1867. This assumed that atmospheric temperature was constant at all altitudes, and 50° F. was taken as the standard. At the instance of a Committee composed of representatives of the various Government Departments interested, research work has been carried out on the variation of temperature with height and corrections for this as well as other factors are published in a set of *Aneroid Tables* produced at the War Office. [93]

The aneroid was chiefly used previously as a means of determining approximate heights during reconnaissance and exploratory surveys and as a result its performance had not been examined with regard to accurate determination of very small differences of level, and detailed investigation into the corrections required had not been made. The necessity for using such instruments for aircraft altimeters made it desirable to go into the matter more fully and these tables are intended to be used for aircraft heights as well as ground heights, in any part of the world. Their basis is a height scale based upon a standard atmosphere, so that the aneroids used are graduated in feet. All the corrections are given in feet.

The Standard Atmosphere is composed of dry air of invariable com-

position at all altitudes; temperature at Mean Sea Level of 15° C. or 59° F.; standard pressure reduced to 0° C. is 760 mm.; and the force due to gravity is 980·62 c.g.s. units.

This Standard Atmosphere is, the Report states, that which has been adopted for the assessment of aircraft performance, and is based upon the assumption of a uniform decrease of temperature with height (or "lapse-rate"). Three tables are given:

(A) Correction to be applied to the height reading of an aneroid in order to allow for the variation of gravity with latitude and height.

The corrections are in feet, being zero for all heights within a few hundred of Mean Sea Level. In latitude 10°, the correction is as much as +30 feet, at a height of 10,000 feet.

(B) Standard Fall of Temperature for a given difference of height.

This is given at intervals of 100 feet from 0 up to 10,000. The rate of fall is uniform and is approximately 3·6° F. per 1,000 feet of height.

(C) Temperature and Humidity Corrections.

This table occupies most of the space, and gives corrections to be applied to heights at intervals of 100 feet up to 10,000 feet for humidities of 0, 30, 60, 90 per cent. The humidity correction is quite small, and in previous tables has been neglected.

It is possible to adapt the tables where it is known that the temperature lapse-rate differs from the standard, and for fuller information the reader is referred to the introductory notes of the *Aneroid Tables*.

## Investigation of the Scope and Limitations of the Simple Stereoscopic Method of Contouring and Levelling

While the simple method as described above is admirably applicable to work on scales up to 1/20,000, it is clear that there is considerable room for improvement. Thus it becomes necessary to discuss how various imperfections affect the accuracy of the method, and how recent improvements are making it possible to reduce the amount of control on medium scales, and to enable accurate contours to be plotted on larger scales.

*The Parallax Equation: Effect of Errors in Parallax Difference on Height.*

The following equations were deduced in Chapter VI (p. 166):

$$p = \frac{f.B}{(H - h)} \qquad \text{[Equation VI.1]}$$

$$\delta p = p_d - p_a \qquad \text{[ „ VI.2]}$$

$$\frac{\delta h}{\delta p} = \frac{-(H - h)^2}{f.B} \qquad \text{[ „ VI.3]}$$

Where $p$ = parallax; $\delta p$ = difference of parallax; $f$ = focal length of camera lens; $B$ = length of air-base between exposures; $H$ = height of aircraft above the datum plane; $h$ = height of point measured above the datum plane; $p_d$ and $p_a$ are the parallax for two points a and d referred to $\delta h/\delta p$ = rate of change of height of ground with difference of parallax.

Any of the terms in equation (VI.1) may be in error. Even the picture plane may not be exactly in the focal plane of the lens if the temperature drop, at extreme flying heights, is not controlled in order to prevent contraction of the camera body. In some cases this effect may be neglected, while in others special precautions must be taken.

Stereoscopic measurement implies measurement of difference of parallax, and in order to consider the effects it is convenient to consider two points quite near the datum plane, so that equation may be written,

$$\delta h = \frac{H^2.\delta p}{f.B} \text{ approximately} \qquad \text{(VIII.2)}$$

$$= \frac{f}{s^2.B}\,\delta p \text{ where } s = \text{scale of photograph} = \frac{f}{H} \qquad \text{(VIII.3)}$$

The rate of change of height with parallax is found from equation (VI.3). This may now be reduced to more specific terms.

Assuming that the photographs are of one standard size taken with the Eagle IV cameras, the photographic measurement in the direction of flight is 165 mm., and assuming a longitudinal overlap of 60 per cent, the air-base $B = 0{\cdot}4 \times 165 = 66$ mm. and this distance on the ground is $0{\cdot}066/s$ metres.

Hence $\dfrac{\delta h}{\delta p} = \dfrac{f}{0{\cdot}066 \times s,}$ where $h$, $p$, and $f$ are in metres. Parallax, $p$ is usually measured in millimetres, $f$ in centimetres, and $h$ in feet.

Equation (VIII.3) may be rewritten in such a form that the expression $\delta h/\delta p$ = gives the difference of level in *feet* per 0·01 mm. difference of parallax, and it therefore becomes

$$\frac{\delta h \text{ (feet)}}{\delta p \text{ (0·01 mm.)}} = \frac{f \text{ (cm.)}}{201{,}100.s} \qquad \text{(VIII.4)}$$

It will therefore be seen that variation in height due to a parallax difference is dependent both on the scale and the focal length of the camera lens.

Hence if $s = 1/25{,}000$, $\delta h/\delta p = 0{\cdot}1243f$ feet per 0·01 mm.; and if $f = 20$ cm., this becomes 2·48 feet per 0·01 mm.

Table VIII.8 gives the relationship between scale, focal length, flying height, and effect of parallax error of 0·01 mm. for the possible scale range of vertical photography.

TABLE VIII.8

| *Scale.* | 1/50,000 | | 1/37,500 | | 1/25,000 | | 1/15,000 | | 1/10,000 | | 1/5,000 | | 1/2,500 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Focal Length f. cms.* | *H* | $\delta h$ | *H* | $\delta h$ | *H* | $\delta h$ | *H* | $\delta h$ | *H* | $\delta h$ | *H* | $\delta h$ | *H* | $\delta h$ |
| 5 | 8,210 | 1·24 | 6,130 | 0·93 | 4,105 | 0·62 | 2,460 | 0·37 | 1,640 | 0·25 | 820 | 0·12 | 410 | 0·06 |
| 10 | 16,420 | 2·48 | 12,260 | 1·86 | 8,210 | 1·24 | 4,925 | 0·75 | 3,280 | 0·50 | 1,640 | 0·25 | 820 | 0·12 |
| 15 | 24,630 | 3·72 | 18,400 | 2·79 | | | | | | | | | | |
| 20 | | | 24,620 | 3·72 | 16,420 | 2·48 | 9,850 | 1·50 | 6,560 | 1·00 | 3,280 | 0·50 | 1,640 | 0·25 |
| 30 | | | | | 24,630 | 3·72 | 14,750 | 2·25 | 9,850 | 1·50 | 4,925 | 0·75 | 2,462 | 0·37 |
| 40 | | | | | | | 19,690 | 3·00 | 13,120 | 2·00 | 6,560 | 1·00 | 3,280 | 0·50 |
| 50 | | | | | | | 24,620 | 3·75 | 16,420 | 2·50 | 8,210 | 1·25 | 4,105 | 0·62 |

*H* — Flying height (feet).
$\delta h$ = Difference of height per 0·01 mm. difference of parallax.

In Fig. 94 a graphical representation of this table is given, no values being given for flying heights above 25,000 feet, which is about the practical limit of height for photography, even when special precautions are taken against the low temperatures experienced.

From equation (VIII.4) it is seen that $\delta h/\delta p$ increases with the focal length,

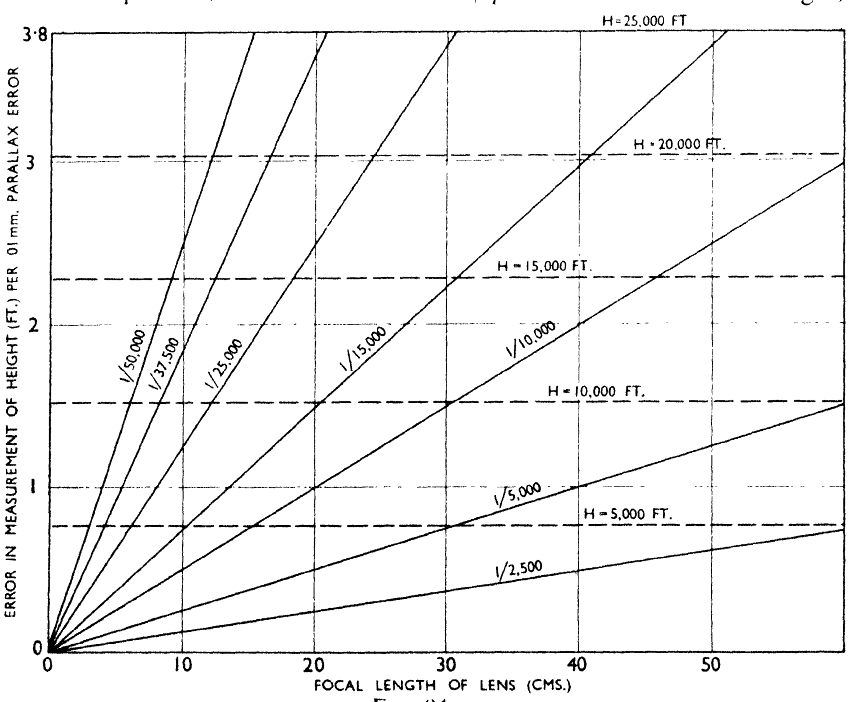



FIG. 94.

so that for levelling and contouring, other things being equal, the camera fitted with a lens of the shortest possible focal length should be used. Height distortion in plan is, however, increased inversely as the focal length, and for large-scale photography a short-focus lens obscures much detail required in plan.

The question of flying height must also be considered and it is usually necessary to make some compromise with regard to photographic requirements for planimetry and contouring, since in the former case the focal length should be as great as possible and in the latter as short as possible. The Ordnance Survey 1/2,500 plan is not concerned with contours and the photography at 1/5,000 for this plan is taken at a height of just over 8,000 feet with a camera fitted with a lens of 50 cms. focal length. These photographs have much less obscured detail when the usual 20 cm. focal length of the R.A.F. cameras is used for photography from a height of just over 3,000 feet. Moreover, the atmospheric conditions are generally steadier at the greater height, thus enabling greater accuracy of navigation to be maintained along a strip.

*Factors affecting the Accuracy of Height Determination.*

(i) *General.* Where stereoscopic measurement is used, the accuracy of determination of difference of level will be affected by displacements of the image position caused by lens, shutter, paper and film distortions.

In addition, where photographs are examined as though each exposure of a stereo-pair were truly vertical and at a constant height above the datum, the tilts and inclination of the air-base which are almost invariably found, introduce false parallax, owing to the fact that the parallactic grids, or floating mark, define a horizontal plane of reference. The effect is the same as that obtained when using a dumpy level with a collimation error to define the horizontal plane of reference.

Until a few years ago the many sources of considerable error made it appear hopeless to expect that contouring on scales larger than about 1/25,000 by simple methods could give the desired accuracy. Research work by the Air Survey Committee in the development of the Arundel Method as published in 1934 [86] showed that contoured maps could be produced at the military scale of 1/25,000 of accuracy comparable with that obtained by ground methods, if adequate ground height control were available, and with no other instruments than the Barr and Stroud Topographical and Precision Stereoscopes, even with tilts up to 2°. The problem of contouring with few ground heights has led to the development of such instruments as the Fourcade Stereogoniometer and other precise stereoscopic instruments.

Since that time very rapid developments have been made in reduction of tilts and errors in materials, and in the greater accuracy of measurements of aircraft heights and of stereoscopic parallax.

(ii) *Tilt.* A pair of tilted photographs examined flat in a stereoscope will be given a false slope from the reference horizontal plane as defined by the floating mark, and, in the case of a lateral slope of 1°, the error across the overlap on a photograph 7 × 7 inches is about 200 feet for a scale of 1/25,000, or 40 feet on a scale of 1/5,000. Even the small tilts still present when automatic control is used give rise to greater errors than are allowable in large-scale engineering surveys, and a routine of observation is necessary to minimize the effect of tilt.

(iii) *Lens.* The chief advances have been in widening the field, and the maximum distortion at the edges of the photograph, which may be neglected, remains at about 0·01 mm. The focal length of a lens is assessed for minimum distortion over the field, and for an ultra wide-angle lens it is not likely to exceed ·06 mm. for a lens of $3\frac{1}{4}$ inch focal length or 0·1 mm. for one of 5 inch focal length. The *differential* distortion between stereoscopically measured points may often be neglected although the employment of a reseau affords a check.

(iv) *Shutter.* Previously the focal-plane shutter was generally used in this country, and gave rise to a false slope up to 4 minutes of arc, owing to the movement of the slit across the film while the aircraft was moving. The resulting error in level is 0·116 per cent of the horizontal distance. The louvred shutter now fitted has eliminated such distortions. Similarly the leaf-type shutters fitted to ultra wide-angle lens cameras avoid appreciable distortion.

(v) *Film.* Probable film shrinkage under the conditions of minimum distortion, i.e., with temperature and humidity control, has been reduced from about 0·3 per cent to about 0·03 per cent. These ideal conditions are, however, rarely attained.

(vi) *Paper.* Recent waterproof and other types of non-distorting paper have enabled the probable effect of shrinkage during processing to be reduced from 0·5 per cent with ordinary papers to about 0·03 per cent. This applies, however, only when humidity variations are eliminated. For example, the waterproof papers will vary by as much as 0·2 per cent when the relative humidity changes by 15 per cent.

(vii) *Measurement of Parallax.* This depends upon the skill of the observer as well as upon the instruments available. It appears that with instruments of the type of the Precision Topographical Stereoscope, differences of parallax can be ascertained to 0·01 mm., under the best conditions. With recent improvements, both in instrument design and

in type of floating mark, the accuracy of routine observation has been improved. Effects of errors in parallax measurement have been shown in Fig. 94 and it will be noticed that if differences of parallax can be measured to the nearest 0·01 mm., errors are not very large from this cause, except when $f$ is large and the ground slope small.

(viii) *Inclination of Air-Base.* Owing to difficulties of navigation and to air pockets, the height of exposure of adjacent photographs is likely to vary by a small amount. If the photographs of a pair are both exposed exactly vertically, the effect in the direction of stereoscopic measurement along the base-line is that the inclined length is used instead of the projected length of the air-base in the parallax equation. Hence if $B_1$ is the horizontal projection of the base-line and $\alpha$ is its inclination, then

$$p = \frac{f.\,B.\cos\alpha}{(H-h)} = \frac{f.\,B_1}{(H-h)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII}.5)$$

$$\frac{\delta h}{\delta p} = \frac{f}{s^2.B.\cos\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII}.6)$$

With manual control, $\alpha$ may be $1^\circ$, causing a variation of 0·015 per cent in the length of base. This has a negligible effect on levels and therefore the slope effect may be altogether neglected. In the case of graphical Arundel plots the length of air-base may be in error by some 0·3 mm., on a length of about 2·75 inches. This is approximately 0·3 per cent, and this does not cause a very large error in height determination. Increased accuracy of base-line determination, by some method of fixing the principal points by computation as described in Chapter IX, will reduce errors from this cause.

(ix) *Altimeter and Statoscope Readings.* From equation (2) $\frac{\delta h}{\delta p} = \frac{H^2}{f.B}$ approximately, and if H is varied by $\delta H$, then $\delta h$ is varied by

$$\frac{[(H \pm \delta H)^2 - H^2]}{H^2} = \pm 2\,\delta H \text{ approximately} \quad . \quad . \quad . \quad (\text{VIII}.7)$$

Consequently, if the altimeter reading is in error by 1 per cent then height differences will be in error by 2 per cent.

Formerly, altitudes were recorded by a small aneroid the image of which appeared at the side of the photograph. These instruments may give altitude readings which are 300 feet or more in error. If this is so when $H$ is 15,000 feet, $\delta H = 2$ per cent and $\delta h = 4$ per cent; for $H =$ 5,000 feet, $\delta H = 6$ per cent and $\delta h = 12$ per cent.

It is obvious that the considerable errors introduced in this manner are intolerable for large-scales involving less height, and even for scales

such as 1/25,000 a definite routine is necessary to reduce the effect.

Not only is it possible now to determine absolute heights much more accurately by means of special altimeters and by the use of aneroid tables as described above, but the employment of statoscopes, as described in Chapter IV, enables small variations to be measured. It thus appears possible to measure heights to within about 20 feet of the correct value. In this case for 15,000 feet altitude, $\delta H = 0{\cdot}133$ per cent and $\delta h = 0{\cdot}27$ per cent; for 5,000 feet altitude $\delta H = 0{\cdot}4$ per cent and $\delta h = 0{\cdot}8$ per cent. Relative heights are likely to be determined much more accurately than absolute ones.

*Errors in the Measurement of Difference of Level as derived from Stereoscopic Observation.*

It is proposed to consider

(*a*) Photographs taken with manual control, with tilts up to 2°, by a camera fitted with a focal-plane shutter, and exposed and printed on photographic materials available before the newly developed non-distorting films and papers became available. These will be called the "old conditions" and were those of about 1930.

(*b*) Photographs taken with automatic-pilot control by means of a camera fitted with a louvred shutter, and exposed and printed on film and paper least liable to distortion. Tilts are limited to $\frac{1}{4}$°. These will be called the "new conditions."

For purpose of comparison, the width of photograph considered will be that taken with the "Eagle" IV camera, namely 165 mm., and the spacing between two points measured equal to a 50 per cent overlap, i.e., 82·5 mm. This is an extreme case because intermediate height control points will usually make the maximum distance much less than this. The spacing of these points on the ground $= {\cdot}0825/s$ metres $= 0{\cdot}2705/s$ feet.

It should be noted that in both cases errors in parallax measurement which may be as high as 0·02 or 0·03 mm. will affect the accuracy of height determination. Errors in height for 0·01 mm. difference of parallax are given in Table VIII.8.

The following values are intended to give an indication of the order of the errors introduced. In practical working it will always be necessary to employ a routine which will enable such errors to be distributed.

(i) *Old Conditions.*

(*a*) *Tilt.* $\pm$ 2°, giving rise to a difference of level equal to 3·5 per cent of the horizontal distance.

(*b*) *Focal-Plane Shutter.* $\pm$ 4′ of arc giving 0·12 per cent of horizontal distance.

(*c*) *Film Shrinkage.* —0·3 per cent; giving error of —0·3 per cent of horizontal distance.

(*d*) *Paper Shrinkage.* —0·5 per cent; giving error of —0·5 per cent of horizontal distance.

Maximum Error = —4·42 per cent = —0·0442 of horizontal distance.

Maximum Error (Excluding tilt) = —0·92 per cent = —0·0092 of horizontal distance.

Table VIII.9 gives errors in height over the 50 per cent overlap.

TABLE VIII.9

| *Scale* 1/ | *Error in height* (*ft.*) (*including tilt*). | *Error in height* (*ft.*) (*excluding tilt*). |
|---|---|---|
| 50,000 | 597 | 124 |
| 37,500 | 447 | 93·3 |
| 25,000 | 298 | 62·3 |
| 15,000 | 179 | 37·3 |
| 10,000 | 119 | 24·9 |
| 5,000 | 60 | 12·5 |
| 2,500 | 30 | 6·2 |

(ii) *New Conditions.*

(*a*) *Tilt.* $\pm\frac{1}{4}^\circ$ giving an error 0·438 per cent of horizontal distance. It is possible that this tilt might be reduced somewhat under very favourable conditions.

(*b*) *Shutter Distortion.* Eliminated.

(*c*) *Film Shrinkage.* May be reduced to —0·03 per cent of horizontal distance provided that care is taken in handling the film.

(*d*) *Paper Shrinkage.* May be reduced to —0·03 per cent of horizontal distance provided that humidity conditions are controlled from exposure. For a variation of 15 per cent in relative humidity there is a variation in size of the order of 0·2 per cent. Take this as the worst value.

The maximum error becomes (*a*) —0·668 per cent including tilt and humidity change; (*b*) —0·230 per cent excluding tilt, including humidity change; (*c*) —0·06 per cent excluding tilt, excluding humidity change.

Table 8/9 gives the height errors over a 50 per cent overlap as before.

Where accuracy is required, control can be provided by the employ-

TABLE VIII.10

| | *Maximum error in height (feet).* | | |
|---|---|---|---|
| *Scale 1/* | (*a*) *Including tilt and humidity.* | (*b*) *Including humidity.* | (*c*) *Excluding tilt and humidity.* |
| 50,000 | 90·2 | 31·1 | 8·0 |
| 37,500 | 67·6 | 23·3 | 6·0 |
| 25,000 | 45·1 | 15·5 | 4·0 |
| 15,000 | 27·1 | 9·3 | 2·4 |
| 10,000 | 18·1 | 6·2 | 1·5 |
| 5,000 | 9·0 | 3·1 | 0·8 |
| 2,500 | 4·5 | 1·55 | 0·4 |

ment of a reseau on the glass pressure-plate, which prints accurate squares on the photograph, or by measurements between calibrated collimating marks. It is important that it should be realized that the figures quoted are relative only, and the effect in practice of measuring on the plate to detect distortions may produce results of even greater accuracy, provided that distortion is uniform over the area of the plate.

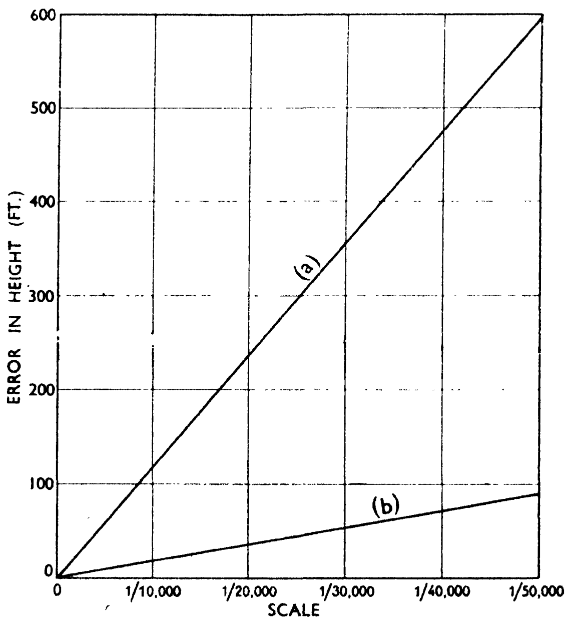



FIG. 95.

The curves plotted in Figs. 95 and 96 show errors in difference of level with and without tilt respectively.

It is clear that the progress has been considerable and under present conditions, even when the effect of tilt is present, probable *maximum* error has been reduced to one-seventh of its former amount. When contouring by the Arundel Method under the old condi-

tions, it was not unusual to have to make an adjustment for height error due to tilt which was considerably greater than the difference of level to be measured. There is much to be said for the efficiency of the routine evolved which enabled accurate contours to be plotted at 1/25,000 despite this obstacle.

To the errors given above must be added those due to (i) inaccuracy of measurement of difference of parallax and (ii) inaccuracy of measurement of height of aircraft.

It must be remembered that in all the above instances errors have all been assumed to be in the *same* direction, thus giving the worst case. Actually, of course, this rarely occurs, and the maximum error will often be much less than those given above. As has been mentioned, a reseau gives a measure of control, and by working in circuits errors may be reduced.

It should therefore be possible on a scale of 1/5,000, under the best conditions, to obtain stereoscopically the difference of level across the 50 per cent overlap, to within a few feet, and by judicious adjustment in circuits between ground control points it should not be impossible to obtain levels to within two feet or three feet.

*Contouring from Rectified Photographs.*

It will be seen from Tables VIII.8, VIII.9, VIII.10, and those which give the effect of small errors in parallax measurement, that by far the greatest error is due to tilt. Hence the possibility of rectification of photographs must be considered. Here the necessity of providing four planimetric controls per photograph makes the method unduly expensive, unless rectification is being used for plotting the plan, or sufficient control is already available.

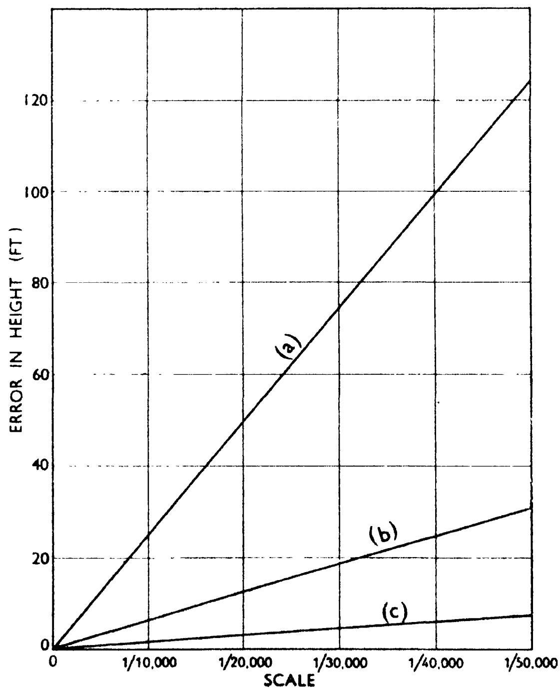



FIG. 96.

On the other hand, if the photographs are rectified to ground control to eliminate tilt distortions, it is possible that levels may be obtained to within a few feet, with a reduced amount of height control on a plan at a scale of 1/5,000.

*Tracing Stereometer.*

The tracing stereometers previously described provide a very convenient method for contouring from rectified photographs, particularly when these have been brought to a mean photographic scale during printing. When the floating mark is set to a reading corresponding to a particular height, the mark may be brought to ground level and kept in constant contact with the ground while the stereometer is moved parallel to the base-line. The plotting pencil seen in the centre will trace out the contour at the mean scale. This is repeated for other contours.

When the stereometer is used for contouring on an existing map the photographs are rectified and printed to a mean scale which corresponds to that of the map.

The Brock process (described in Chapter IX) has also been used to produce contours from rectified photographs.

## Determination of Levels from Oblique Photographs

*Crone's Method.*

Oblique photographs taken for the determination of levels are not usually tilted more than 25° from the horizontal. The process evolved by Major D. R. Crone, R.E., [26] involves a graphical construction with reference to distant points of known height, in order to ascertain the position of the camera in space, so that additional levels can be found. He likens the problem to that of levelling with an unadjusted clinometer where reciprocal readings are not possible.

In Fig. 97, WXYZ represents an oblique photograph on which are the images A, B, C, of three ground points *a*, *b*, *c*, of known level. P is the principal point; L′L″, the horizon line as shown on the photograph or its estimated position and PL is the perpendicular from the principal point on to the horizon line. VV′ represents the elevation of the photograph when looking along its plane in the direction of the arrow and perpendicular to the principal line PL. The projection of the horizon line on VV′ is $L_1$, and that of the principal point P is $P_1$, while the projections of the control points on this line are at $A_1$, $B_1$, $C_1$. $P_1O$ is set out perpendicular to VV′ and made equal to the principal distance of the

camera, so that O is the camera station. Join $OL_1$, which represents the elevation of the horizon plane, $\angle OL_1V'$ being the apparent angle of tilt $\theta$. Perpendiculars $A_1A_2$, etc., are dropped on to $OL_1$ so that $OA_2$, $OB_2$, $OC_2$ represent the plan distances of the control points A, B and C respectively from O, as projected into the principal plane.

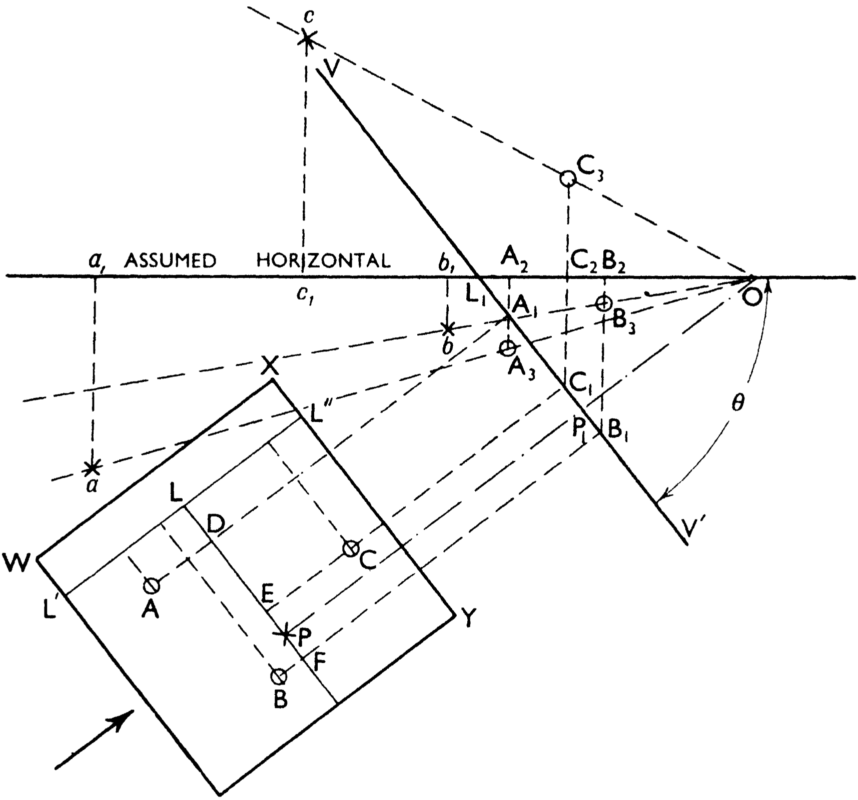



FIG. 97.

Horizontal distances of the picture positions of the control points A, C, B from the principal plane, as measured in the plane of the photograph, are AD, CE, BF respectively; and these distances are set off from $A_2$, $B_2$, $C_2$, perpendicular to $OL_1$, thus fixing $A_3$, $B_3$, $C_3$. These are the plan positions of the photographic points A, B and C plotted in the assumed horizon plane in their correct relationship to O. If the construction is plotted on kodatrace or other transparent material each ray can be superimposed over the correct position of its corresponding point at the plotting scale of the map. Thus points *a*, *b*, *c*, plotted on the master plan are shown

in Fig. 97 in correct position with relation to the previous construction which has fixed $A_3$, $B_3$, $C_3$, and which is superimposed on the master plan. The camera station is resected in exactly the same way as a station in ordinary plane-tabling is resected by the tracing-paper method.

Even if the horizon line which was chosen was somewhat in error, it is unlikely that there will be any appreciable error in plan, but for levelling, the slope of the horizon line must be checked.

Referring again to Fig. 97, $OL_1$ is the elevation of the line of intersection of principal plane and assumed horizon line, and $a_1$, $b_1$, $c_1$ are the projections of $a$, $b$, and $c$ respectively into this line. That is, they are the elevations in the principal plane of the projections into the horizon plane of the points in space.

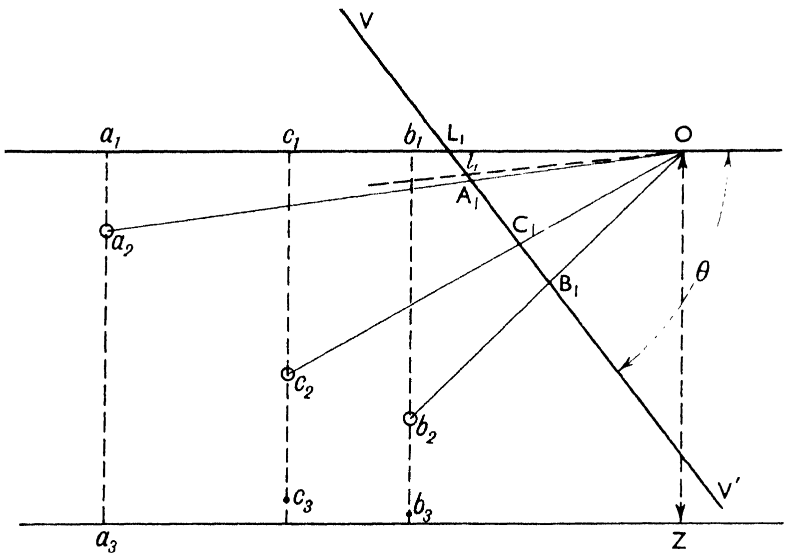



FIG. 98.

In Fig. 98, the same letters are used, but the diagram is separated here for the sake of simplicity. The diagram again represents a section in the principal plane. Lines drawn through $A_1$, $B_1$, $C_1$ from O meet projections perpendicular to OL from $a_1$, $b_1$, $c_1$ respectively at $a_2$, $b_2$, $c_2$ and these represent the elevations of the ground control points, if the horizon line chosen on the photograph is in the correct position and at the correct slope. This may be checked as follows: Set down from $a_2$, the known height ($h_a$) of the point, making allowance for curvature of the

earth and atmospheric refraction.* This fixes $a_3$. Similarly $b_3$, and $c_3$ are fixed. If the assumed horizon were drawn correctly, then a line from $a_3$ parallel to $L_1O$ would pass through $b_3$ and $c_3$ so that the perpendicular projection from O to the Datum, fixes OZ as the height ($H$) of aircraft at exposure.

This condition is unlikely to be achieved from the first assumed horizon line and this must now be adjusted. Crone has developed a graphical method [6, 26] but the following outline of a method of computation will show the general principle.

In Fig. 99, let L′L″ be the assumed horizon line as before and l′l″ its true position displaced from the assumed line by $y$ at the centre measured along the assumed principal line LP, and having a slope of $\alpha$ in relation to the assumed horizon. The photograph is tilted through $\theta$ from the horizon as seen in Figs. 97 and 98. Also the horizon line from O will pass through point $l_1$.

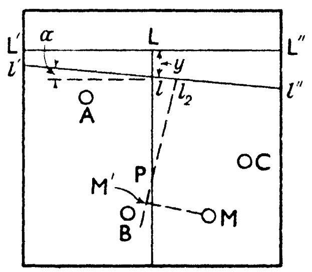


FIG. 99.

The problem is to find values of $y$ and $\alpha$ so that the horizon line can be plotted in its correct position, when it will be found that the points corresponding to $a_3$, $b_3$, $c_3$ will be in a straight line and parallel to the true horizon.

Let $H$ be the height of the aircraft, then if $a_3$, $b_3$, $c_3$ are collinear, since $a_2a_3 = h_a$, etc., it follows by reference to Fig. 98 that

$$H = a_1a_2 + h_a = b_1b_2 + h_b = c_1c_2 + h_c.$$

Now $a_1a_2 = A_1A_2 \cdot a_1O/A_2O$ from Fig. 97

also from Fig. 97, $A_1A_2 = (DL - y + AD.\ \tan\alpha).\ \sin\theta$

and similarly for points $b$ and $c$.

$$\left.\begin{aligned} \text{Hence } H &= (DL - y + AD.\ \tan\alpha).\ \sin\theta.a_1O/A_2O + h_a \\ &= (FL - y + BF.\ \tan\alpha).\ \sin\theta.b_1O/B_2O + h_b \\ &= (EL - y - EC.\ \tan\alpha).\ \sin\theta.c_1O/C_2O + h_c \end{aligned}\right\} \text{(VIII.8)}$$

The unknowns are $H$, $y$ and $\alpha$, which can all be found by solving the above equations. The horizon can now be replotted in its correct position and the whole plotting process checked over. This will usually involve adjusting the original plot in another colour.

First of all the corrected principal line is drawn by projecting from P

* For values of the corrections to be applied for curvature and refraction refer to standard text-books on surveying, such as *Topographical Surveying*, by Col. Sir Charles Close. If the sights are over great distances special coefficients are necessary and these may be obtained from tables published by the Survey of India.

on to $l'l''$ thus fixing $l_2$ (Fig. 99). The process is then exactly similar to that used before up to the fixation of the datum line.

In Fig. 100, the points are shown in elevation in the principal plane, and plotted in relation to the correct position of the horizon OL. Heights of the points *a*, *b*, *c*, are shown above the datum plane. Suppose it is required to find the height of any other ground point *m* the image of which is M in the photograph (Fig. 99). M′ is the projection of M on to the correct principal line $Pl_2$. If this point appears in two photographs, its position can be fixed in plan and its projection will appear at *m′* in Fig. 100, the projection of its image in the photograph considered being $M_1$. By producing $OM_1$ to meet in *m* a line projected vertically from *m′*, the height $h_m$ can be measured directly from the datum line. After allowing for curvature and refraction, the corrected height of the point is found.

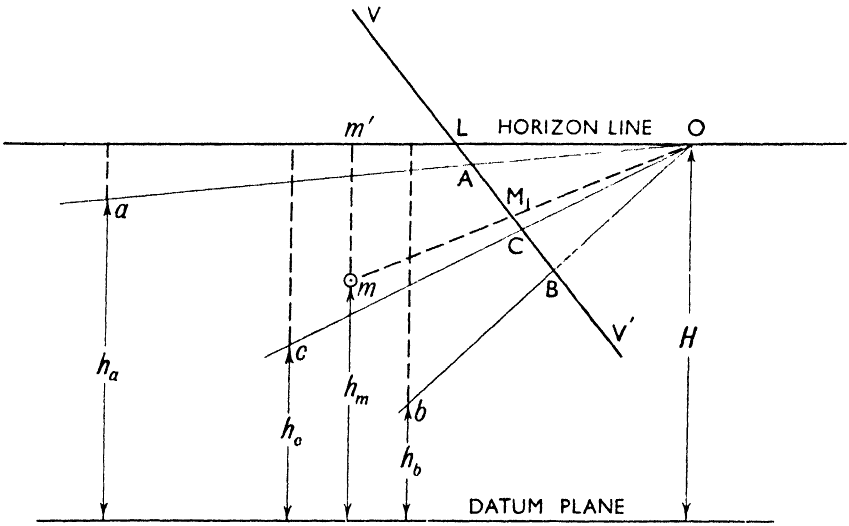



FIG. 100.

It has been found convenient here to separate some of the various diagrams for the sake of clarity, but in practice all the plotting can be done on one diagram.

For a complete explanation of the method the reader is referred to the articles by Crone in the Second Report of the Air Survey Committee [6] and in the Proceedings of the Conference of Empire Survey Officers, 1935. [26]

*Scope of Levelling from Obliques.*

In the second Report of the Air Survey Committee 1935 [6] Crone's method is introduced as follows: "Contours as well as planimetry can be determined from a minimum of fixed ground points and no natural horizon

is required on the photographs. These factors render it a particularly suitable method for the reconnaissance survey of inaccessible country, or as an auxiliary to mapping from vertical photographs, for enabling a suitable number of spot heights to be fixed for the contouring of the vertical photographs."

Much of the pioneer work in India has been done by Crone. It is important to note that he has been concerned in the main with North-West India, a different type of country from that found in Britain; and that the largest scale for publication of maps is 1/63,360 or one inch to one mile, or 1/50,000 for military purposes. Crone has concluded that the Arundel Method meets Indian requirements for planimetry, but that the photographic level control points should be evaluated from obliques by the method outlined above instead of by the parallax measurement method previously described. Finally, contours are sketched in by stereoscopic examination of both verticals and obliques. It should be remembered that the area will be partly covered by verticals and partly by obliques.

The control points in the oblique photographs are often at a considerable distance away in inaccessible country, while points to be evaluated are comparatively near. This improves the possibilities of accuracy in determination of heights. Identification without stereoscopic examination would be well nigh impossible in areas where the topography consists of many hills and valleys, all looking much the same.

Crone's method is very suitable for contouring on scales of one inch to one mile, or smaller, as very little ground control is required. It has been estimated that twenty fixed points should be enough to contour adequately 5,000 square miles at 1/100,000.

This method naturally depends upon the identification of points at considerable distances, and its useful application is really limited to scales of the order mentioned above, in mountainous or very hilly country. Many of Crone's original objections to the application of the Arundel Method have become of less importance with the possibility of tilt reduction to about $\frac{1}{4}^{\circ}$ by means of the automatic pilot.

For scales of 1/25,000, or larger, contours would be produced, either by the simple method from verticals, as described above, together with adequate ground control, or by more elaborate apparatus requiring less ground control, and some of these will be described in Chapters IX and X.

### Application of Simple Methods: General Conclusions

It appears that the ordinary Arundel Method of plotting, coupled with simple stereoscopic measurements, will probably enable adequately

contoured maps to be produced up to a scale of about 1/15,000, or rather larger, provided that height control is present at the rate of some nine spot-levels per overlap. For larger scales, stereocomparator measurements are necessary on rectified photographs; or a more elaborate instrument must be used, enabling the tilt to be set in the instrument and thereby eliminated in measurement. These instruments are also useful in carrying forward an aerial triangulation over areas with little or no ground control.

The Air Survey Committee in its second Report in 1935 [6] concluded: "The elaboration of improved methods of plotting or the design of improved apparatus which will enable heights to be determined accurately from the same points as those used for the horizontal plotting, would add greatly to the value of the Arundel Method." The Committee also considered that the question of height control is dependent upon automatic-plotting machines, of which the present precision justifies the hope that levelling can be carried forward in such machines without known heights in every overlap.

Levels for construction surveys cannot be obtained satisfactorily by this method without expensive ground surveys, and even then one must be content with the nearest "foot or two."

Hotine, in a paper before the Institution of Civil Engineers [61] stated: "Levels of sufficient accuracy for detailed engineering projects cannot be provided from air photographs, and it is unlikely that they ever will be, whatever the system of measurement adopted. The fraction of a foot required, when translated into the stereoscopic parallax used for its measurement, is so small a quantity as to be confused with the grain size of the photographic emulsions. The air photograph can supply contours of sufficient accuracy for any purpose to which a contour may be legitimately put, but it cannot supply spot-levels."

How far it is advisable to use air photographs for engineering levelling is a matter not only of cost but of required accuracy. For surveys which approach construction survey scales, namely 100 feet to 1 inch or larger, either a specialist organization having available the most accurate plotting instruments must be employed, or the engineer should be content to obtain his plans from air photographs and carry out his levelling as before. This will be made easier by the availability of separate prints for stereoscopic examination.

It is held by many in this country that while accurate large-scale engineering plans can be produced by air survey, levels are best determined by ground methods for construction surveys. This opinion is undoubtedly largely influenced by the fact that British engineers have

available the excellent six inch to one mile Ordnance Survey maps which they can use for their preliminary work. There is now a growing tendency here to employ rather more elaborate methods for contouring.

In the United States it is not customary to attempt contouring by the simple methods outlined above.

When suitable instruments are not available for contouring from air photographs and the engineer wishes to produce his own contours, the advantages of tacheometric surveying for levelling and contouring at the stage just prior to construction, become apparent. Tacheometry deserves much closer attention from British engineers than it has previously received. A combination of theodolite traversing with tacheometric observations will enable ground work to be considerably accelerated.

In countries less developed and where the maps of the country are not adequate for the preliminary plans, the advantages of levelling from air photographs become obvious.

CHAPTER IX

# PREPARATION OF MAPS FROM VERTICAL PHOTOGRAPHS WITH SPECIAL REFERENCE TO LARGE-SCALE PLANS AND REDUCTION OF GROUND CONTROL: ELIMINATION OF GRAPHICAL PROCESSES AND ANALYTICAL METHODS OF CONTROL: RECTIFICATION AND PLOTTING FROM RECTIFIED PHOTOGRAPHS: REVISION OF LARGE-SCALE PLANS

## Preparation of Large-Scale Plans

The radial-line method and its application to medium-scale plans has been discussed in Chapter VII, and it remains to consider the applications of the method to larger scales. The engineer is concerned with scales of the order of 1/2,500 rather than those of 1/25,000, except for reconnaissance and preliminary surveys.

As the scale of mapping increases, so does the normal error of plotting approach more nearly to the allowable limit of error. On a scale of 1/25,000, 0·01 inch on the paper represents about twenty feet on the ground, while at 1/2,500, 0·01 inch represents approximately two feet. The engineer, using the latter, would endeavour to scale to the nearest foot, but those using the 1/25,000 map would probably be contented with the nearest fifty feet. Hence 0·01 inch is of more consequence on the large scales. This does not, of course, take into account the accumulated effect of plotting errors. Moreover the limits of scale must be controlled by the conditions of flying, namely, constant height, accurate measurement of the accuracy of line of flight, and verticality of exposure.

It is now possible with improvements in photography and materials to use the radial-line method for direct graphical plotting on scales up to 1/2,500. With reduction of tilt to $\frac{1}{4}^\circ$ or so, and ground control points spaced a mile or so apart, a good result can be obtained for that detail which can be identified. Wire fences, details obscured by oblique rays to buildings, etc., are measured up on the ground and the plotting is completed.

The following table gives a list of values obtained for the co-ordinates of the principal points on a strip of six photographs. The plotting was done by the Arundel Method between two triangulation stations about a

mile apart, one at each end of the strip. The photographs were taken with automatic-pilot control at a scale of approximately 1/5,000 and the scale of plotting was 400 feet to 1 inch. The plotting was carried out, as a normal part of their surveying course, by the final year Civil and Municipal Engineering Degree students at University College, London. The variations obtained are relatively quite small when one remembers that (*a*) at 400 feet to 1 inch, 10 feet is represented by 0·025 inches, (*b*) errors of plotting are likely to be introduced while learning a method, (*c*) students at this stage are not mature engineering or surveying draughtsmen. There is little doubt that had time permitted a second plot by the same men, very much closer values would have been obtained. Similar values have been obtained by subsequent classes.

## AIR SURVEY PLOT, 1936

### Municipal Engineering Department, University College, London

Comparison of Students Plots for Co-ordinates of Principal Points
Scale 400 feet to 1 inch

| | *Photograph No.* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
| *Student.* | *N* | *E* | *N* | *E* | *N* | *E* | *N* | *E* | *N* | *E* | *N* | *E* |
| A | 9222 | 17232 | 8688 | 18496 | 8294 | 19784 | 7866 | 20918 | 7447 | 22108 | 7086 | 23084 |
| B | 9224 | 17235 | 8687 | 18517 | 8278 | 19804 | 7860 | 20924 | 7444 | 22120 | 7093 | 23090 |
| C | 9222 | 17270 | 8690 | 18510 | 8294 | 19784 | 7865 | 20918 | 7446 | 22138 | 7086 | 23096 |
| D | 9223 | 17238 | 8676 | 18505 | 8267 | 19795 | 7845 | 20920 | 7432 | 22110 | 7085 | 23090 |
| E | 9235 | 17240 | 8688 | 18515 | 8283 | 19790 | 7861 | 20920 | 7446 | 22117 | 7088 | 23081 |
| F | 9220 | 17234 | 8689 | 18499 | 8289 | 19791 | 7862 | 20922 | 7442 | 22121 | 7078 | 23089 |
| G | 9227 | 17250 | 8696 | 18495 | 8297 | 19793 | 7880 | 20916 | 7460 | 22101 | 7092 | 23073 |
| H | 9210 | 17230 | 8680 | 18500 | 8282 | 19792 | 7860 | 20932 | 7448 | 22116 | 7088 | 23094 |
| I | 9215 | 17271 | 8686 | 18502 | 8285 | 19797 | 7868 | 20928 | 7451 | 22122 | 7093 | 23095 |
| J | 9237 | 17251 | 8683 | 18521 | 8278 | 19811 | 7847 | 20939 | 7440 | 22123 | 7102 | 23101 |
| K | 9220 | 17242 | 8680 | 18503 | 8280 | 19807 | 7860 | 20922 | 7446 | 22115 | 7097 | 23090 |
| L | 9200 | 17230 | 8680 | 18500 | 8280 | 19800 | 7860 | 20920 | 7400 | 22100 | 7080 | 23080 |
| M | 9238 | 17250 | 8690 | 18508 | 8289 | 19796 | 7854 | 20914 | 7432 | 22106 | 7071 | 23068 |
| N | 9220 | 17230 | 8675 | 18500 | 8270 | 19791 | 7850 | 20916 | 7430 | 22100 | 7078 | 23075 |
| O | 9215 | 17260 | 8690 | 18500 | 8280 | 19803 | 7870 | 20920 | 7440 | 22100 | 7080 | 23080 |
| **Mean** | **9222** | **17244** | **8685** | **18504** | **8282** | **19797** | **7860** | **20923** | **7443** | **22114** | **7086** | **23087** |

*N.B.*—Each point was measured twice and the values meaned.

The Arundel Method depends upon the radial assumption, namely, that corresponding lines on photograph and ground radiate from the principal point. Since tilt distortions are radial from the isocentre and height distortions from the plumb point, these points have been used for isocentral and plumb point triangulation. Since the isocentre is about

midway between the principal point and the plumb point, tilt distortions are about equal for principal and plumb-point triangulation.

Neither isocentre nor plumb point can be marked on the photographs mechanically at exposure and must be determined on the photographs by finding the angle of tilt from ground control points. As a result neither method has been used extensively. In Britain the efforts have been towards improvement of photography so that the simple methods of plotting could be retained, while on the Continent the tendency has been to resort to plotting machines, which not only allow the stereoscopic pair to be set to their respective angles of tilt, but also enable contours to be accurately plotted.

Where, however, the ground height variation in relation to flying height is too great for the principal point triangulation to be effective, then a plumb-point triangulation may be used, as described by Hotine in the Air Survey Committee's Professional Paper *Extensions of the Arundel Method.* [58] Since that time (1929), improvements in flying and instruments have so widened the scope of the Arundel Method that resort to plumb-point triangulation is unlikely, except in special methods such as the Brock process (p. 269).

It appears that in certain circumstances, where height variations are small and automatic control is used to maintain the tilt at a small value, the simple graphical method may be used for planimetry for scales of 1/5,000 and possibly somewhat larger. Usually, however, the amount of ground measurement required will make it comparatively easy to establish a closer ground control, and some form of rectification may then be used in printing the photographs to eliminate the tilt effect, unless automatic control has kept it at a very low figure.

In large-scale planimetry it is more satisfactory to employ an instrument which will enable some of the graphical processes to be eliminated, thus ensuring a more rigid solution. Such instruments are the Cambridge Stereo-comparator, Zeiss Stereo-comparator and the Radial Triangulator, all of which will be described later.

Such instruments also widen the scope of small-scale surveys by enabling the amount of ground control to be reduced.

For large-scale engineering surveys, particularly those for roads and railways, it is often not possible to fly in straight strips because of the winding route. Straight strips would result in considerable wastage of photography and the automatic pilot cannot be used to fly along the route on account of the turns. For this reason the commercial air photographers often prefer in such instances to fly without automatic control and rectify when printing.

Hotine in a paper before the Institution of Civil Engineers [61] remarked: "For the preparation of large-scale plans, the air photograph is almost invariably cheaper than ground methods if a sufficient area is in question, in spite of the fact that a greater proportion of the work, in comparison with smaller scales, remains to be done on the ground. It is, of course, only possible to plot such detail as can be seen on the photograph, and correctly interpreted with the help of a stereoscope; any further detail which may be required must be supplied on the ground."

At this stage only the preparation of plans is considered and the problem is rather different when levels are required. Different methods must be employed if reasonably accurate contours are required and some of these methods are described later in this chapter. In many cases levels for construction surveys are taken by ordinary ground methods, and for setting out work, ground measurements must be made.

Hotine in the same paper says: "It is unlikely that the air photo will ever be used for providing levels of the accuracy usually associated with large-scale engineering plans. . . . In a few words the air photo can replace the clinometer, the plane-table and to a large extent, the chain, but it cannot replace the spirit-level and the theodolite."

## ELIMINATION OF GRAPHICAL MINOR-CONTROL PLOTTING

Difficulty of ensuring accurate identification and fixation of points, and of drawing lines in the right places on the photographs and when tracing from them in large-scale plotting from vertical photographs, has led to the development of instruments which enable the co-ordinates of minor control-points to be determined. In most cases such instruments employ stereoscopy. With them it is possible to take measurements so that co-ordinates of minor control-points can be computed from a minimum number of ground control-points whose co-ordinates are known. In addition, such instruments may be applied to the plotting of medium and small-scale maps with a minimum of control.

When a triangulation between these auxiliary points* is made for planimetry only, it is called a *radial triangulation*; when heights are also concerned the third dimension is introduced and the process is known as *aerial triangulation*.

An instrument such as the Barr and Stroud Precision Topographical Stereoscope (Fig. 77) for making co-ordinate measurements on the photographs, could be used for this purpose, but for this class of work it is found more convenient in practice to employ a single floating mark,

* These are often called "pass-points" in the United States and in Germany.

similar to one of those described in Chapter VII rather than to depend upon parallactic grids. It is considered that greater precision can be obtained from the single mark. (Perhaps it is clearer to say that the two separate half-floating marks fuse into a single complete floating mark.)

When the single mark is used, the photographs can be "base-lined" by setting the floating mark to ground where required, instead of setting a convenient grid-cross to ground in the vicinity of the principal point. When this has been done the direction of the common base-line on a pair of photographs provides the $x$-axis of co-ordinates, with the principal point as the origin. A scale is provided to register movements along the $x$-axis, and another to record the $y$-ordinates (see Fig. 101). When the

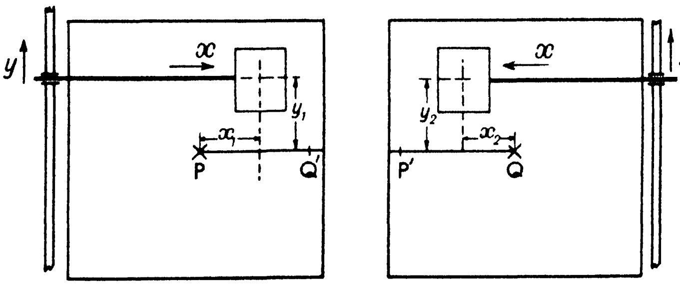



FIG. 101.

photographs have been set in correspondence in the instrument, $x$ and $y$ ordinates are measured to various points by setting the floating marks to ground level at each in turn. This establishes position, and from the difference of parallax of pairs of points [as given by the change of the sum of $(x_1 + x_2)$ for different pairs of corresponding points] variations of level can be determined. Also if the photographs are slightly tilted the want of correspondence $(y_1 - y_2)$ of a series of points enables tilt to be calculated.

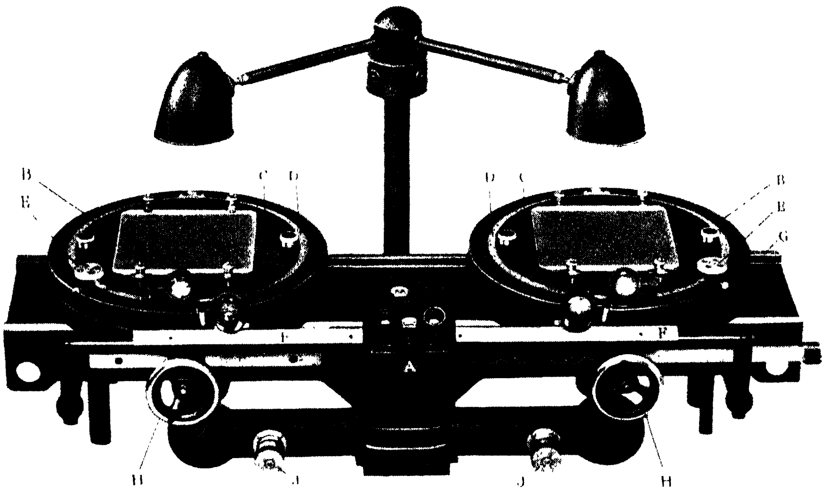
Before discussing analytical methods of control, it is convenient to describe the more important of the stereoscopic measuring machines using this method.

*The Zeiss Radial Triangulator* (*Fig.* 102).

This is an apparatus for measuring the polar co-ordinates of points, in which measurement is made of angles, which are plotted by tracing in a minor control-plot. It is claimed that positions can be determined more rapidly by radial triangulation than by angular measurement on the ground, and, moreover, that better points can be selected from the photographs. This is particularly useful when the radial triangulator is being

used preparatory to rectification of the photographs to ground control, a process which often becomes necessary when large-scale plans are being prepared, and particularly when levels are required.

Stereoscopic observations are made on the photographs, using either negatives or diapositives. These latter are transparent positives, and are used so that the light may be transmitted through the picture. They are a feature of Continental apparatus, whereas it is more usual in Great Britain to use prints and reflected light.




[*Courtesy of Carl Zeiss (London), Ltd.*

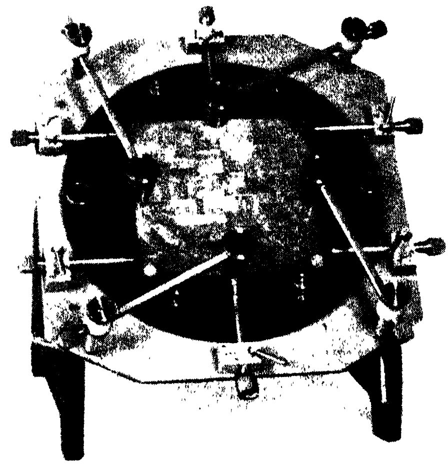
FIG. 102—RADIAL TRIANGULATOR.

The radial triangulator may be described briefly, referring to Fig. 102. Light is transmitted through the pictures (i.e., negative or positive) which are set in the plate-carriers B, which can be rotated, the angle being measured on the divided circles D. These plate-carriers can be altered in relative spacing by movement along the guide G. The stereoscopic viewing is done from below through an optical system to the double microscope A, which is fitted with built-in Amici prisms.*

The plate-carrier frames are detachable and each has a glass plate with a central point mark and frame marks, so that the photograph can

* An Amici prism is one which deviates the rays of light through 90°, and owing to its shape, which is tetrahedral, the image is inversed, as is the case, for instance, when a convex lens forms an image of a point at a distance.

be set centrally, Fig. 103. When the loaded plate frame is replaced in the plate-carrier, the centre of rotation coincides with the principal point of the photograph. The angles through which the photograph is turned are read in the same manner as with a theodolite, and the graduation is to one minute with estimation to thirty seconds.*



[*Courtesy of Carl Zeiss (London), Ltd.*

FIG. 103—SETTING DESK FOR RADIAL TRIANGULATOR.

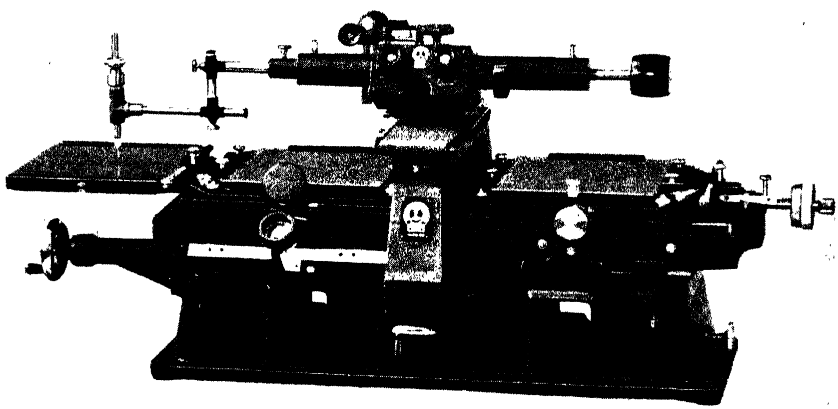
Lateral movement of the plate holders is measured on the scale F to 0·02 mm. by means of verniers, the movement being actuated by the hand-wheels H.

The knobs J enable the Amici prisms to be rotated independently so that an apparent turning movement can be given to each of the plates. The observer looking through the double microscope, therefore, can make the images appear in any direction from the centre and this direction does not necessarily coincide with the setting of the plate-carriers. The positions of agreement are marked by stops. The independent adjustment is of importance where the air-base between the two points of exposure is not horizontal. Also by reversal of the prisms a pseudoscopic image can be obtained which is often of value in checking the setting of a pair of photographs.

* All Continental photogrammetric instruments are graduated on the centesimal system for the measurement of angles—i.e. 100 grades to the quadrant; 100 centesimal minutes to the grade; and 100 centesimal seconds to 1 centesimal minute. Thus an angle of 45 grades, 35 minutes, 88 seconds, is written as $45^{g}\ 35'\ 88''$, or $45{\cdot}3588^{g}$.

The photographs are base-lined by rotating the photographs until the line joining the principal points, serving as the "straight-edge," cuts similar points of detail in each photograph. This position is established with reference to the measuring marks in the double microscope. The employment of stereoscopy in this process makes it similar to correspondence setting. Finally the reciprocal directions of the photographic pair are read off on the circles D.

When it is desired to measure the direction of selected points in the overlap, the photographs are turned until the point in question is set accurately in relation to the measuring mark.



[*Courtesy of Carl Zeiss (London), Ltd.*

FIG. 104—ZEISS STEREOCOMPARATOR.

Experiments at the Geodetic Institute of the Technical High School of Stuttgart have shown that in two triangulation nets each of twenty-six triangles, the mean error of an observed direction was $\pm 0{\cdot}9$ minutes. For a flying height of 9,000 feet on ground where the height variation was 600 feet, the mean error of positions fixed by radial triangulation was $\pm 10$ feet.

*The Pulfrich-Zeiss Stereocomparator* (Fig. 104).

The stereocomparator was one of the earliest stereoscopic measuring instruments to be designed for the measurement of ground photographs and has been used extensively in astronomical work. It has been found

to have a limited application to the measurement of air photographs which are almost vertical or which have been rectified. Plotting mechanisms similar to that seen on the left in the figure are described in Chapter X.

This stereocomparator has had an important influence on the development of stereoscopic measuring instruments. The two microscopes give a magnification of six diameters, and each contains a reference mark which will fuse with the other to form a floating mark of type similar to those shown in Fig. 71. Movements in the $x$ and $y$ directions are provided so that co-ordinates of points can be established in the common overlap.

Here again it is the custom on the Continent to use the negatives or diapositives, although there does not seem to be any objection to using photographic prints, provided that a reseau is used as in the Thompson Comparator.

The photographs are first "base-lined" by inspection and by making use of a method similar to correspondence setting, so that the common base-line of the two photographs coincides with the $x$-direction. A turning device is fitted so that the photographs can be rotated about a centre to which the principal points of the photographs are set. The instrument, while very convenient to use for ground photographs, is less so for vertical air photographs because of the limited horizontal turning movement of the turn-tables on which the plates rest.

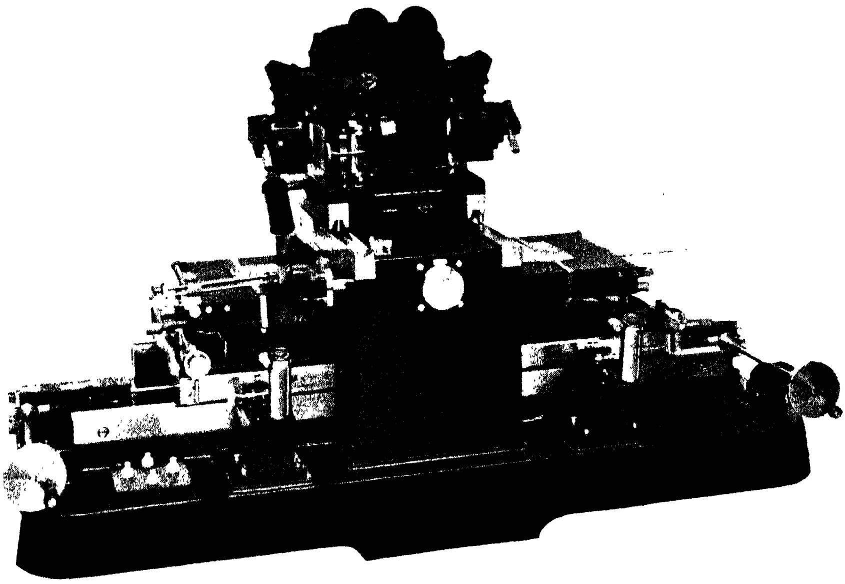
The microscopes are set so that co-ordinates are measured with reference to the principal points.

The main slider movements in the $x$ and $y$ directions can be read by scale and vernier to 0·02 mm., while the independent $x$ and $y$ movements of the right-hand photograph are read by micrometer to 0·01 mm. and by estimation to 0·001 mm., the ultimate accuracy of a mean reading being probably of the order of 0·01 mm. These two measurements, representing horizontal and vertical parallax, must be read with some such accuracy if precise results are to be obtained.

*The Thompson Comparator* (Fig. 105).

This instrument has been recently designed by Captain E. H. Thompson, R.E., late Research Officer to the War Office, Air Survey Committee and now Air Officer to the Ordnance Survey, and has been constructed by the Cambridge Instrument Company of London and Cambridge. The object is to determine aerial triangulation with speed and accuracy, the method being to combine the measurement of rectangular co-ordinates on photographs with computation. The photographs can themselves be used to provide additional control points related to a few ground control points, which can be some miles apart.

Extreme accuracy of reading on a stereoscopic instrument becomes of value only when there is provided a stable basis of measurement, such as a reseau, a device commonly used in astronomical measurements from photographs. This reseau is engraved on the register plate in the camera in centimetre squares with fine lines of thickness 0·001 inch, but which, however, appear somewhat thicker on the photograph. It provides a means of correcting for the distortions which may arise between the



*Courtesy of Cambridge Instrument Co., Ltd.*

FIG. 105—THOMPSON, OR CAMBRIDGE COMPARATOR.

moment of exposure and examination of the photograph in the instrument, since these lines will be just visible on the photograph. Although great improvements have been made in the reduction of distortion of films and papers, there remains the necessity of some such check if the most accurate results are to be obtained. It is considered that a special feature of this machine is its suitability for measurement of paper positives, upon which appears the image of a reseau instead of having to use negatives or diapositives.

Among the many ingenious solutions which Thompson has provided in this compact machine, is the incorporation of an optical device for fixing a point on any photograph, which avoids the usual marking of the photographs.

In the design of the instrument full use of kinematic principles is made, and it is provided with three tables, each of which is mounted on ball bearings. The central upper table can be moved along the $y$-axis and carries prism binoculars having a magnification of four diameters so that photographs mounted on the two tables below and left and right respectively of the upper one can be viewed stereoscopically. The $y$ ordinate of the table is read on an invar scale to 0·01 mm. The table is provided with free movement and slow motion, in the same way as in the case of the circles of a theodolite.

The two lower "photograph tables" are each fitted with two micrometer adjustments at right angles, so that the photographs can be set to the centre of rotation of the tables and this rotation enables a pair of photographs to be set in correspondence. These tables have movements either independently, or together, along the $x$-axis which is parallel to the baseline when the photographs are oriented. Clamps and slow motions are given to all the movements and readings are made with microscopes on separate invar scales.

For various reasons the $y$-ordinates of the photographs may not be quite the same for a pair of corresponding points and therefore, in addition, tilting parallel plate micrometers* have been fitted just below the object glasses of the microscopes, so that the "want of correspondence" of the points can be measured and consequently tilt of the photographs can be found.

The instrument will accommodate photographs up to 225 × 225 mm. (9 × 9 inches), and as long as the pictures are smaller than this, measurements are completely independent of the camera and lens used for the photography.

The photograph tables are free to rotate completely so that after completing measurements on the first pair, the second photograph of the pair may have the control point for continuation marked on it with the optical marker. This photograph is then turned through 180° and the next photograph is then placed on the other table, making the second stereoscopic pair.

This instrument has been designed specifically for use by the Ordnance Survey in Great Britain, for fixing additional points to those fixed by the new secondary triangulation. Since this country has already been accurately levelled, and no increase of the scope of contours is contemplated,

* This is a similar device to that which is fitted to the object glass of an ordinary level when it is required to take staff readings to 0·001 foot instead of the usual 0·01 foot. The parallel plate of glass in front is rotated, and although the ray to the staff remains parallel to its original direction, it is moved through a distance which depends upon the refractive index of the glass. By calibrating the displacement of the ray upon a scale, the required measurement can be made.

the instrument is being used principally for radial triangulation. Its design is such, however, that aerial triangulation may be successfully fixed by its use in undeveloped areas or in difficult places for ground levelling, so that aerial co-ordinates may be computed. It is expected that its use will enable tertiary triangulation to be eliminated by working between secondary triangulation stations as control points.

The instrument is simple in its construction and it seems likely that it will prove of great value in all cases where accurate surveys are required from air photographs, and will become particularly valuable on the larger scales.

For accurate work it is desirable that tilts should be limited by automatic-pilot control, and, where there is sufficient ground control, the photographs may be rectified in printing. Such rectification is made in the case of Ordnance Survey photographs during the process of enlarging from the photographic scale of approximately 1/5,000 to a mean scale of 1/2,500. By making use of Hotine's "want of correspondence" method of stereoscopic observation to determine tilts [58] (see later in this chapter), it is expected that contouring will be possible with little more ground control than for planimetry.

## Analytical Methods of Control*

Improvements in Arundel Method plotting are made by making use of instrumental measurements on the photographs in order to substitute a computation process in place of the graphical minor control plot, while keeping the number of ground control points to a minimum.

There are several of these methods in use and it is proposed to deal in outline with one or two of them only. It has been shown in the previous section that there are two main types of measuring instrument for use when the photograph is assumed vertical, namely, those which measure polar co-ordinates and those which measure rectangular co-ordinates.

### *Radial Control by Angular Measurements.*

Where photographs have been taken with a small tilt it has been shown that no appreciable error arises by assuming that corresponding lines on photograph and ground are radial from their respective principal points. In Fig. 106 a control diagram for a strip of photograph is shown. The strip is made short for convenience. P, Q, R, S, T, U, are the principal

* It is assumed that the reader has a working knowledge of ordinary traverse correction and of the solution of triangles. For detailed information he is referred to one of the standard text-books on surveying.

points of a strip of six overlapping photographs thus making the principal point traverse, and *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, are the minor control points selected on the photographs, as before, each in the common overlap of three adjoining photographs in the strip. X and Y are two ground control points, one at each end of the strip. Angles between the principal point base-lines are measured to all minor and ground control points with an instrument such as the Radial Triangulator.

*One method of solution is then as follows*: Assuming that the co-ordinates of X and Y are known, then the length between X and Y and the bearing of the line can be found with respect to the standard direction for the measurement of co-ordinates. No exact distances are known on the

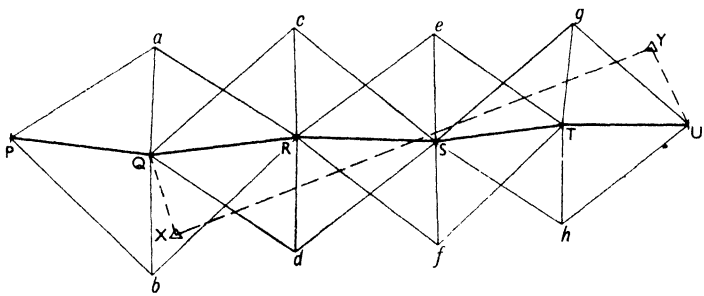



FIG. 106.

individual photographs although these have been taken approximately to the desired scale. Automatic control of the aircraft for line and height makes this scale more uniform, while increased accuracy of altimeter or statoscope readings enables it to be more easily determined. Owing, of course, to ground height variations, scale is variable over the photographs so that only an approximate value is available at, say, the mean level of ground.

Therefore as in the case of the Arundel Method minor control plot, no scale is immediately determinable, and the first computation must be to an unknown scale. If QX is measured on the photograph, then knowing the focal length of lens *f* and approximate height of aircraft *H*, the approximate length of QX may be found. Two angles have been measured in each triangle from the ends of the air-base to within about one minute of accuracy, so that each triangle may be solved.

Taking this value of QX, solve the chain of triangles,* XQR, QR*c*,

* By the sine rule—in triangle XQR, $QR = QX.\dfrac{\text{Sin X}}{\text{Sin R}}$, and $RX = QX.\dfrac{\text{Sin Q}}{\text{Sin R}}$ and so on.

*c*RS, RS*e*, *e*ST, ST*g*, *g*TU, YTU, and then coming back on the other side, UT*h*, T*h*S, ST*f*, S*f*R, RS*d*, R*d*Q, RQX.

There will thus be a check on the working and any small discrepancy may be adjusted.

Perhaps a better method of computation is to take a series of quadrilaterals which can be adjusted to satisfy the triangulation conditions. In this case the triangle QRX is solved giving QR at the assumed scale.

Then the first quadrilateral as in Fig. 107 may be computed as follows:

(i) Add up the angles at R and adjust if necessary to make the total = 360°.

(ii) From the measured angles, and allowing for the corrections above, find the angles at *c* and *d* so that the angular total in each triangle is 180°.

(iii) If the polygon were now computed, it is quite likely that there would be a closing error; i.e., if the computation were started from QR, the final value of this length would differ somewhat from the initial value. If a triangle (say QR*c*) is viewed outwards from the centre station R, ∠ *c*QR is a left-hand angle, and ∠ R*c*Q is a right-hand angle, and it can easily be proved for the whole polygon that

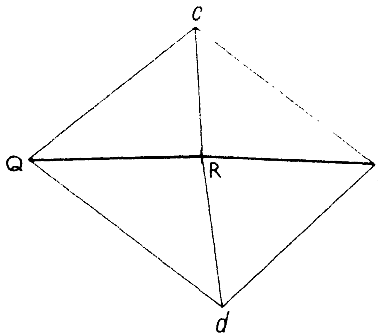



FIG. 107.

Sum of log. sines of left-hand angles = Sum of log. sines of right-hand angles.

It is then possible to adjust the sums of log. sines to satisfy this condition while maintaining the angle conditions.

Von Gruber remarks [46] that rigorous adjustment of a triangulation net by the method of least squares is not justified because some of the discrepancies which occur in the calculated lengths will be due to inequalities of ground height accentuated by tilt. Since this was written (1930) the automatic pilot has enabled these tilt distortions to be much reduced, so that distribution of errors on the lines suggested above probably becomes justified. A satisfactory method which is also simple is the Method of Equal Shifts*, or a trial-and-error adjustment may be made.

When the angles have been corrected to satisfy these conditions and

* For a complete proof of this formula, and for a discussion of the Method of Equal Shifts, and examples, the reader is referred to *A Treatise on Surveying*, vol. II, Middleton and Chadwick, revised by Professor M. T. M. Ormsby.

the sides are computed, QR should check back exactly. The process is continued through the other polygons until TU is found. Finally, the triangle TUY is solved.

A traverse may now be worked out along the principal points. Since all angles are known in the traverse YXQRSTYX, and QX, QR, RS, ST, TY are known accurately relatively, but to an unknown scale, the departure and difference of latitude between X and Y is determined from the photographic measurements, but to an unknown scale. Hence XY is known to this scale and may be compared with the true value so that the scale ratio may be found. All other lengths are then adjusted in the same proportion, and co-ordinates may be worked out for the principal points, and later for all the minor control points. True plan positions are thus obtained. For contouring, either a considerable number of known height control points must be fixed or some stereoscopic method on the lines of the want of "correspondence method" must be employed.

Detail plotting is then continued with these points as a basis.

The area covered is considerable, and, with a proper routine of computation the reduction of these points takes no longer than control calculations for ground survey. Moreover, on large scales the additional accuracy obtainable is very desirable.

*Radial Control by Comparator Measurements.*

Instruments such as the stereocomparator may be used similarly except that in this case the points are measured stereoscopically. Rectangular co-ordinates are found with reference to the principal point as origin and with the air-base as direction of $x$ co-ordinates. The ordinates can be measured to 0·001 mm. by estimation and with a mean accuracy of about 0·01 mm. and hence the radial angles may be found by computation with probably greater accuracy than with the radial triangulator.

This method of measurement allows of the determination of parallax, so that the instrument lends itself to the determination of levels and to the fixation of aerial co-ordinates.

When co-ordinate measurements are made with a comparator, it is possible to compute the positions of the minor control points along the strip, and, at the same time to make allowances for variations in positions of such points from photograph to photograph, as a result of varying height or tilt distortions. Such a method has been devised by Thompson, and the Cambridge Comparator has been designed primarily for the use of the Ordnance Survey in breaking down the new secondary triangulation of England, and for other control work in connection with the revision

and repair of the 1/2,500 plans. Experiments have proved satisfactory and it is anticipated that results will be entirely satisfactory for Ordnance Survey requirements, and also that the instrument will have great value in carrying forward control on almost vertical photographs, so that contours can be determined accurately, at least for medium scales, with a minimum of ground control.

The method used involves the measurement of want of correspondence for the computation of tilts by Hotine's method. It is found that tilts computed from the measurements agree to within a few minutes. This method will now be described briefly.

*Determination of Tilts by Want of Correspondence Method.*

This method, evolved by Hotine, is being used in conjunction with the Cambridge Stereocomparator for bridging over considerable distances with little control. A full theoretical discussion of the method is given in Professional Paper No. 6 of the Air Survey Committee. [58] This paper was written before the automatic pilot was a practical proposition, and at a time when stereoscopic measuring instruments were not capable of producing such accurate and consistent results as the Thompson Comparator. Moreover, considerable improvement has been made in photographic materials, while the grain size of emulsions has been reduced so that greater magnification is possible before observing.

It was not easy to obtain consistent results within the narrow limits required for large scales with an instrument such as the Barr and Stroud Precision Topographical Stereoscope (Fig. 77), and it is interesting to note that recent instruments give results of remarkable consistency. Hotine stressed at the time that the method depends upon measurements on the photographs, and particularly differential measurements in the $y$-direction to 0·01 mm.

In Fig. 108, P is the principal point of the left-hand photograph of a stereo-pair and P′ its image on the right-hand photograph. Similarly, Q is the principal point of the right-hand photograph and Q′ its image on the left-hand photograph. The direction of base-line is the basis of $x$-ordinates, positive directions being as shown in the figure. $y$-ordinates are measured at right angles to the base direction, and if the two photographs are free from tilt and taken from the same altitude, these ordinates to any point appearing on two adjacent photographs will be the same. When, however, it is found during stereoscopic observation that the right half-floating mark has to be adjusted in the $y$-direction, there is then a "want of correspondence", $k = (y - y')$.

If the want of correspondence is measured to four carefully chosen points on a stereoscopic overlap, it is possible to determine the tilts of the photographs relative to the air-base and the lateral tilt of one photograph with respect to the other. Later, knowing the ground heights of these points the absolute tilts of photograph and air-base may be computed. Actually three points provide a solution, but it is usual to measure to four points as this not only provides a check, but also enables a simpler routine to be followed.

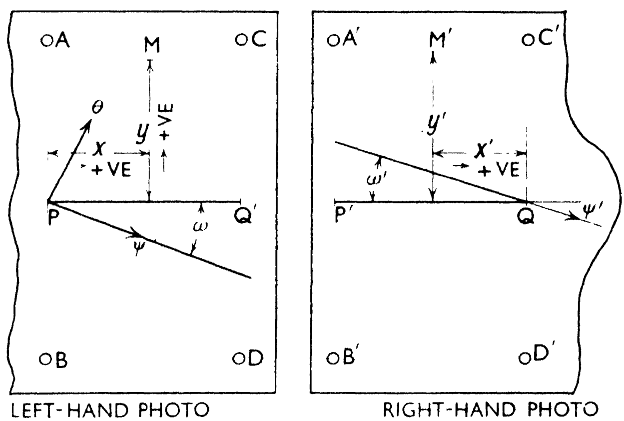



FIG. 108.

Recent improvements in stereoscopic instruments and in the more general use of automatic pilots for survey photography where this method is likely to be required, enable the working to be considerably simplified.

In Fig. 108, the lengths of air-base as measured on the photographs are PQ′ ($= b$) and P′Q ($= b'$) for the left and right photographs respectively. Points A, B, C, D, are chosen so that ACQ′P and PQ′DB form approximate rectangles. Where there is a fair amount of detail on the photographs, which is usual in the case of photographs taken for economic or engineering purposes, it is convenient to make PA and PB perpendicular to PQ′, and QC′ and QD′ perpendicular to P′Q. This can easily be done in the case of the Thompson Comparator by running out the carriage at right angles to the base-line until a suitable point is found. A further simplification, sometimes possible, is to take the points on opposite sides at exactly the same distance from the base-line.

The pair of photographs is set accurately in the Comparator and the $x$- and $y$-ordinates carefully observed, the readings being repeated to ensure

a check. Hotine remarks that if it is required to find tilts to one minute, want of correspondence must be measured to 0·01 mm., and the remaining quantities to 0·1 mm.

Before computation is commenced, corrections for distortions should be made. These include corrections for lens distortions and refraction due to the glass pressure-plate. When the lens has the normal field of 70°, then lens distortions are generally negligible with lenses of the best type, while distortion due to the pressure-plate is balanced by an adjustment of the principal distance. When an ultra wide-angle lens is used these factors may have some significance and while corrections can be made from known distortions at definite angles of field from the centre of the photograph, the use of a reseau enables the effect of all distortions to be determined at once.

The actual direction of base-line is dependent upon the relative tilts of the photographs, and in Fig. 108, using Hotine's symbols, it is assumed that the left-hand photograph has a tilt of $\theta$ relative to the right-hand one, the direction of this tilt being perpendicular to the true air-base, which makes angles of $\omega$ and $\omega'$ with the principal point bases. The left and right photographs are tilted $\psi$ and $\psi'$ with respect to the air-base.

Then it can be shown that for any point such as M,

$$y\,\theta + x\,\psi + x'\,\psi' + \frac{f^2.\theta}{y}\left(1 - \frac{x}{b} - \frac{x'}{b'}\right) = \frac{f.k}{y} \quad . \quad . \quad . \quad \text{(IX.1)}$$

If A and B are perpendicular to PQ′ at P, then $x = 0$, and $x' = b'$

$$\text{so that for A, } y_A\,\theta + x'_A\,\psi' = \frac{k_A.f}{y_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.2)}$$

$$\text{and for B, } y_B\,\theta + x'_B\,\psi' = \frac{k_B.f}{y_B} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.3)}$$

Now for C and D, $x = b$ and $x' = 0$.

$$\text{so that we have } y_C\,\theta + x_C\,\psi = \frac{k_C.f}{y_C} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.4)}$$

$$\text{and} \quad y_D\,\theta + x_D\,\psi = \frac{k_D.f}{y_D} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.5)}$$

Hence $\theta$, $\psi$ and $\psi'$ may be found and checked.

There are certain second order terms given by Hotine, and these will necessitate adjustment to the values found above if the area photographed has considerable height variation and the photographs are taken with manual control. When an automatic pilot is used, relative tilts are not

likely to exceed 15′. In such cases the second order terms may be neglected.

It can also be shown that the base-line directions are found accurately enough from the following equations:

$$\omega = \theta\left(\frac{f}{b} + \frac{\psi}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IX}.6)$$

$$\omega' = \theta\left(\frac{f}{b'} - \frac{\psi'}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IX}.7)$$

When these relative figures have been found, it remains to determine the absolute fore and aft tilts and lateral tilts, i.e., the horizontal must be introduced into the calculations. Although three points are actually sufficient it is customary to use the four points observed previously so that there is a check.

The absolute orientation of a pair of photographs is effected in several stages by Hotine's Method. For details of the computation, the reader is referred to the paper mentioned above, but the successive steps may be mentioned: (i) The rectangular photo co-ordinates are transferred to an $X$-axis, which is parallel to the true direction of the air-base and passes through the right-hand principal point, (ii) The left-hand photograph is "rectified" with respect to that on the right, to eliminate the relative tilt $\theta$, (iii) Determination of the depths of the points observed below the air-base.

At this stage the heights of points are measured from, and their positions fixed in the plane containing the air-base in the direction of its maximum slope.

Then if the true heights of the control points are known, the absolute photographic tilts and base-line slopes may be determined.

Although this method of computation takes some time, the process offers certain advantages.

When using an instrument such as the Comparator the accuracy of readings taken by a skilled stereoscoper is such that tilts can consistently be determined to a few minutes.

One method is then to use the tilt values determined for the pair of photographs, in order to find other levels so that contours may be plotted. This, however, involves the determination of four spot heights per overlap, which is desirable if large-scale contouring is contemplated.

For mapping purposes and particularly for scales of the order of one or two inches to the mile, the amount of control can be reduced considerably. If the minor control points are chosen in approximately the positions suitable for tilt determination, it is possible in the Cambridge

instrument to take one set of readings for both radial control and want of correspondence to the same points.

Then, not only can the minor control points be fixed by rectangular co-ordinates, but relative fore-and-aft, and lateral tilts may be determined along the strip. As an example of the reduction of control, it has been found recently that a strip of photographs some twenty miles long, taken at a scale of 1/40,000 with an ultra wide-angle lens and controlled by automatic pilot could be completely plotted and contoured from three ground control points.

The advantages of such a method over the original contouring process employed with the simple Arundel Method, when applied to medium and small-scale mapping, and for military purposes, hardly needs emphasizing.

## Ground Measurements for Radial-Line Plotting

Ground control points will often be tertiary triangulation stations which have been previously fixed. If these happen to be conveniently placed with respect to the strips, then the stations themselves may be marked in some way so that they will show up in the photographs and give a permanent record. Usually, however, it is more convenient to take the photographs first. In this case points, easily identifiable on the photographs, may be fixed accurately in relation to the known stations by careful traversing or instrumental intersection. Where the existing triangulation stations are not conveniently placed, either the process of compilation of blocks as described in Chapter VII must be resorted to, or a local triangulation or accurate traverse is necessary to extend the control. It is always necessary for the ground survey party to ascertain how much of the essential information cannot be plotted from the photographs and then go out with a set of photographs to take the necessary measurements. Chaining from definitely established points on the photographs is usually sufficient.

In closely built-up areas and where roads run through woods, there will always be a number of obscured details. In wooded areas it may be considered desirable to photograph at a period when the trees are devoid of leaves to eliminate some of the ground measurement. The lengths of shadows, which vary with the season and the time of day, will affect the interpretation problems, some of which have been dealt with in the section on interpretation (Chapter II).

In plotting houses on large scales the overhang of eaves has presented some difficulties, since it is the plan at ground level which is required. The Ordnance Survey has concerned itself with this problem in the 1/2,500 revision and information as to the standard practice is given by Lloyd

Brown [10]. He remarks that eaves are generally about a foot wide and that a standard allowance is made in penning the photographs. "It is seldom that an eave departs sufficiently from this standard to introduce a plottable error at this scale. An excessively large eave, though rare, is an object which the ground examiner would naturally look out for; while a house with no eaves is usually apparent on the photograph."

He also remarks that the displacement of buildings, due to their height, presents a problem when near the edge of an enlarged photograph. Actually it is possible to plot accurately the side nearest the centre, and either an allowance is made for height distortion or certain ground measurements are made. "The height displacement for 25 feet of height at 6 inches from the centre of the photograph is 6 links (4 feet), and a 2° tilt will only alter this by one link. . . . Taller buildings present a similar problem . . . which . . . is not difficult of practical application."

Although the ground measurements which are required on these large scales may take considerable time, it will always be much quicker and often cheaper to utilize air-survey methods than to complete the survey by ground methods alone. It is necessary, however, to realize that air survey cannot provide the complete solution for large scales.

## Rectification of Air Photographs

Photographs taken with the axis inclined from the vertical may be rectified graphically, with respect to ground control points, but this method is rarely employed now except in the case of high obliques. (See Chapter XI.)

It is more usual for photographs which have been taken with a tilt to be reproduced as though they had been taken vertically, by projection through a suitable rectifier. There is much confusion as to the exact meaning of rectification and the process may be defined as follows: Rectification converts a tilted photograph into the photograph that would have been taken if, at the moment of exposure, the negative had been truly horizontal and the lens axis truly vertical. Some methods of plan production on large scales employ the process of rectification (with or without enlargement or reduction) from slightly tilted photographs. In other cases where multi-lens photographs are used, the side obliques must be rectified through much greater angles than photographs which have been taken almost vertically.

Although many processes involve work on the tilted photographs without rectification, particularly for medium and small scales, for plotting large scales and for contouring, some form of photographic or optical

projection is necessary unless tilting can be kept very small. Optical projection is involved in plotting machines while rectification is used for large scales where a simple plotting method is desired.

It has become a policy among the British air survey firms, when producing large-scale plans of areas where there is ample ground control, to rectify the nearly vertical photographs during the printing process. It is an essential condition for accuracy that the photograph shall have been taken with a narrow-angle lens, e.g., a 20-inch lens on 7 × 7 inch photographs. In this way, if the variations of ground height are small, it is possible to produce a photograph very nearly free from height distortions. Unless the area to be covered is extensive and the amount of ground control scanty, this method has much to commend it. The disadvantage where there is no previously accurately mapped detail is that there must be at least four accurately known points on each photograph so that rectification can be carried out. Actually three points are enough if the interior orientation (i.e. focal length and relation of plate position to that of the lens) of the camera is known. Four points are generally employed, but if they all lie on or near the surface of a cylinder there is no determinate solution.

The efficacy of many rectifying processes depends upon the requirements of the rectification. For example, a number of rectifiers, adequate for plan rectification of nearly vertical photographs, would be quite inadequate if levels were required or the tilts became large.

*Perspective Transformation.*

The problems are dealt with here as those of rectification, although they also arise in the case of projection and during the setting of plotting machines. In both cases the process amounts to a *perspective transformation* where the plane of a picture is altered while maintaining the correct perspective conditions that obtained at exposure.

*Requirements of Optical Rectification.*

In photography the lens serves the purpose of reproducing on a screen a perspective view of the object space as covered by the field of the lens (Chapter IV). The condition for the formation of a clear image in focus by a convex lens system is that the sum of the reciprocals of image and object distances $u$ and $v$ respectively should be equal to the reciprocal of the focal length $f$ of the system, i.e.

$$1/v + 1/u = 1/f \qquad \text{(IX.8)}$$

In the process of rectification it is required to reconstruct the picture which would have been taken had the exposure been exactly vertical.

Rectification without change of scale through a lens of the same focal length as the camera lens is fairly simple, particularly when tilts are small, but the general solution is more involved when it is required to satisfy conditions for (*a*) enlargement or reduction, (*b*) large tilts, or (*c*) focal length of enlarger lens different from that of camera lens.

When the plane of projection is inclined to the plane of the negative a further requirement for clarity of focussing is that the line perpendicular

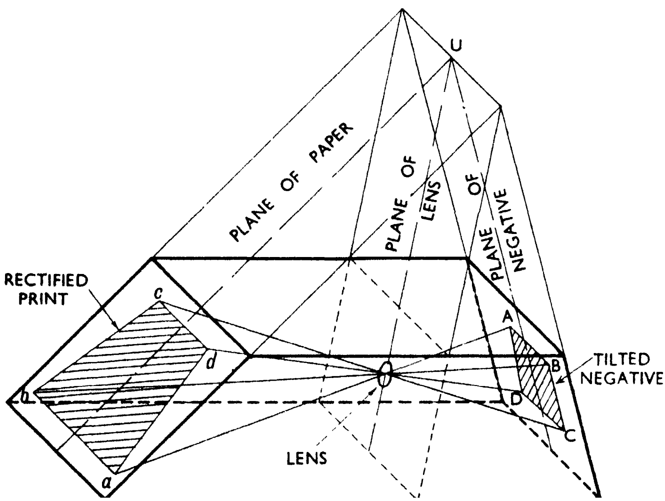



FIG. 109—SCHEIMPFLUG CONDITION FOR RECTIFICATION OF PHOTOGRAPHS.

to the optical axis of the lens must be in the same plane as the principal lines on the negative and photograph, and must intersect in the same point. In other words, the principal planes of negative, lens and projection must intersect in a common line. This basic condition was first given a practical application by Scheimpflug, and is usually called the *Scheimpflug condition.*

This principle is illustrated in Fig. 109* where the tilted photograph ABCD (with AB as the small-scale end and CD the large-scale end) is rectified in projection on to the paper plane. It will be noticed how the shape of the area changes in projection owing to rectification.

Thus in Fig. 110 which is drawn in the plane of the principal lines of the negative and projection planes, the relative position of the three planes is controlled by the lens formula given in equation (IX.8) and the Scheimpflug condition. Lettering of this diagram corresponds with that of

* From Fairchild Aerial Camera Corp. Publication—*Multiple-Lens Aerial Cameras in Mapping.*

Fig. 49, so that T is a point of the horizon trace, i.e. ST is parallel to BU. If SL is parallel to UA, then it has been shown by equation (V.4) that an essential condition of reconstruction is that the product of adjacent sides of the parallelogram UTSL must be constant, i.e. TS $\times$ SL is constant, and TS and SL are homologous distances. Any process of rectification must ensure that this product remains constant, although distortion in the *shape* of this parallelogram is permissible.

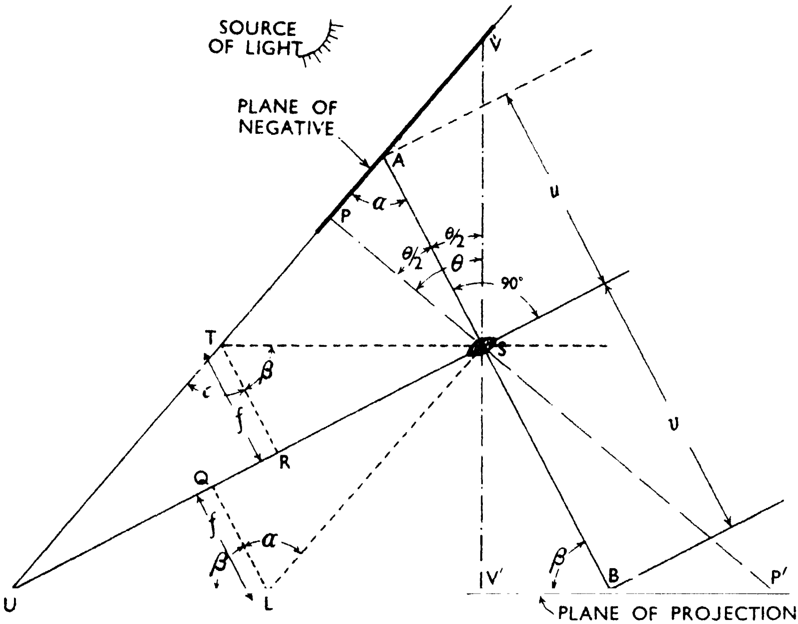



FIG. 110.

In the full-scale model TS $= f.\text{cosec } \theta$ and SL $= H.\text{cosec } \theta$ from equation 4, Chapter V. The rectified picture must take account of scale. Suppose the scale of reproduction is to be $1/r$, then the plane of projection will be $r$ times nearer to the perspective centre than the original view was at the moment of exposure. In the parallelogram UTSL, LS becomes $\dfrac{H.\text{cosec } \theta}{r}$

For this reason it would be impossible to reproduce a clear view while preserving the shape of the parallelogram, and advantage is taken of the fact that the shape may be changed while preserving an essential condition of rectification.

*Rectification without Change of Scale with Lens of Same Focal Length as the Camera Lens.*

This is the simplest possible case:

In Fig. 110 let TR, LQ be perpendiculars to the principal line of the lens US. Since AS $= u$, and SB $= v$ then $\frac{TR}{u} = \frac{UR}{US}$ and $\frac{LQ}{v} = \frac{QU}{US}$, and since TR $=$ LQ by the property of a parallelogram, then

TR $\left(\frac{1}{v} + \frac{1}{u}\right) = \frac{(UR + UQ)}{US} = 1$, so that if $f =$ focal length of projector lens $=$ focal length of camera lens it is clear that

$$TR = LQ = f \qquad \text{(IX.9)}$$

This is a condition of clear focus.

Since the scale of reproduction is to be the same as that of the photograph, i.e., the rectification is to be effected by rotations of the photograph and projection plane about the plate parallel passing through the isocentre (Chapter V), and the scale will be still $\frac{1}{r} = \frac{f}{H}$, so that since LS $= \frac{H.\text{cosec}\ \theta}{r}$

then $LS = f.\text{cosec}\ \theta$ . . . . . . . . . . (IX.10)

In order to preserve the Scheimpflug condition, the principal lines of negative picture and lens must all meet at U. If $\alpha$ and $\beta$ are the inclinations of the projecting and projected planes respectively to the optical axis AB of the lens then since UTSL is a parallelogram, and TR and QL are parallel to AB, angles $\alpha$ and $\beta$ are as indicated on the diagram.

$$\cos\ \alpha = \frac{QL}{LS} = \frac{f}{LS} = \frac{f}{f.\text{cosec}\ \theta} = \sin\ \theta \qquad \text{(IX.11)}$$

$$\text{and}\ \cos\ \beta = \frac{TR}{TS} = \frac{f}{TS} = \frac{f}{f.\text{cosec}\ \theta} = \sin\ \theta \qquad \text{(IX.12)}$$

Hence in this case $\alpha = \beta = 90 - \theta$ . . . . . (IX.13)

The distances of negative and picture plane from the perspective centre are easily determined, since SL $=$ ST (i.e. $u = v$), and it has been shown that Q and R will be coincident, so that $u = v = 2f$ . . (IX.14)

This distance is measured along the optical axis of the lens. In order to take account of the nodal distance of the lens system (Chapter IV), distances in the object space are measured from the outer node, while those in the image space are measured from the inner node. It has been shown that the perspective diagram is unaffected (Fig. 22).

It has also been shown that the isometric parallel, along which there is

no change of scale, passes through the isocentre of the photograph and is perpendicular to the principal line. Hence in Fig. 110, A and B are the negative and projected isocentres respectively, while P and P′ represent the negative and projected principal points respectively, and V and V′ the corresponding plumb points. When the setting is correct for rectification then SP and SV′ are perpendicular to their respective picture planes, so that

$$AP = BV' = f.\tan \tfrac{1}{2}\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IX.15)$$

Any rectifying apparatus designed for these conditions is arranged so that the lens axis is maintained in a position where Scheimpflug's condition is satisfied, while the negative and projection planes are turned so that both are inclined equally to the lens axis. At the same time the projection planes may be moved in the direction of the principal lines, so that four points on the negative are projected into co-incidence with their corresponding points previously known in position and plotted on to the plane of projection. The plumb-point may be marked on the rectified print from equation 20.

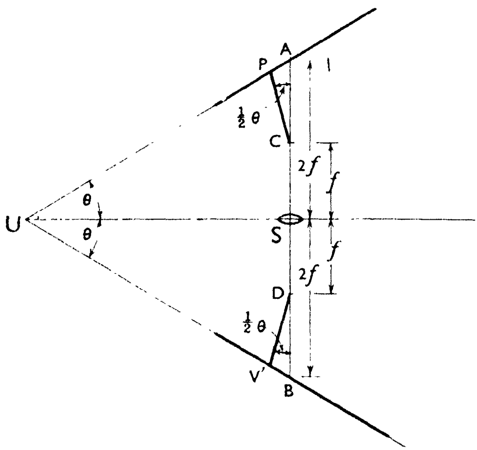



FIG. 111.

The basis of a simple automatic rectifying apparatus is seen in Fig. 111 [55] where (from equation IX.14) SA = SB = $2f$.

Also since $\alpha = \beta = (90 - \theta)$ from equation (IX.13), A and B becomes respectively the negative and projected isocentres, while ∠AUS = ∠SUB = $\theta$, the angle of tilt. ASB is of constant length and ∠USA is a right angle. The lens axis ASB may be slid in or out normally to US.

If C and D are pivots each at a distance $f$ from S, and levers CD and DV′ are pivoted at P and V′, so that when the angle $\theta$ is altered by sliding the lens axis ASB in or out, the angles PCA and BDV′ are maintained at $\tfrac{1}{2}\theta$, and the plumb point V′ of projection is at once known in relation to the principal point P.

Photographs are truly rectified by such a process, and may be used for contouring as well as planimetry.

Eden [29] has described a process of rectification for planimetry patented by the Indian Air Survey and Transport Ltd., which depends upon four independently measured lines in the common overlap of four adjoining

photographs. This appears to be somewhat similar to the American Scale-Check Method described by Anderson. [4]

The Indian Method is applicable only in flat country and for photographs taken at heights which do not vary by more than 100 feet or so for a flying height of approximately 9,000 feet.

Eden points out that for this method the "distance of negative from the lens must be approximately the same in both camera and enlarger, hence we must know the approximate scale of the photograph and the required rectified scale before designing the enlarger. If this condition is not satisfied, the two projections will only be similar if they are both on a horizontal plane."

He mentions that those enlargers which rectify by tilting the projection and negative plane in a manner which satisfies the conditions of enlargement and change of focal length of lens are not suitable for rectification on measured lines.

*Reduction or Enlargement with a Lens of Different Focal Length from that of Camera Lens.*

This becomes the general case and is mentioned here because the significance of alteration of projection angles as a result of variations in conditions of reproduction may not be readily understood.

Suppose that the rectifying camera has a lens of focal length $f'$ and that the photograph is to be enlarged $e$ times. Since scale is everywhere varying over a photograph where there are variations of height, some definite height must be chosen for correct rectification.

This problem arises in those cases where, for purposes of photographic economy, the photographs are taken at a smaller scale than that required, or as in the case of the Zeiss Multiplex Aeroprojector where the negatives are reduced before projecting. In such cases it is common for the photographs to be rectified to the required scale at a selected height during the printing process. In this way the photograph scale is as nearly equal to the required scale as possible.

The scale of the rectified photograph is now $e/r$; and in the parallelogram UTSL, SL $= H.e/r. \sin \theta$ and TR $=$ LQ $= f'$.
From this it follows, in the same way as before, that

$$\cos \beta = \frac{f'}{f.\text{cosec } \theta} \qquad \text{(IX.16)}$$

and $$\cos \alpha = \frac{r.f'}{H.e.\text{cosec } \theta} \qquad \text{(IX.17)}$$

Thus it will be seen that $\alpha$ and $\beta$ are now *different angles* and are not each

equal to $(90 - \theta)$. Also the angle of obliquity of the line joining the negative and projected principal points must be calculated. The conditions for accurate reduction or enlargement can still be satisfied by maintaining the parallelogram condition.

*Simple Rectification with Lens of Different Focal Length from that of Camera Lens.*

Here again a simplification results. In the previous case, the scale of rectified photograph is $e/r$ and since there is to be no change of scale along the isometric parallel, then $\frac{e}{r} = \frac{f}{H}$ and hence in equation (IX.17),

$$\cos \alpha = \frac{f'}{f.\text{cosec}\,\theta}.$$

i.e. $\alpha = \beta$ by equation (IX.16) so that

$$\cos \alpha = \cos \beta = \cos \gamma = \frac{f'}{f.\text{cosec}\,\theta} \quad . \quad . \quad . \quad . \quad . \quad (\text{IX}.18)$$

Hence a somewhat similar mechanism to that sketched in Fig. 111 might be employed. The angle made by the negative and projected plane with the lens plane is $\gamma$ which is not equal to the tilt so that the distances SC and SD are not now equal to the focal length of the lens.

It is found that an approximation which is adequate for small values of $\theta$, is to make

$$\text{CA} = \text{DB} = \frac{(f)^2}{f'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IX}.19)$$

Most enlarging rectifiers are adequate for all conditions when the tilts are small, while others are satisfactory for large tilts. In the latter class two may be mentioned.

*The Zeiss Automatic Rectifier.*

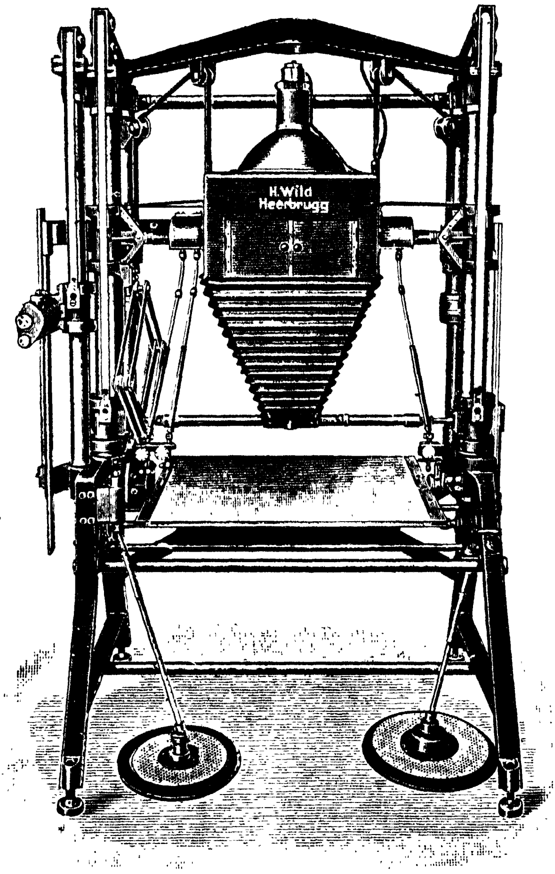
This allows of enlargement up to four times and reduction to one-half. Forty degrees of tilt can be rectified. The instrument is constructed on the parallelogram principle described so that rectification can be effected very quickly to four ground points.

*The Odencrants-Wild Rectifier.*

The instrument is illustrated in Fig. 112 and is somewhat similar in essential principles to the Zeiss type. The parallelogram which controls the relationships of negative, lens and projection plane is seen on the left, and examination of the diagram will give some idea of the working of the

instrument. Enlargements are allowed up to four times and reductions to one-third. The maximum tilt which can be rectified is about 35°.

Both this instrument and the Zeiss instrument are designed for the general case, which includes rectification of multi-lens photographs.




[*Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 112—ODENCRANTS-WILD RECTIFIER.

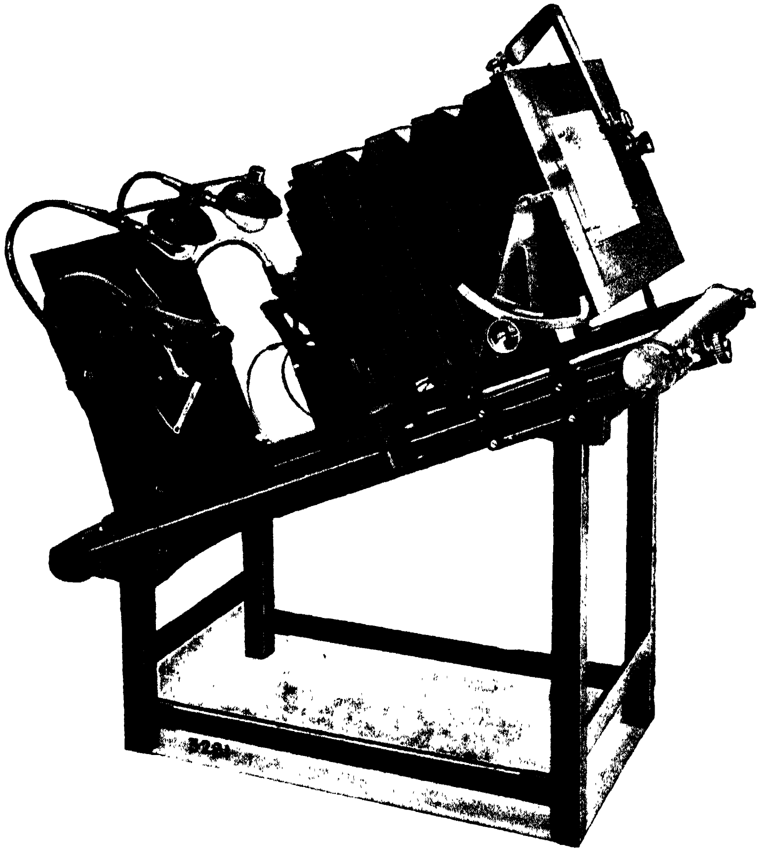
*Rectifiers for almost Vertical Photographs.*

These are naturally much simplified instruments which will deal with the small tilts in manually controlled strips for vertical photography. For small degrees of tilt it can be shown that the rigorous solution need not be followed and as a result tilting of the plane of paper will suffice to give required accuracy. There is a Zeiss rectifier which is adequate up

to about 8° of tilt, while another is the Williamson Automatic Enlarger. The range of this enlarger is quoted as from 1/3 to 3 times the photographic scale.

*Approximate Rectification for almost Vertical Photographs.*

The Barr and Stroud Epidiascope (Fig. 113) provides a solution where it is required to rectify photographs for planimetry, particularly for revi-



*Courtesy of Barr and Stroud Ltd., Glasgow.*

FIG. 113—BARR AND STROUD EPIDIASCOPE.

sion purposes. In principle it is similar to the projector epidiascope used for illustrating lectures. The copy-board can be given a tilt equal to that at exposure and the lens is adjusted along the optical axis for variations of scale. Either the plan to be revised or a plot of the control points is

placed on the tracing-board. The copy-board and lens are adjusted by trial and error until the projected photographic images of any three, or preferably four, control points coincide with their correct positions as fixed on the tracing-board. The detail may now be traced off within these limits. Here it is assumed that tilt distortions in any direction are a linear function of the tilt. Hotine points out [58] that such an assumption without reference to the conditions of rectification mentioned above, will give good results for planimetry where the tilts are very small but that false parallax is introduced which affects adversely the determination of levels.

The Epidiascope has been used extensively in the past, where Ordnance 1/2,500 plans are being revised locally, the map to be revised being placed in the focal plane of the projector lens. Where it is required to re-plot a large area there are other methods which prove more suitable.

It will be seen from the principles stated above and the instruments mentioned, that it is not sufficient to specify that the photographs are to be rectified. The purpose of the photographs and the precision which is expected from them will influence the choice of the type of enlarger to be used. Clearly it is possible, by using a different focal length from that of the camera lens, to achieve some sort of rectification without fulfilling the fundamental conditions, but the resulting print will probably give rise to appreciable errors. Where enlarging or simple rectification is proposed, by using a lens of the same focal length as the camera lens, there may be small variations in the actual focal lengths, but if the lenses are both made from the same batch of glass casting these are likely with care to be kept small. In some cases, provision has been made for the use of the camera lens itself during the rectification process.

In addition to the optical problems of rectification and enlargement, the photographic conditions must be such that accuracy is maintained to the standard required. Naturally, economy of photography is obtained by photographing on a small scale and then enlarging, during the process of printing, by projection. The extent to which enlargement may be used depends upon the fineness of grain of the emulsions used for films. If the ratio of enlargement is too great then a point of detail will be indicated on the print by a blur circle instead of a well-defined point, so that possible accuracy of settling is lost. Until recently it was considered that an enlargement of two to two and a half times maintained accuracy of image, but improvements by reduction of grain sizes of emulsions may allow this to be increased. Most enlargers allow for an enlargement of between three and four diameters.

### Plans from Rectified Photographs

When photographs have been rectified, the plumb point may be marked so that height distortions may now be taken correctly, as radial from the plumb point, instead of from the principal point. Tilt distortions radial from the isocentre will be of the same order as before, and it has been shown in Chapter VI that these amounts are generally negligible.

Hence, once the photographs have been properly rectified a more accurate plot can be made than with unrectified photographs, and plans up to 100 feet to 1 inch or larger plotted satisfactorily. Also it is easier to determine levels. Ground control must be provided at the rate of four points per photograph, so that if the survey is all new work, the ground work will be considerable, and, with automatic control reducing the tilt, it would appear more satisfactory to employ the radial method in conjunction with a Comparator or Radial Triangulator.

An account of a method of producing plans on a scale of 1/1,584, i.e., two Gunter's chains to one inch, developed by the London, Midland and Scottish Railway Company in conjunction with Messrs. Aerofilms is of great interest, because in similar form it is being largely adopted for surveys of roads, etc., on scales as large as 100 feet to 1 inch. An account of the experimental survey which influenced this decision was given by W. H. Christy Clay, before the Fifth International Congress of Surveyors in 1934, in a paper entitled "Air Surveys Especially in Relation to Railways."[19] The air photography was carried out by Messrs. Aerofilms, Ltd., at a scale of approximately 1/3,600 with 60 per cent longitudinal overlap, and at an altitude of 6,000 feet, using a 20-inch lens. The ground control and plotting were carried out by the railway staff, the work being quite independent of the Ordnance Survey plans.

Some of Major Clay's conclusions for air surveys at the above scale were as follows:

"Control points should be fixed in ordinary circumstances at approximate intervals of 300 to 400 feet parallel to the line of railway and at a distance from it of about 450 feet on either side. The object should be to get equally spaced, well-conditioned triangles.

". . . Time is saved and control points more easily selected if this work is delayed until contact prints of the photographs are to hand, and in cases where old plans which are known to be thoroughly reliable are available these can be used to form the basis of the new plan and control points selected by reference to the contact prints. Unless the plan is of proved accuracy it is, of course, useless, and it then becomes necessary to fix the points on the ground by instrumental means."

At first sight, this makes the work more expensive, but any one who has attempted to plot a survey where some of the old detail was inaccurate will realize that the added expense in the field will result in considerable saving in office time, and besides, the completed new plan will be reliable, while that utilizing old detail is less likely to be so, especially when it is not certain which of the old points of detail are inaccurate. In the 1/2,500 revision the Ordnance Survey has found numbers of "old errors" and in quite a number of cases the areas are being completely resurveyed without reference to old detail, except for checked control points.

The L.M.S. experiment was a length of $1\frac{1}{4}$ miles at Upminster on the London to Southend line. The Company has now established an air survey department, and a considerable mileage has been plotted.

When the first set of prints was received, ground control points were selected by inspection on the ground and with reference to the photographs. These points were fixed by triangulation from the traverse. Points such as roof corners, lamp-posts, chimneys, identifiable on the photographs, were chosen and each was fixed by rays sighted with a theodolite from three known stations along the traverse. Fig. 10 illustrates a photograph and plan with ground control marked from this survey.

This ground survey was plotted as accurately as possible to a scale of 1/1,584 on a material subject to the least possible distortion. Kodatrace or mounted Whatman paper would be suitable for this purpose. The control points were marked and pricked through as precisely as possible on to the print, and were numbered and ringed with coloured chalk.

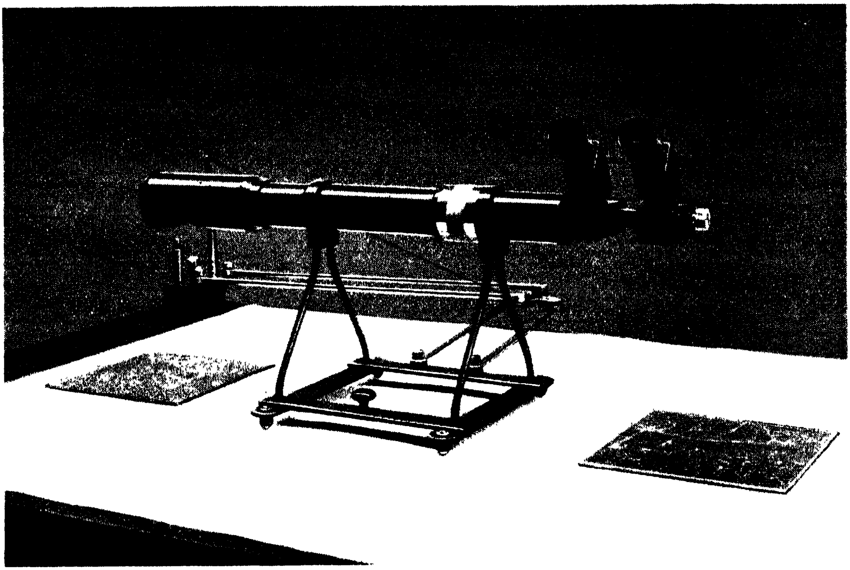
In the example quoted, the detail was plotted from photographs using a tracing-table illuminated from below. The variation in ground height over the area was small, and the rectified prints were nearly true-to-scale. The ground control points used for rectification were plotted accurately on to tracing material, which was placed on to the tracing table, and fitted exactly over the corresponding points on the photograph. The photographs had been rectified over two-thirds of their width, but to eliminate possibility of distortion only about one-half of this was used when tracing. This also allowed sufficient overlap for joining up detail from one photograph to the next. It is generally agreed that stereoscopic examination is essential during the plotting process to ensure precise fixation of points.

The photographs for this survey were taken by flying with manual controls and the tilt was specified to be less than $2^\circ$.

It appears that results of accuracy comparable to those obtained by the above method and with much less ground work, can be obtained with photographs having tilts limited to $\frac{1}{4}^\circ$ by use of the automatic pilot, and

advantage should therefore be taken of it whenever possible. It is then unnecessary to provide sufficient ground control to rectify the photographs, since the small tilt errors can be eliminated during plotting by the use of the Arundel Method.

Another method of preparing plans from rectified photographs is by one of the stereometers illustrated in Chapter VI, or by a Tracing Stereoscope of the type shown in Fig. 114, which may be used either with negatives or diapositives, when a sheet of Kodatrace (which is very trans-



[*Courtesy of G. de Koningh, Arnhem, Holland.*

FIG. 114—TRACING STEREOMETER.

parent) is placed over one photograph of the pair, the detail may be traced off while keeping the stereoscopic impression by looking through the stereoscope.

*The Brock Method.*

This method has been used extensively in the United States, and is a variant of the radial-line method which is particularly applicable to large-scale engineering surveys. It was developed while the ordinary radial-line method was as yet inadequate for scales of engineering size. The preparation of a contoured plan from the photographs is divided into several different stages. Planimetric control is provided by a measured base-line

at each end of a strip of photographs, and four height control points are required per photograph so that it may be rectified.

First of all the photographs are examined in a measuring stereoscope and observations similar to the "want of correspondence" observations previously described are made to height control points which are marked. This enables the height displacement of each point to be determined with respect to the datum plane. A slide-rule or graph is employed for this purpose. Actually the tilt and its direction can be calculated, but these are found in the method by trial and error.*

Each negative is now rectified without enlargement with reference to the four control points, by trial and error, until each point has a displacement in projection equal to that previously found, while the isometric axis is unchanged in length. A transparent positive is made of the rectified image and while the plumb point is marked on it as the centre of a glass grid engraved with squares, which is set along the direction of the air-base. This latter is now the line joining the adjacent plumb points.

The next stage is to set these rectified positives in the Brock Stereo-comparator, the centre of each being at the centre of the turn-table so that the micrometer reading is at zero. They are then set in correspondence by the usual stereoscopic correspondence method.

Contours are then drawn by setting the floating marks at the spacing required for a particular contour, and tracing direct on to tracing-paper placed over the transparent positives, by moving the fused floating mark at a constant parallax setting so that it appears to remain in contact with the ground. The intersections of points of detail with the contour are noted, but this plotting is not continued on either side because of the constant variation of scale owing to changes of ground height.

Next, an equalizing projector is used to vary the scale according to the height. The correct plan positions of the control points are plotted on tracing-paper to the desired scale, and the projector is set so that the scale may be checked by making control points on the first tracing coincide with their correct positions as plotted on the second sheet of tracing-paper. In this manner the projecting-table can be raised or lowered until a contour is at the required scale, i.e., allowance is made for height distortion.

* It should be noted that what has been described as the *isometric parallel* in Chapter V, i.e., the plate parallel through the isocentre, and the only one along which the scale is $\frac{f}{H}$, is called the *axis of tilt* in the United States. This is actually the *true axis of tilt* although by the definition of the Air Survey Committee it is defined as the *plate parallel through the principal point* as a matter of convenience. This approximate definition appears to have been adopted because of the assumption made in the Arundel Method in that all distortions are radial from the principal point.

When a contour is set, it is traced off together with the detail up to about half-way to the contours on either side. This is repeated for all other contours and finally the detail is transferred together with the contours to the master plot.

Hotine [55] remarks that this method, while well suited to large-scale engineering surveys, is not economic for small scales and considers that "when trimmed of many superfluous processes" for this purpose would reduce probably approximately to the Arundel Method.

The method was designed to produce accurate contoured plans at large scales at a time when the radial-line method could not provide them owing to distortions of materials, inaccurate recording instruments and non-stabilized flying. At the present time it appears that some of the stages could be eliminated with little or no loss of accuracy, by employing the radial method, together with stereocomparator measurements.

## Revision of Large-Scale Plans

In this country a considerable amount of Air Survey had already been done for local authorities before the Ordnance Survey used this method for the revision of the 1/2,500 scale plan. In districts where the Ordnance Survey plans were some years out of date, the subsequent development had made the plan unrecognizable, and the air photograph had provided the means of gathering information for the town-planning scheme.

Wills [96] describes the method commonly used:

"One method is to trace directly from the 25-inch rectified photographs the outline of buildings, etc., with a steel stylo through carbon paper on to tracing paper laid directly under the photograph. This can only be done by the aid of the stereoscope and two neighbouring pairs of photographs, and by the careful inspection and interpretation of relief of the individual houses, fences, etc., before each line can be drawn. . . . The tracing of each centre section of scale photograph is laid and oriented in position on the 25-inch O.S. sheet. All corrections are marked and old field boundaries, etc., are erased from the original sheet; after which the new work is traced on to the sheets, and eventually fair drawn inked in to the Ordnance Survey standard of draughtsmanship."

Another method used is by the Barr and Stroud Epidiascope described previously. This has been much used for revision of Ordnance Maps, but it has been found in a number of instances where it is desirable to use air photography for revision, that it is cheaper to re-plot the plan entirely where there has been much development and some of the old detail is of questionable accuracy.

The Ordnance Survey Department has had air survey under consideration for some time and contracts for air photographs were let to a civil firm in 1935 and 1936. In the Annual Report of the Ordnance Survey for 1937–8 [73] an account is given of the progress of revision of 1/2,500 plans by air survey. The experimental revision is almost completed; the plotting of two hundred and seventy-two plans have been completed in the Birmingham area, of which eighty-five have been ground checked, while a further eighty-five plans were in process of checking at the end of March 1938.

The report states: "The experiment has established that, with photographs of the best quality, the accuracy of the method is sufficient. A few mistakes in plotting have been discovered during the examination on the ground, but these have proved to be due mainly to inexperience on the part of the plotting staff, and not to defects in the method. Incidentally the photographs have revealed a number of small errors which had previously escaped notice.

"Taking the area as a whole the method has shown a substantial saving in time as compared with the normal procedure and the saving in cost is, of course, proportional to the saving in time."

### ADAPTATION OF THE ARUNDEL METHOD TO REVISION OF ORDNANCE SURVEY 1/2,500 PLANS

**This method has been described in the Journal of the Chartered Surveyors' Institution [10] by Lieut.-Col. R. Ll. Brown, R.E., who is in charge of this work.**

The process is somewhat shortened because there is usually sufficient existing detail to enable the control points to be plotted direct, without making a minor control plot to an unknown scale. The tilt can be eliminated by rectification in printing when the print used for plotting is an enlargement. Messrs. Aerofilms, Ltd., the contractors to the Ordnance Survey, take the photographs on a plate $8\frac{1}{2} \times 6\frac{1}{2}$ inches from 9,000 feet with a 21-inch lens, and enlarge to approximately 1/2,500. Photographs will be $9 \times 9$ inches for use with the Thompson Comparator. Aerofilms Ltd. have also undertaken a considerable amount of work on similar scales for local authorities and other public bodies.

A number of *ground control points* are chosen. These are points of old detail on the plan which can be precisely identified stereoscopically on at least two photographs. It may be necessary to select, in addition, a number of *minor control points* as in the normal method. These need not appear on the old plan.

After the photographs have been base-lined, a *minor control trace* is made of each photograph showing the base-line and rays through ground and minor control points. Also, at this stage, it may be considered desirable to select and mark the points required for plotting detail. These are called *slope control points* because they are chosen to reduce height distortion effects in plotting. All points used are selected and fixed after stereoscopic examination.

The *minor control trace* for the first photograph is now placed on the master plan showing *ground control points* and shifted about until each ray passes through its appropriate point and the base-line and control points are pricked through. Short rays are then drawn through these points as before. This is precisely the same as the tracing-paper method of resection in plane-tabling. Any ray which does not quite fit may be in error or there may be an "old error." It is usually possible to tell if the old point is in error after plotting the adjacent photographs.

Having established all the control points the detail is traced off from the photograph within the limits of each separate small triangle, formed by three *slope control points*, by using an epidiascope to project the image of that part of the photograph on to a glass screen over which is placed the plan. Tilting and lateral movements are provided so that the apices of the triangle can be made to coincide exactly with the corresponding points on the plan. The detail is then traced off within the limits of this triangle. The operation is repeated for other triangles until the whole area covered by the photographs has been plotted.

Later, the work is revised in the field, additional details noted and plotted where necessary, and the finally revised and up-to-date 1 2,500 Ordnance Survey plan is produced.

Brown mentions one difficulty which arises in modern large-scale mapping which was not met in the early days of the Ordnance Survey. Modern town-planned lay-outs are much more difficult to survey on account of the increase of curves in roads, and irregular building lines with separated dwellings. He stresses the greater ease of obtaining such detail from the air and is inclined to the opinion that the results may sometimes be more accurate than those obtained by ground methods. The errors which have arisen in the plotting from air photographs appear to be partly due to lack of training or lack of experience of the personnel.

Finally Brown mentions that future developments will be to employ a reseau (page 245) to check distortions and the Thompson Comparator for increased accuracy of measurement.

The Ordnance Survey Report for 1937–8 [73] mentions a re-survey experiment carried out on these lines over an area near Petersfield, for

which the photography was completed by the Royal Aircraft Establishment in 1937. For plotting from these photographs the Thompson Comparator has been used and the Report adds "... The general construction and working of the machine have proved very satisfactory, and the results so far show that the accuracy of the Arundel Method of plotting can be considerably increased by the proposed methods of working."

CHAPTER X

# STEREOSCOPIC INSTRUMENTS AND PLOTTERS

## INTRODUCTION

A VARIETY of types of apparatus has been produced for the automatic plotting of points from the stereoscopic observation of pairs of photographs. The development of such instruments has been traced in Chapter I and it is proposed to give here a general idea of the principles underlying their construction and operation, together with brief descriptions of some of the more important ones in use at the present time. A number of them are expensive, and descriptions of the construction, operation and setting of some of the latest types are available only in the writings of those interested in that particular machine, so that minor details of real or supposed superiority are sometimes rather exaggerated. The operation of setting a pair of photographs varies with the different instruments, and much of the controversy is in connection with the actual routine and time taken to set a pair of photographs in the machine for plotting and contouring.

The British policy of concentrating on simple methods has led to a considerable measure of success in planimetry, but contouring has proved much more difficult. E. H. Thompson [91] has stated that the problems of planimetry may be said to have been completely solved by the Arundel Method, but that heights are much more difficult to determine because six spot heights are required per overlap for reasonably accurate contouring.

This necessity for the provision of large numbers of spot heights reduces the value of the simple Arundel Method for military purposes where the provision of adequate ground height control is difficult. For instance without ground height control the contoured map cannot be extended accurately enough for the direction of artillery fire.

As a result, the Air Survey Committee decided that it was necessary to develop an automatic plotting apparatus, which would allow for the difficulties of tilt, inclined air-base and unknown height of aircraft. Most of the work in connection with these plotters has been done on the Continent and in the United States, the only previous attempts in the British Empire (and then before the War) were made by Deville in Canada and Major F. V. Thompson at the War Office.

E. H. Thompson [91] says:

"Some of the Continental firms have had such an immense experience of the construction of this type of instrument that, but for one happy fact, it is doubtful whether we in the Empire could ever hope to catch up, or excel them in the production of a machine. Fourcade has treated air survey as a separate problem and his basic principles are sounder than any produced before. We have reasonable grounds for confidence that provided Fourcade's ideas can be successfully materialized, we will produce a machine in the Empire superior for our purpose to anything on the market." Since 1935 (the date of this paper) such a machine on the Fourcade principle has been constructed. This instrument is known as the Thompson Plotter and is constructed by Barr and Stroud.

Brigadier McLeod in a discussion on the above paper pointed out that the makers of the Continental machines, "who generally know what they are talking about, make great claim for the accuracy obtained by them. So far as we are able to judge, the accuracy obtained is based upon a relatively dense ground control, but there is no doubt as to the precision of workmanship of some of these machines, and I think we cannot deny that what they claim is more or less correct or that such machines are capable of map making of great accuracy, sufficient even for large scales. The objection to them is, I think, partly the present difficulty of operating and partly the time required for adjusting."

The British commercial operators, who have made considerable use of a rectification process for work in this country, have embarked on a bolder policy for surveys abroad and for large-scale contour surveys. After due consideration, Aerofilms, Ltd. have installed a Wild A.5 Autograph. On the other hand it is considered that the Arundel Method, with fixation of control by the Thompson Comparator, has a very wide scope. For instance, if photographs are taken with an ultra wide-angle lens camera from an aircraft controlled by automatic pilot, measurements on these photographs with the stereocomparator, enable excellent results to be obtained by the Arundel Method both for plan and level, with ground control even wider spaced than tertiary triangulation stations.

The modern tendency is for the more elaborate stereo-plotters to be used primarily for control work.

## Principles of Stereoscopic Reconstruction and Plotting

The principles of perspective and stereoscopy have been discussed in Chapters V and VI and it now remains to consider how these principles are

applied to the reconstruction of the space impression from a stereoscopic pair of pictures.

The condition to be achieved is that the pictures should be set in the same relative position to each other as at exposure, and any mechanism for plotting must make use of one of two basic principles.

(i) Intersection of radial planes.

(ii) Fixation of epipolar planes.

These principles will now be considered.

*Fixation of Points of Detail by the Intersection of Radial Planes.*

In Fig. 115 a sketch is shown of the two adjacent photographs of a

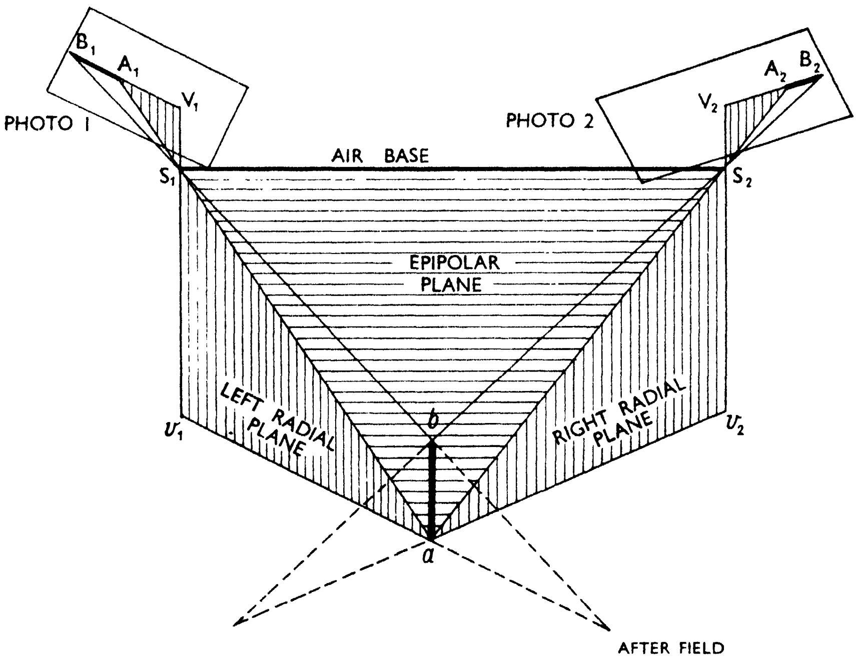



Fig. 115.

strip, forming a stereoscopic pair, which are set in their correct relative positions. If these photographs are tilted, then $V_1$ and $v_1$ represent the photo and ground plumb points respectively for the left-hand photograph while $V_2$ and $v_2$ are similar points for the right-hand photograph.

The ground points $v_1$ and $v_2$ are contained in a horizontal plane in which is also contained the base $a$ of a vertical chimney. The vertical plane containing the plumb-line $V_1v_1$ and the point $a$ is called by Field [35] the *left radial plane*, and contains also the image $A_1$ of the point $a$. The

corresponding plane $V_2v_2a$ for the right-hand picture is called the *right radial plane* and contains also $A_2$. Clearly the line of intersection of these planes is a vertical line, which in this case will contain also the top b of the chimney. Hence the intersection of the radial planes will fix the plan position of a point no matter what its height above the datum plane may be.

Field, who has done considerable research work in connection with plotting instruments in Canada, has described the fundamental principles of stereo-plotters very clearly in a paper before the Canadian Institute of Surveying. [35] A successful instrument based on this principle has been produced in Canada by Field and Burns. [13]

Field sums up the possibilities as follows: "Imagine a pair of stereo-vertical photographs to be restored to their air positions at the time of exposure, and take two vertical planes, free to rotate about the plumb-lines through the perspective centres. If each plane be rotated until its trace on the photograph passes through the image of the points we wish to plot, the intersection of the two planes will be the orthogonal projection line of the point in question. Hence we could plot in this way, point by point, the plan positions of the detail recorded on the overlapped portion of the two photographs. As the position of a radial plane is independent of the altitude of the point photographed, this method without extension, would not yield information regarding the elevation of the plotted detail." This is, of course, the basis of the radial-line method which has been described previously and which in practice involves using the principal points instead of the plumb-points. The validity of these assumptions has been fully discussed in Chapter VI, and with automatic control of flying, there should not be much difficulty with the plan positions. A mechanism can be provided which enables plan positions to be plotted with reference to the setting of a floating mark.

The problem of heights must be dealt with by "want of correspondence" observations which do not lend themselves to automatic plotting.

*Fixation of Points in Space by establishing their Epipolar Planes.**

In Fig. 115, $S_1$ and $S_2$ are the two camera stations, so that points $S_1$, $S_2$ and *a*, are in the epipolar plane of the point *a* which includes also the image points $A_1$ and $A_2$. The epipolar plane containing *b*, vertically above *a* is different from that for the base and is that plane which contains $S_1$, $S_2$ and *b* and similarly as before $B_1$ and $B_2$. It now becomes possible to determine the positions of various points both in plan and height, by rotation about the epipolar plane.

* Refer to Chapters V and VI for perspective and stereoscopic definitions respectively.

Field describes the essentials of a plotter employing this principle as follows: "Make two pointers passing through the perspective centres, to remain always in a plane free to rotate about the air-base; let the upper ends of the pointers pass through the images of a point on the negatives and the intersection of the pointers will determine the position and elevation of the point on the ground."

This second method is that which is applied in principle to all the more elaborate and accurate plotting machines, although the optical and mechanical details of construction and operation differ considerably between the various types.

*The Porro-Koppe Principle.*

One of the greatest difficulties in the early days of photogrammetry was the appreciable distortion in the projection of images through lenses. In order to eliminate the distortion, Porro and Koppe [46] used a method which consisted of reversing the direction of the refracted bundle of rays in the image space, and projecting it back into the object space, either through the camera lens or through one of the same focal length and distortion. By this means, the photographic distortion could be much reduced, and the requirement of distortion-free projection has become known as the Porro or the Porro-Koppe Principle.

Until very recently it was considered essential to incorporate this principle in any precision-plotting machine. There are, however, certain practical difficulties which may arise when it is used, as pointed out by Field [35]—"As one or other of the observed images approaches the margin of the photograph, the sighting telescope in question is directed obliquely through the lens and the visual definition falls off, resulting in some difficulty in placing the floating mark. This objection carries very much more weight with the wide-angle lenses now being produced for air cameras." The distortion of a modern survey lens system is practically negligible and the principle can be abandoned. When plotting from ultra wide-angle photographs where the distortion at the extreme edges may be 0·06 mm. or so (see Chapter IV), some necessity for adhering to the Porro-Koppe principle still arises. Opinions differ as to whether the principle can now be abandoned. This has, however, been done in the case of the most recent model of the large Wild Autograph A.5, one of the most successful of the stereo-plotters where a special glass plate of variable thickness is employed to give a refraction distortion equal and opposite to the ultra wide-angle lens distortion.

*The Zeiss Parallelogram.*

The origin of this device, which is employed in almost all plotting machines, appears to be somewhat obscure, but as it was first applied practically in the construction of instruments by Zeiss, it is usually given the above title. It is essentially a mechanism for producing a similar motion to that of the setting of a floating mark on a pair of stereoscopic photographs. Here again Field has given a very clear account of the mechanism. [35]

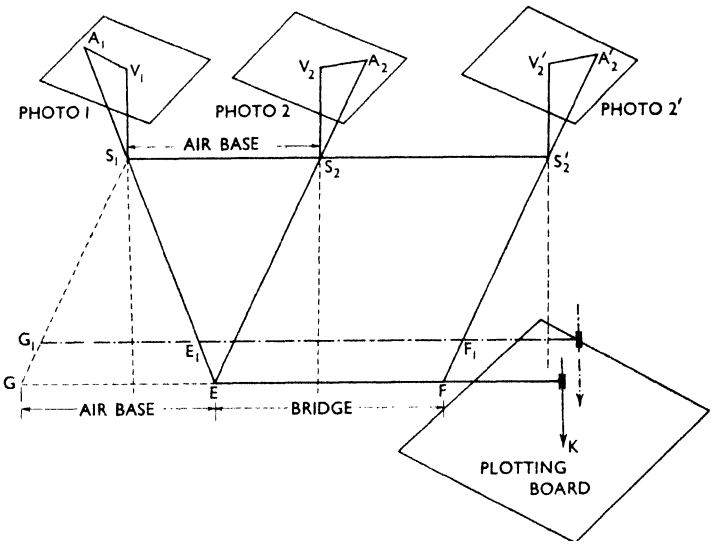



FIG. 116—ZEISS PARALLELOGRAM.

On referring back to Fig. 115 it can be seen that if a plotting mechanism were fitted in the form of rods along the lines from the ends of the air-base to the point, this might easily get in the way of the photograph holders. These holders, which reverse the original photographic process by projecting the picture back into the object space, are usually called goniometers. The likelihood of such interference arises in particular when the plotting scale is of the same order as the photographic scale. The scale of the plot is fixed by the scale of the base-line, so that such a simple mechanism would make it necessary to alter the photograph spacing whenever flying conditions caused a variation of photographic scale.

The practical solution is to provide a means whereby the relative setting of the pair is not altered when once established; any adjustment required being effected by the mechanism. In Fig. 116, photographs 1

and 2 are set in their correct relationship to one another, the camera stations (or perspective centres) being $S_1$ and $S_2$. The length of air-base $S_1S_2$ fixes the scale of the plot. $S_1E$ and $S_2E$ are two "space rods" in the epipolar plane, which fix the position of E.

In order to avoid congestion, the photograph 2 is moved to 2′, he extended air-base being $S_1S_2'$. Let EF be parallel and equal to $S_2S_2'$. Then EF is called the "bridge" and it may be seen from Fig. 116 as Field remarks: "Any point of the bridge will plot the same relative positions for stereo-images as the original point E, which was the intersection of the space-rods in our original hypothetical plotter." Since E plots both plan and elevation of detail, the whole bridge must be moved vertically when, to direct the rods correctly on to corresponding images of a point, it is necessary to spread the rods in the epipolar plane, i.e., when there is relief, or, as it may be put, a change of parallax in the images. In other words the bridge moves so that all points in it remain in horizontal planes so long as level details are being plotted, but it must be moved vertically when there is a change of elevation of the plotted detail—the bridge length remaining constant. Usually a pencil is attached to the bridge . . . which plots the relative plan positions of the points on which the sighting rods are directed. Elevation distances are found from the amount which it is necessary to raise or lower the bridge to restore correspondence between the stereo-images."

If EG and $S_1G$ are drawn parallel to $S_2S_1$ and $S_2E$ respectively, then GE = $S_1S_2$, the air-base. It follows that GE = $S_1S_2'$ − EF. This "inset" of the bridge, will, if altered in length, cause the scale of the plot made by the pencil to be changed. If the length of the bridge is altered to $E_1F_1$, then the inset becomes $G_1E_1$ and the scale of plotting is thereby also altered.

Since all points at the same height above the datum have the same value of absolute parallax, the scale will be constant along any particular contour line. To plot a contour, the floating mark is set by stereoscopic observation on to a point of detail at the height of a required contour and this setting will enable the observer to trace out the contour on the photograph by keeping the bridge at constant height, and moving the floating mark so that it appears to be constantly in contact with the ground. The plotting mechanism will automatically trace out the plan of the contour to the scale at which the bridge is set.

When the length of bridge has been set for a particular plotting scale at the datum plane, variations of photographic scale are allowed for by raising or lowering the bridge during the process of observing.

Thus for other contours, the height of bridge is set to allow for height

distortions and the process repeated, the mechanism being so arranged that the plotting pencil is in contact with the paper whatever the height of the point in space.

When plotting detail, the bridge is simply moved horizontally and vertically as required so that the floating mark traces the outlines required.

## RADIAL PLOTTERS

*The Canadian Radial Stereo-plotter.*

This instrument was designed by Lieut.-Col. E. L. M. Burns, R.C.E. and Mr. R. H. Field of the National Research Laboratory, Canada.

The simple radial-line method has been used to a considerable extent in Canada for medium-scale work. Elaborate stereo-plotters are considered too costly for mapping at these scales; and also the time taken in setting is considered to be excessive for medium scales. The intention was to develop the ordinary Arundel Method and to increase accuracy and accelerate plan production, by the introduction of a simple plotting machine.

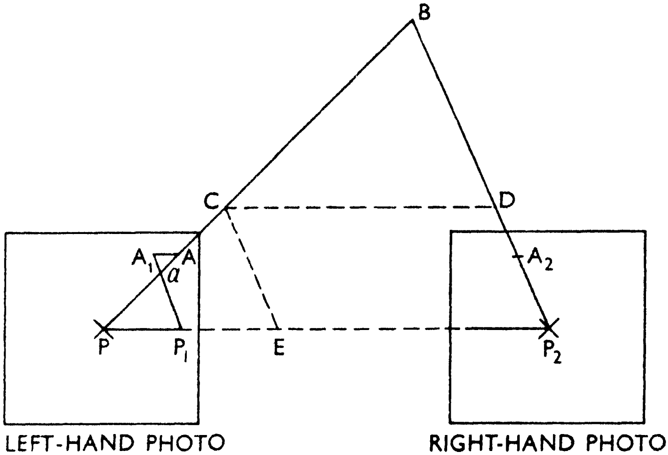


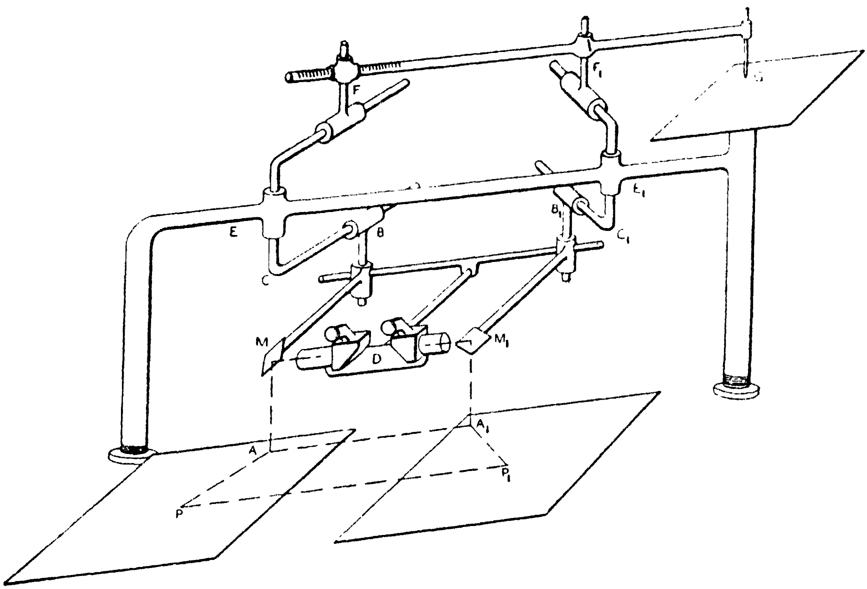
FIG. 117—PRINCIPLE OF RADIAL PLOTTER.

The instrument depends upon an application of the Zeiss parallelogram working in a horizontal plane, and has been described recently in a paper by Burns and Field. [13]

The fundamental principle of the plotter is seen in Fig. 117. The two photographs of the stereo-pair, when superimposed, have a base length of $PP_1$. If it be assumed that tilts are sufficiently small so that the radial-line method is valid, then all rays to detail on the left-hand photograph radiate

from the principal point P, and from $P_1$ for the right-hand photograph. On the first photograph A is the image of a point of detail on the ground and $A_1$ its image on the second photograph. The intersection of these rays at point a gives the position of the point to scale with height distortions eliminated.

In order that the overlap may be viewed in a stereoscope, the photographs are separated, and when accurately base-lined, $P_2$, the position of the principal point of the second photograph, will be on the line $PP_1$




[*Courtesy of National Research Council of Canada.*

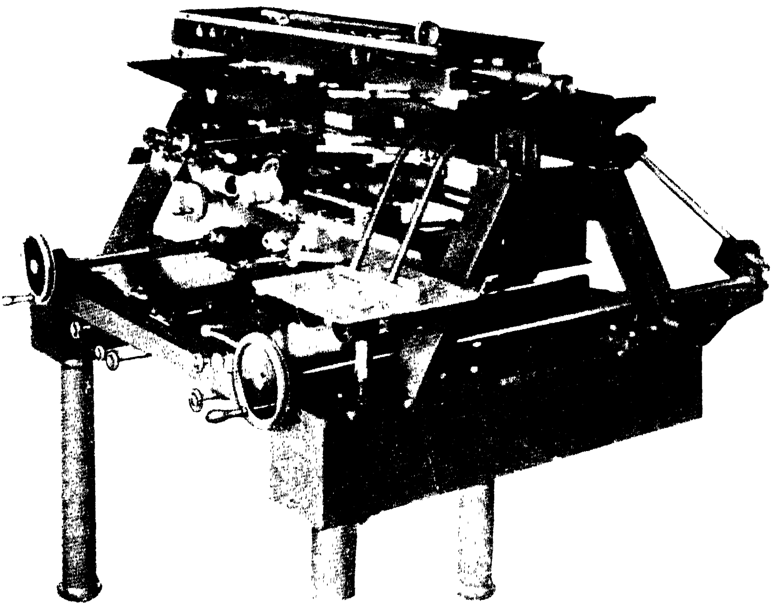
FIG. 118—DIAGRAM OF RADIAL STEREO-PLOTTER.

produced. Thus $P_1$ is moved to $P_2$, and $A_1$ moves parallel to $P_1P_2$, to point $A_2$ through a distance $P_1P_2$. The intersection of PA and $P_2A_2$ produced is at B, and it follows that the triangle $BPP_2$ is similar to the triangle $aPP_1$. Thus a plotting machine may be devised in which the radial arms PB and $P_2B$, when directed through the corresponding point of detail on the left and right photographs respectively would plot points of detail at B.

The enlargement of scale is in the ratio $PP_2/PP_1$. This simple scale-adjusting device is not practicable and the Zeiss parallelogram is introduced by making CD parallel to $PP_2$ and EC parallel to $P_2B$, and point C fixes the detail to a scale of $PE/PP_1$. The air-base is represented by the length ($PP_2$ − CD) so that by adjusting the length CD, the scale of plotting is altered.

The mechanism cannot be conveniently placed in the plane of the photographs, since this would interfere with stereoscopic observation.

A diagram of the essential points of the radial stereo-plotter is given in Fig. 118. The photographs are first set on the two tables, which are somewhat similar to those on the Barr and Stroud Precision Topographical Stereoscope (Fig. 77), and the respective principal points brought to the centres about which the tables rotate. The mirrors M and $M_1$ are set at a fixed angle so that by adjusting the spacing in the $x$-direction



[*Courtesy of National Research Council of Canada.*

FIG. 119—RADIAL STEREO-PLOTTER.

(parallel to the air-base) and their relative positions in the $y$-direction, the images A, $A_1$ of a ground point *a* are set on the left and right parts of the floating mark. In this case, the floating mark is set at the outset to correspond to the base-length, and the points of detail are brought into coincidence with it.

The plotting mechanism is actuated by the movement of the mirrors, the arm $FF_1$ being constrained so as to be always parallel to $PP_1$ while the plotting pencil on the continuation of this arm describes a path corresponding to the intersection of the radial rays from P and $P_1$ (namely PA

and $P_1A_1$ produced). The scale is fixed by the length $(EE_1 - FF_1)$ which is adjustable.

The instrument itself, Fig. 119, was constructed by the Canadian authorities and, with the addition of the plotting-machine, it somewhat resembles the Thompson Comparator, which has been described in Chapter IX. Over the centre of rotation of each table the principal point of one of the stereographic pair of photographs is placed. The photographs are brought into the correct relative orientation or base-lined in a similar manner to that previously described. The tables are spaced at the fixed distance of 18·5 inches, and may be rotated freely about their axis, or clamped and adjusted by slow-motion screw. The right-hand table can be moved independently in a direction perpendicular to the base-line, i.e., the $y$-direction, and this enables "want of correspondence" to be measured. A scale and vernier is provided to measure this displacement and this facilitates the re-setting of a pair of photographs in the instrument.

Above the photograph tables is the lower carriage, which is capable of movement in the $y$-direction. The stereoscope is attached to this carriage and can be moved as a whole in the $x$-direction.

Each telescope of the stereoscopic pair contains a graticule consisting of half of the floating mark. In order that this may be set to ground level at a particular point of detail and the plan position recorded, a definite observational procedure is necessary. First the lower carriage and the stereoscope are adjusted so that fusion is obtained near the point and the floating mark appears above or below the ground. Any want of correspondence is eliminated by the fine adjustment of the right-hand table in the $y$-direction. The $x$-movement of the stereoscope carries the mirrors with it, and any adjustment required for difference of parallax is obtained by altering the spacing of the mirrors by means of a right-and-left-hand screw. This difference of parallax can be read by scale and vernier to 0·02 mm. Other movements can be read to 0·1 mm.

Above the lower carriage is fitted the upper, or plotting, carriage which is supported on four rollers running on two rails which are bolted to the main I-beams. A scale and vernier is provided so that the "inset" of the bridge defining the scale may be set to the required amount. The plotting arm is actuated by movement of the mirrors, so that its pencil describes a similar motion to that of the floating mark. It is considered that future models of this instrument should be fitted with a special microscope, which could be substituted for the plotting pencil when it is required to re-establish a position. A useful result of the use of this instrument has been to show that a single floating mark is more satisfactory than

a parallactic grid, and since the floating mark can be moved in the $x$-direction, the accuracy of base-lining can be readily checked by moving the mark along in this direction.

The process of producing the minor-control plot is similar to that employed in the simple case of the Arundel Method (Chapter VII). First the pair of photographs is set in correspondence (or base-lined, for vertical photographs) and the base-lines are ruled in. If there are two known points in the first overlap, the plotting mechanism can be set, by proportion, at the correct scale after finding the distance between the points in terms of an arbitrary base-length. The minor control points used should be in similar positions to those in an ordinary Arundel plot. If there should be a want of correspondence in the floating mark, due to tilt, the two parts of the floating mark may appear at different levels. In such a case each half should fit over its own point of detail if possible, so that the accurate $y$-ordinate may be determined.

Having finished with the first pair, the first photograph is removed and the third one put in and base-lined, but since the direction of flight is not quite straight due to changes of direction of the aircraft, the plotting-table is now turned through an angle so that the floating mark will traverse along the base-line joining the principal points of photographs 2 and 3. The base length must now be adjusted, so that the plot can be continued at the same scale. The floating mark is set to one of the control points fixed previously and the plotted position should check; if coincidence is not obtained, the base length is adjusted until the plotting pencil is over the previously plotted position of the same point.

If the photographs have appreciable tilt or the photographic materials have distorted, there may be discrepancies. In such a case the magnitude and direction of tilt may be determined by the "want of correspondence" method evolved by Hotine and described in Chapter VIII.

The Radial Stereo-plotter, although barely past its experimental stage, shows great promise, and a number of minor improvements have been proposed for future models. The authors [13] point out that the effects of tilts less than 3° in flattish country are negligible.

Their conclusions are that photographs can be set to 0·03 mm. so that "base-line orientation should be accurate to two minutes of arc. The thickness of the base-lines, scratched by needle, is about 0·1 mm., and any departure from straightness may be detected by moving the floating mark in the $x$-direction over the principal point base."

"Experience shows that plotting by the machine holds azimuth accurately as a rule, in flattish country. Long straight roads or railways, extending through several photographs, preserve their alignment. More

difficulty has been found in keeping the scale constant throughout a strip; in many that were plotted in the early stages, variations of ·025 of scale between the beginning and end of a strip were found. . . . The same sort of errors have been met with in radial-line plotting by ordinary graphical methods, and it was in the hope of eliminating them, and lessening the amount of adjustment during compilation, that the Radial Stereo-plotter was devised.

"It is therefore not possible to state finally to what degree the Radial Stereo-plotter will increase the accuracy and speed of radial-line plotting. However, experience to date indicates that, when plotting at 1/15,840 (4 inch to 1 mile) azimuth can be held correct in strips of photographs where the tilt is small, so that the centre of a strip 750 mm. between ground control points will not be bent laterally by more than 1 mm. The scale can be held correct to about 1 per cent from one end of the strip to the other."

It is indicated that the instrument will reduce appreciably the time taken in fixing control and plotting detail.

## EPIPOLAR PLANE PLOTTERS: GENERAL PRINCIPLES AND OPERATION

*General Principles.*

Plotting machines employing this principle are able to deal easily with both plan and contours. The two photographs of a stereoscopic pair are set so that each is in the same relative position as at exposure with respect to the air-base or the horizontal reference plane. A stereoscopic viewing device enables the observer to appreciate the three-dimensional picture, and by moving a floating mark in apparent contact with the ground over the area the plotting mechanism, which is actuated by the movement of the floating mark (or corresponds with it), is enabled to make the plot automatically on a drawing-board.

It is the process of setting the photographs accurately and rapidly into their correct perspective relationships which calls for a complete understanding of the principles of perspective reconstruction, and for considerable precision in the construction of the instrument.

This process of setting involves three stages: (i) Internal setting for each photograph; (ii) Mutual setting of the pair; (iii) Absolute setting of the three-dimensional model to the ground control.

Any instrument designed for plotting by this method should provide a quick and simple routine at each stage of the setting without disturbing previous settings. Some of the instruments on the market do not allow of absolute setting without disturbing the mutual orientation of the

photographs. This means that a trial-and-error method of setting must be introduced, or computation used. In usual cases, with the established instruments this additional adjustment is easily made.

Nevertheless, even though opinions differ as to the best principles for the design and operation of these plotters, there is no doubt that all those which are established enable great accuracy to be attained, both for planimetry and contours when adequate ground control is provided for large scales. Plans and levels may be carried forward with a minimum of control for smaller scales.

*Elementary Principles of Correspondence Setting and Absolute Orientation.*

The term "correspondence setting" has been used previously in connection with photographs which are almost vertical, and where the photographs are set in the stereoscope in a common plane, so that the third dimension is neglected. This process of base-lining, which involves local correspondence setting, is a very simple case of the general problem.

Hotine's method of determination of tilt by "want of correspondence" involves measurements by which the amount of deviation from the true epipolar plane setting can be found.

In the plotters where pairs of photographs are set so that all pairs of points are in their correct epipolar planes, reference is preferably made to the epipolar axis, or air-base. Some instruments are constructed for setting to the datum of plotting, rather than to a plane of which the inclination air-base defines the maximum slope, and this former method is considered by some authorities to be less sound than the latter.

Whichever method is adopted, the problem reduces to the determination of the position in space of the inner perspective centre, as defined by the rear node of the lens system. This position must be fixed in space by tri-linear or aerial co-ordinates, either with respect to the plane containing the inclined air-base as described above, or a horizontal plane.

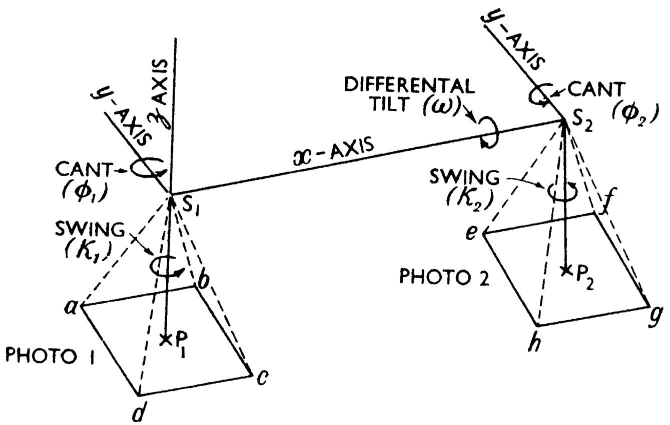
In discussing the setting of photographs according to the Fourcade Theory of Correspondence Setting, [38] it is stated in the Second Report of the Air Survey Committee (1935)—"Among various possible solutions, one alone, originally applied by Dr. H. G. Fourcade, has real practical significance, and is now almost universally adopted where possible."

There are three stages in the process of setting:

(i) *Internal setting.* The goniometer for each photograph is much the same as the original camera in its optical properties, in that the photograph which was taken in the focal plane is now replaced in a similar plane so that the picture can be re-projected into the object space. This part of the process involves the setting of the photograph into the focal

plane of the projector lens, or in a similar position in the case of the Wild Autograph, with the principal point on the principal axis. This is a similar operation to that employed with a simple stereoscope.

Thus in Fig. 120, *abcd*, *efgh* are the planes of the equivalent positives




Note: the three axes with respect to $S_1S_2$ are usually called *bx*, *by*, *bz*.

FIG. 120.

for photographs 1 and 2, while $P_1$ and $P_2$ are their respective principal points set on the principal axes $S_1P_1$ and $S_2P_2$.

(ii) *Mutual setting*. When each photograph is correctly set in its goniometer, these must be adjusted to their correct positions with respect to one another. This setting is done by means of a floating mark, and the result is a true stereoscopic reconstruction of the picture to an unknown scale, and in an unknown direction. The internal setting of each photograph is maintained during this setting.

In Fig. 120, $S_1$ and $S_2$ are the two camera stations for a stereoscopic pair, the respective equivalent positives being *abcd* and *efgh*. In order to bring the photographs into their correct relationships to one another adjustments of the goniometers, and therefore the photographs, must be possible in three dimensions. If $S_1S_2$ is considered to be the main axis, then angular adjustment should be possible about this axis and about two other axes which are mutually perpendicular to it. The only adjustment which is relative is that about the air-base, so that one goniometer only need be adjustable in this direction.

Thus the adjustment must allow of five degrees of angular freedom.

Another method of approach is to consider one goniometer fixed, when the air-base must be capable of adjustment in bearing and elevation

with respect to the fixed goniometer axis, while the other goniometer can be given an angular adjustment about three mutually perpendicular axes. As before there are five elements to be allowed for.

Referring again to Fig. 120, the five elements required may be given as follows:

(*a*) The *swing* of each photograph about its goniometer axis. Here this axis coincides with the principal axis of the goniometer and contains the principal point of the photograph. The angle of swing is called $\kappa$, so that there are *two* movements, $\kappa_1$ and $\kappa_2$.

(*b*) The *cant* of each photograph fore and aft in the epipolar plane containing the principal point. This corresponds to the $y$-axis. The angle of cant relative to the air-base is denoted by $\phi_1$ and $\phi_2$ for the two photographs. Thus again there are two elements.

(*c*) Rotation of one photograph about the air-base gives the *differential tilt** of two photographs. This is one element, and is denoted by $\omega$.

The setting involves movements in three dimensions and it can be shown that the mutual setting is correct when five pairs of corresponding points are set in their respective epipolar planes. If the setting is with respect to the air-base as the $x$-axis, then, in order to differentiate the three dimensions $x$, $y$ and $z$, these are usually referred to as $bx$, $by$, $bz$.

(iii) *Absolute setting*. In order that the stereoscopic model may be set in its correct orientation and to the desired scale, it must be adjusted to fit on to the positions of previously known control points which have been plotted to scale. Three dimensional co-ordinates of these points will be known, i.e., two horizontal ones and the height.

For the absolute orientation, Fourcade records that there are seven elements to be found, and it is here that linear functions should be introduced. These elements are as follows:

(*a*) Fixation of one length to establish scale.

(*bcd*) Three-dimensional adjustment of the perspective reconstruction about the air-base, to establish correct orientation.

(*efg*) Three-dimensional adjustment of the above, so that control points have the correct aerial co-ordinates.

Four of these seven movements can be eliminated by plotting to ground control and setting this correctly on the plotting-board.[6] The other three movements are incorporated in the instrument. These are (*a*) alteration of scale, usually by adjusting the bridge length of the Zeiss parallelogram or similar device (usually called the $bx$ movement), (*b*) setting the space-model for cant, (*c*) setting the space-model for tilt.

* It should be noted that "tilt" as applied to plotters refers to rotation about the air-base, as an axis.

The Zeiss and Wild instruments are provided with $by$ and $bz$ adjustments so that the model may be oriented in space.

*Correspondence Setting.*

Hotine has dealt with the theory of correspondence setting of stereoscopic pairs in some detail, and for a complete investigation of the theoretical principles involved the reader is referred to his book *Surveying from Air Photographs.* [55]

The Fourcade principle states that two photographs of a stereoscopic pair have five degrees of freedom which may be used to destroy the setting of the photographs in correspondence. These five are two swing movements ($\kappa_1$, $\kappa_2$) two cant movements ($\phi_1$ and $\phi_2$), and one differential tilt movement ($\omega$). If five pairs of corresponding points are set in correspondence, then all other such pairs of points will be in correspondence. In practice, six pairs of points are taken, to facilitate routine and afford a check. These points are chosen so that they are affected by some movements and little affected by others. This makes it possible to employ a simple setting routine. The six points on an overlap are in somewhat similar positions to those of an Arundel Minor Control Plot as may be seen from Fig. 121.*

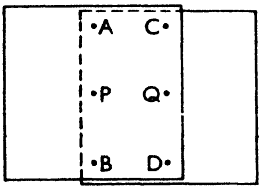


FIG. 121.

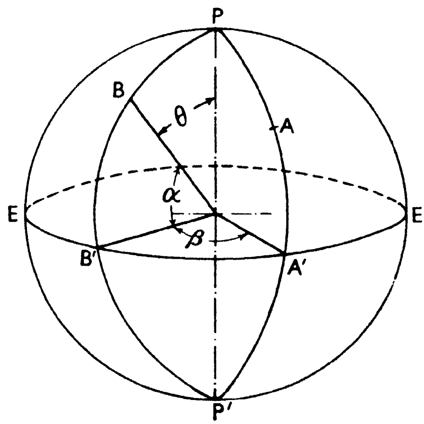


FIG. 122.

* The principle may perhaps be best understood by considering some elementary properties of a sphere. In Fig. 122 PP′ is the polar axis of a sphere which is divided in half by the equator EE′. Any diametrical section through the sphere has as its outline a *great circle*, e.g., the equator is a great circle. An important property of a sphere is that the shortest distance between two points on its surface is along the great circle connecting them. Any other section which does not pass through the centre has as its boundary a *small circle* e.g. a section through B and parallel to the equator forms a small circle known as a parallel of latitude.

On a sphere, distances are measured in terms of the angle subtended at its centre. Thus the arc BP is fixed in size, for a particular diameter of sphere, by the angle $\theta$. In order to fix a point on the surface of a sphere with respect to the centre, two angles are necessary, (i) the angle between the plane of the polar great circle of the point concerned and that of the reference plane, e.g., if the reference great circle is defined by PAP′, then the *longitude* of B is given by the angle $\beta$; (ii) the angular distance of the point from the equator—for B in this case the angle is $\alpha$. This corresponds to *latitude* for points on the earth or the *declination* of heavenly bodies. The latter term is used in connection with plotting machines.

Referring to Fig. 123 SS′ is the polar axis or air-base of the stereoscopic model, with which SP the camera or goniometer axis makes an angle of $\alpha$. The axis of cant, SU, is perpendicular both to the polar axis and the principal axis SP. Suppose a pair of photographs is set in correspondence, then if the cant of the goniometer axis at S is varied by rotation about SU, the declination is altered and correspondence over most of the photograph will be destroyed.

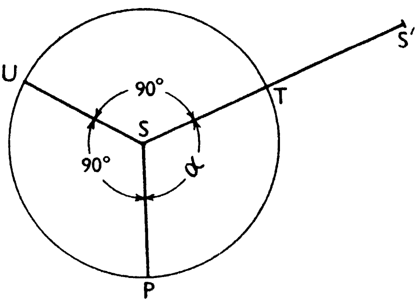



FIG. 123.

The polar axis for the sphere of radius SP is defined by ST and it can be shown that a movement of the goniometer axis in declination, disturbs the correspondence of all points except that of the principal point and points on the equator of this sphere. All other points move on a small circle of the sphere and correspondence is thereby destroyed.

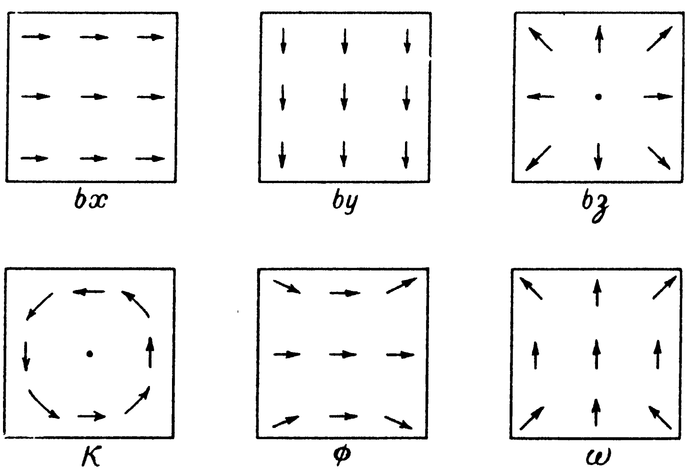



FIG. 124—EFFECT OF SETTING MOVEMENTS.

*Effect of Setting Movements.*

In the Zeiss and Wild instruments there are six degrees of movement which can effect the setting of a pair of photographs, namely the three-linear movements $bx$, $by$ and $bz$, and three-angular ones $\kappa$, $\phi$ and $\omega$.

The diagram in Fig. 124 shows the effect of each of these, on nine points arranged symmetrically round the principal point. If the length

of air-base is altered by adjusting $bx$, all the points will move in a direction parallel to the base direction. If the $by$ screw is adjusted, the points will move at right angles to the base, and if the photographs were previously in correspondence, this will be destroyed over the photograph.

The effect of a $bz$ adjustment to the photograph is to change the size or scale and therefore all the points will either move in or out radially with respect to the principal point as centre.

An adjustment about the $\kappa$-axis for swing causes all the points to move in a circular path about the principal point as centre.

The cant and differential adjustments have effects which are not quite so apparent at first. When an adjustment is made for cant or longitudinal tilt about the $\phi$-axis, the only points for which the correspondence is not disturbed are those along the equator of the sphere, i.e., along the base line passing through the principal point. All other points move on a small circle of the sphere, and their paths are curved as seen in the diagram.

When a differential tilt adjustment $\omega$ is made, correspondence is generally disturbed over the photograph, but not in quite the same manner as for the $by$ movement, because the points clear of the central meridian now move in a curved path.

### THE FOURCADE STEREOGONIOMETER AND THOMPSON PLOTTER

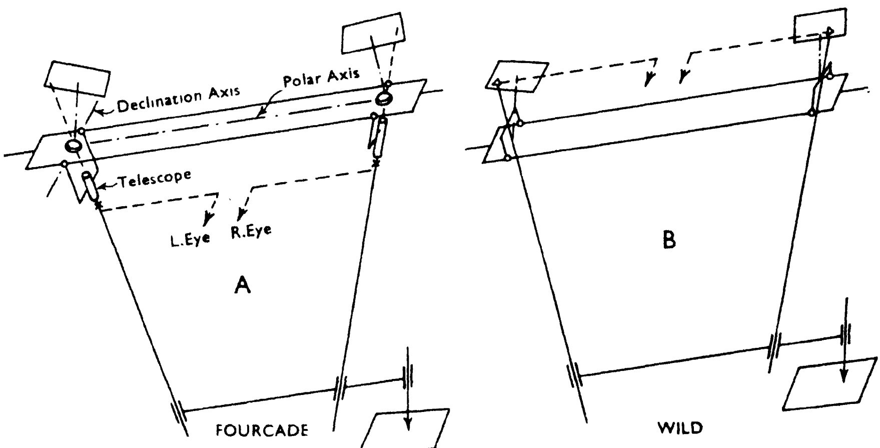
This instrument has been designed and built according to the principles laid down by Fourcade. The preliminary work on the stereoscopic observing system was largely due to Hotine at the time he was Research Officer to the Air Survey Committee, and the instrument and experimental work on it have been described by him in a special professional paper of the Committee,[59] and in his book *Surveying from Air Photographs*.[55] The plotting mechanism has been designed by Thompson during his period as Research Officer to the Committee. A preliminary model was described by him at the Conference of Empire Survey Officers in 1935,[91] while the plotter itself, built by Barr and Stroud, has been described by General McLeod, the Director-General of the Ordnance Survey[68].*

Burns mentioned during the discussion on Thompson's paper that the Canadian authorities, after reading all the available literature on the subject, had been persuaded by Hotine's descriptions that the Fourcade machine was based on the best principle and that "a machine constructed on these lines should be easier to operate and easier to understand than any of those which are in existence and cannot make use of Fourcade's principle because of patent rights. . . . We think that we have the best

*** Destroyed during recent war.**

principle here to work on, yet we must realize that the Continental machines and especially the Wild and Zeiss machines have reached a very high pitch of excellence."

Field, who, with Burns, has been responsible in Canada for the construction of plotters, described the principle of the latest design for the Fourcade Plotter at the thirtieth annual meeting of the Canadian Institute of Surveying in 1937.[35] He says with regard to this instrument that it has been "designed in part by Dr. Fourcade of South Africa and in part by officials of the British War Office, after some consultation with our




Porro principle: Photographs observed through lenses similar to camera lenses by means of telescopes attached to space rods moving in epipolar planes, about air base, or polar axis.

Space rods actuate small prisms, moving parallel to planes of negatives giving normal observation of all parts of overlap.

[*Courtesy of National Research Council of Canada.*

FIG. 125.

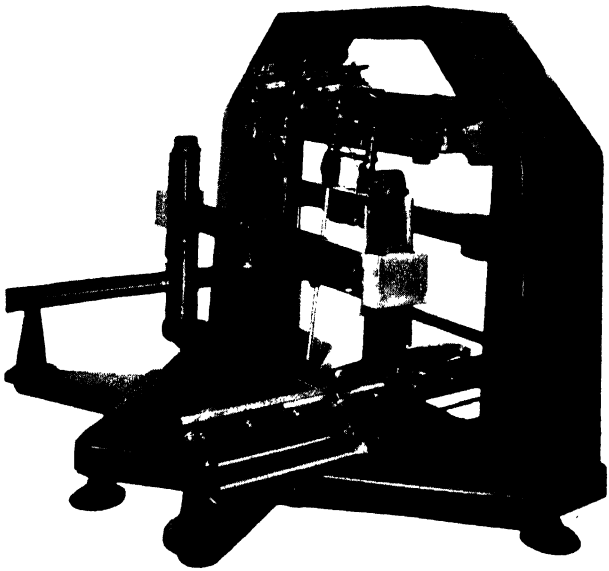
own Survey Research Committee and with the co-operation of the firm of Barr and Stroud of Glasgow, who are building the instrument."

One of these instruments has been built for Canada and one for Great Britain. The instrument which has been finally constructed after considerable modification is called the Thompson Plotter.

A diagrammatic lay-out of the instrument is shown in Fig. 125A, as given by Field, who describes it as follows:

"The photographs are set up in two skeleton cameras, or goniometers. These can be tilted together or independently about the polar axis, corresponding to the air-base, and about axes perpendicular to this—the declination axes (i.e. cant, Fig. 120). Each photograph can also be rotated in its own plane, and these three movements permit complete correspondence to be restored by an appropriate systematic setting programme.

"Sighting rods are used in epipolar planes swinging about the polar axis and forming a Zeiss parallelogram, with a plotting pencil actuated by the bridge. Lenses of the same focal length as the camera lenses are fitted at the intersection of the polar and declination axes,* and the direction of the sighting rods is continued optically (through the lenses) by telescopes at their upper ends as indicated in the diagram. The semi-floating marks are placed at the foci of the objectives of these telescopes, and the lines of sight are continued and brought to two fixed eye-pieces by means of a suitable optical train. The observer guides the bridge



[*Courtesy of Barr and Stroud, Ltd., Glasgow.*

FIG. 126—THOMPSON PLOTTER.

along the direction of the polar axis by means of a control actuated by one hand, and in the perpendicular horizontal direction by means of the other hand, while a foot disc changes the elevation of the bridge. By suitably operating these three controls, the floating marks can be apparently moved about the landscape in three dimensions and plan positions or contours plotted at will."

A photograph of the Thompson Plotter is given in Fig. 126.

*Correspondence Setting.*

The basis of the method of setting used on the Fourcade Stereogoniometer is as follows:

* Porro-Koppe principle.

(i) The photographs are set in the focal plane of the goniometer and with the principal point of the photograph on the principal axis of the goniometer. The two photographs are oriented by reference to the floating mark with respect to detail in the neighbourhood of each principal point in turn, in the same way as with a Barr and Stroud Precision Topographical Stereoscope (Chapter VI), except that any want of correspondence of the floating mark can be taken out by adjusting one goniometer for differential tilt, and both for cant or fore-and-aft inclination.

(ii) In order that the setting may be completed, four other points are chosen in the common overlap. These are selected as closely as possible in the photographic equators, and at approximately equal polar angles from the principal point as measured at S.

The cant direction of the left goniometer is set to zero, i.e. into the equator, and A and B (Fig. 121) found at approximately equal angles on opposite sides of P by swinging out both goniometers together until suitable points are found. C and D are found in the same way with respect to Q.

In setting the photographs for P and Q, it will be found that the swing adjustment $\kappa$ has been made practically correct, and for the other points cant (declination) $\phi$ and differential tilt $\omega$ can be eliminated by a simple routine.

Finally each point of the six should be checked with the floating mark to make sure that all "want of correspondence" has been eliminated, and it will be found also that this has been eliminated for all other pairs of corresponding points. Hence the two photographs are set in the same relation to one another and to the air-base as at exposure.

*Setting a Strip of Photographs.*

In many cases observations are required along a strip to establish setting and orientation before any detail is plotted. The Fourcade Method involves the determination of the three-dimensional co-ordinates of the perspective centres, with respect to the inclined air-base of the first photograph. This means that the co-ordinates are fixed relatively on an unknown scale. Then, knowing three points in space (i.e., horizontal co-ordinates and height), one at each end of the strip, and two as widely spaced laterally as possible to give a control for level in both fore-and-aft and lateral directions the co-ordinates of the camera stations can be transformed into the horizontal plane. Provided that the absolute tilt and position has been fixed for one pair of photographs, it is theoretically possible to continue indefinitely on these lines without further ground control. By taking one strip with ground control, parallel to an area where

ground control is difficult to obtain, a considerable area may be mapped in plan and level by running out cantilever strips, with a cross tie strip parallel to the first one.

One of the major differences between instruments arises from the manner in which the space-model is "fastened" to the ground control. Thus the Fourcade Method excludes any reference to ground control until the mutual orientation along the strip has been established. In other instruments, while the importance of correspondence setting to the base-line is now recognized, it is considered desirable, when working along a strip, to establish the setting to the datum plane and so determine the true space co-ordinates with reference to the datum. Thus in the Wild Autograph handbook, it is recommended that the setting should be completed with respect to the air-base for a pair of photographs, but that for a strip, instead of employing the five-angular elements, it is recommended that three only should be used, together with two linear ones, for relating to the datum plane. In practice it seems that this method is adequate, and, although perhaps not so theoretically perfect as the Fourcade principle, it has the advantage of setting the model to the required horizontal datum at an early stage in the work, although the setting process may require rather more trial and error.

When the various instruments are described an outline of the special setting method will be given.

*Detail and Contour Plotting.*

When the photographs have been set in the instrument in their correct relationship or re-set after computation, the detail plotting follows. If the floating mark is set to any point of detail, it will be free from parallax and "want of correspondence" when the mark appears to touch the ground at that point. The plotting mechanism must be so constructed that the plotting pencil will describe a similar motion to the apparent motion in space of the floating mark. The instrument must have, in addition to the setting adjustments, three others by which any pair of corresponding points can be brought into the correct epipolar plane containing the point of detail on the stereoscopic model and the air-base. Obviously, if this setting alters the relative setting of the photographs, setting the photographs will take a very long time. Hence the setting to any point of detail should be quite independent of the correspondence setting. To fix the position of a point, the epipolar plane of the point must first be established with respect to a reference plane; and to fix the precise position in space in this plane, the directions of the rays to their respective perspective centres from the images of the point must be noted.

The usual arrangement is that the plotting pencil automatically traces out the plan as traversed by the floating mark, while the heights are measured off from a vertical scale. When the height is fixed by keeping the bridge of the plotter at a constant height, the plotting pencil traces out a contour when the floating mark is kept in apparent contact with the ground.

Modern tendencies in the allocation of work between various grades of instruments will be given at the end of the chapter.

### THE WILD AUTOGRAPH A5

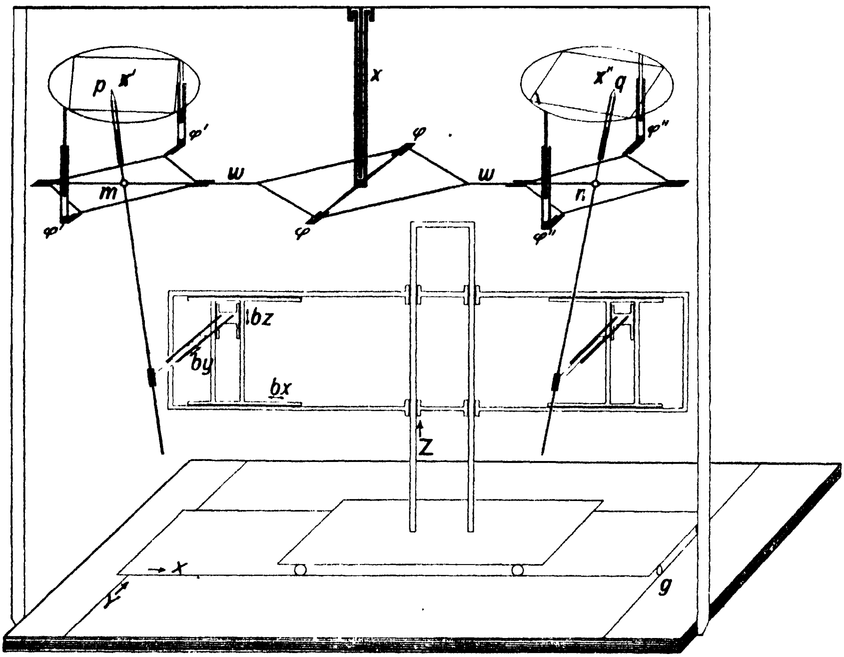
The most recent model of this instrument, Model A5 made by the Wild Surveying Instruments Supply Co., Ltd., of Heerbrugg, Switzerland, is firmly established as a thoroughly practical method of setting and plotting from air photographs. It is also suitable for ground photogrammetry. The instrument has become established not only on the Continent but in many other parts of the world, and has been selected as, in their opinion, the most suitable available instrument for their purpose by Messrs. Aerofilms, Ltd., of Wembley, and will therefore be the first automatic plotter to be installed in Great Britain for routine survey work. In this instrument either negatives or glass positives (diapositives) are used.

The basic construction may perhaps be best seen by referring to the sketch, Fig. 125B, given by Field.[35] Here it will be noticed that the Porro principle is not adhered to, since it is considered that the accuracy of modern lens construction enables this to be neglected without sensible error. With ultra wide-angle photographs a special glass plate of variable thickness is fitted to counteract lens distortions. The sighting rods are carried up through the intersection of the polar and declination axes, and reflecting prisms at the top are actuated in the plane of the photograph by the movement of the rods. The photograph is actually placed a smal' distance away, but parallel to the correct plane, and the stereoscopic view is seen in two fixed eye-pieces to which the line of sight is brought from the photographs. The semi-floating marks when set to ground level at a point enable the sighting rods, coupled to them, to point in space to the point observed. Plotting is effected by incorporating the Zeiss parallelogram mechanism.

For a complete description of the construction and mathematical principles of the Wild Autograph, the reader is referred to the handbook published by that firm in 1938—*Wild Autograph Model A5; Description and Instructions*, by E. Berchtold.

Since the distortions of the modern survey lens may be neglected for

almost all practical purposes, the abandonment of the Porro-Koppe principle enables the apparatus to be at once used for a much greater range of photography without the necessity of changing lenses. Berchtold points out that wide-angle photographs projected through a lens on the Porro principle may result in noticeable curvature since even the best lenses do not produce the image in an ideal plane, and he considers it




[*Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 127—DIAGRAM OF WILD AUTOGRAPH, MODEL A5.

preferable to abandon the principle in universal instruments, such as the Autograph.

The movements provided for setting the photograph internally, mutually and absolutely are very similar to those in the Fourcade instrument. A rather more elaborate diagram than that already given is shown in Fig. 127. It will be noticed that the three mutually perpendicular axes for aerial co-ordinates are two in the horizontal plane of the plan and one vertically. The lower carriage on the table can be moved in the $y$-direction, while the upper carriage can be moved in the $x$-direction. The vertical slider can be moved vertically in the $z$-direction without disturbing the $x$ or $y$ settings. Inside the vertical slider are two other sliders, each connected to the plotting arm of one photograph. Each is capable of three

movements, namely, the $bx$, $by$ and $bz$ directions. When the photographs have been set, the plotting mechanism may be connected to the $x$ and $y$ motions in order that the plan may be plotted. Hand-wheels, foot-wheels and slow-motion adjustments are provided for the various movements.

The main $x$-axis of the instrument is suspended centrally as shown and the casing of this supports the observing system which can be rotated about the axis, the zero setting of the attached $\phi$ axis being in the $y$ direction. This axis will give fore-and-aft tilt to the air-base. At right angles to the $\phi$-axis is the $\omega$ or differential tilt axis for which the zero is along the $x$-axis. The photographs are given three differential adjustments, (i) $\kappa'$ and $\kappa''$ swings; (ii) $\phi'$ and $\phi''$ cants or fore-and-aft adjustment; (iii) $\omega'$ and $\omega''$ differential tilts.

The number of adjustments is greater than in the Fourcade instrument. This is because the instrument is so constructed that setting may be with respect to the horizontal as well as to the air-base.

The pointer of each rod represents the appropriate half of the floating mark, and the rod indicates the direction of the plotting arm, the pointer always being kept in contact with the photographic plane, with the lengths $mp$ and $nq$ equal to the focal length of the camera lens when set at principal point of photograph.

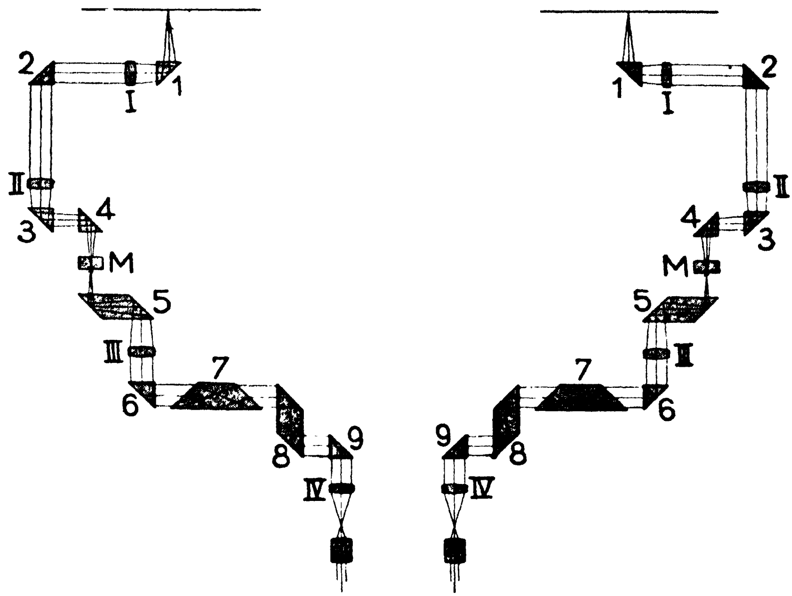
The "inset" of the bridge determines the scale at which the movement is transferred to the plotting mechanism, but this is not the final plotting scale which can be adjusted also at a "gear-box" which is, in effect, an enlarging and reducing device. By this means the scale can be controlled at two points, and the best arrangement is chosen for facility in the instrument and plotter.

When the photographs have been set in correspondence and the scale established with the space-model set with respect to the horizontal plotting plane, the floating mark is moved over the points to be plotted. The length of bridge remains constant, and the height of bridge, or $z$ ordinate, records the height. If the bridge is at a constant height a contour is traced, while the vertical movement of the bridge in the $z$ direction gives a measure of the height, which can be found from the scale setting. The mutual and absolute setting of the photographs is not disturbed by the setting movements of the floating mark to points of detail.

It has not been possible in the diagrams given to show the optical system of the Autograph, which as for other similar instruments requires an elaborate system of prisms and lenses. This system is illustrated in Fig. 128, and shows the complicated path by which the rays come from the photographic plane at the top to the observers' eyes at the bottom. The various optical devices are necessary to allow of observation when all the

various adjustments are made and, on account of lack of space, it is not proposed here to go into detail of this optical system. Fig. 129 shows the instrument.

Some particulars of the instrument are as follows:




[*Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 128—OPTICAL SYSTEM FOR WILD AUTOGRAPH A.5.

(i) The observing system has a magnification of $\times$ 5, $\times$ 8 or $\times$ 10 by simple adjustment.

(ii) Focal distance is adjustable between 100 and 215 mm.

(iii) Negatives up to 180 mm$^2$.

(iv) Angular adjustments are such that oblique photographs may be set in the instrument.

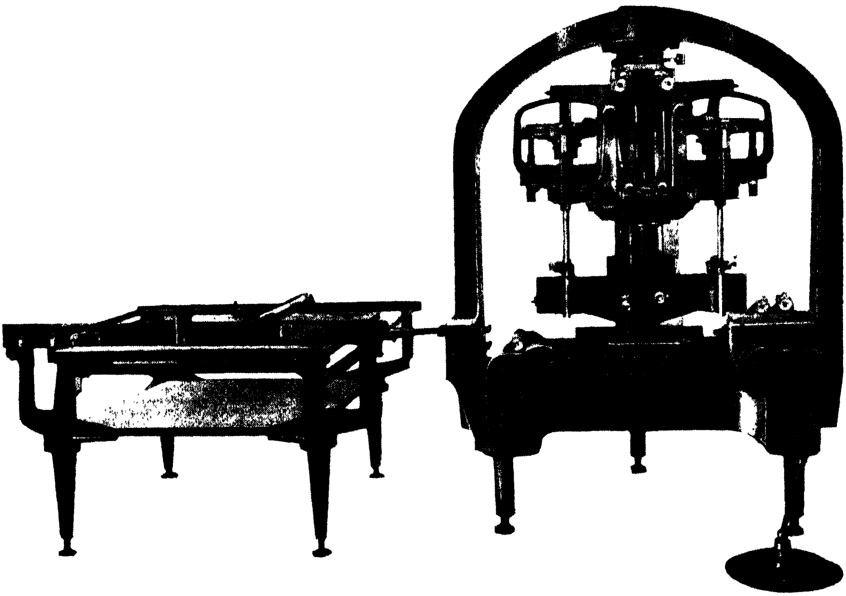
It is claimed that the new Wild Autograph is the only stereo-mapping apparatus on the market which allows of the space-model being rotated about all three space axes without disturbance of the stereoscopic model relief.

*Correspondence Setting with the Autograph A5.*

The pair of photographs are first set in the holders in their correct interior orientation, so that each will rotate in its own plane about the principal point.

The adjustments to be made are $\kappa'$, $\kappa''$ swings, $\phi$, $\phi''$ longitudinal tilt or cant, and differential lateral tilt $\omega$.

The various movements are set to zero and the photographs are first approximately oriented by inspection of detail in the neighbourhood of the principal points, and the measuring mark on the left-hand photograph is then set on the principal point by moving the main $x$ and $y$ adjustments by turning the handles. By operation of the foot-height disc for the main $z$ movement, the other measuring mark is brought into such a position that the image of the principal point of the left-hand photograph, as it



*[Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 129—WILD AUTOGRAPH A5, WITH PLOTTING-TABLE.

appears on the second one, has the same $x$ position but not the same $y$-ordinate. By swinging the second photograph ($\kappa''$) the corresponding image may be brought under the right-hand measuring mark. The right-hand measuring mark is now brought on to the principal point of the right-hand photograph, and a similar adjustment made on the left-hand photograph ($\kappa'$). Although the photographs are still set as though they had been taken truly vertically it will probably be possible to obtain some stereoscopic relief over part of the overlap area.

It now remains to set each photograph to its $\phi$ reading. By reference

to Fig. 130 (which represents the condition when the right-hand photograph is given an adjustment about the $\phi''$ axis) it will be noticed that the effect of $\phi$ is apparent at the corner points. If the measuring mark on the right-hand side is set on the right-hand principal point and the main $y$ adjustment given so that the view point is moved out in a direction perpendicular to the base-line, it will be found generally that, even when the height-disc is adjusted until the left-hand mark is as nearly as possible on the same point, there is a discrepancy in the $y$ direction. This is partly due to $\phi$ and partly due to $\omega$. If this amount is estimated and the view point run across in the $y$ direction to the corresponding point on the other side, the discrepancy will be found to differ because $\phi$ and $\omega$ effects are not of the same sign. The adjustment for $\phi'$ on the left-hand photograph is made the average amount, which should be approximately the amount of $\phi'$. Owing to a diagonal movement of the point, a $bx$ adjustment is now made. A similar adjustment is made for corresponding points with $\phi''$.

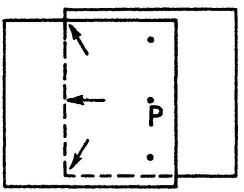



FIG. 130.

Then follows the adjustment for the relative lateral tilt $\omega$, which may be applied to either photograph. The $\phi$ adjustment will have left a small discrepancy between the points in the $y$ direction, and this is corrected by adjusting about the $\omega$ axis. The visual adjustment has to be made greater than that apparently required because of various effects, such as another adjustment to correct $\kappa$, and this amount can be calculated approximately. Thus in one case the writer found it necessary to over-correct $\omega$ six times.

Finally the $\kappa'$ and $\kappa''$ adjustments are made again and it will usually be found that the setting is very nearly correct. By going round again and finishing on the $\kappa$ setting, it will be found, in most cases, that the pair of photographs are now set in correspondence. A skilled observer can complete the setting in approximately ten minutes.

The above is described to give an idea of the process of setting and once the routine is mastered the operation of the instrument is quite straightforward.

*Setting the Model to Ground Control.*

The routine which is commonly used in the operation of the Autograph is to set the first pair of photographs of the strip to ground control before inserting the succeeding photographs. For this purpose it is necessary to have three known points in the common overlap, for two of them the complete air co-ordinates $x$, $y$ and $z$ and for the third the height $z$.

At this stage the plotting-table is brought into operation, and the floating mark set to each of the two fully known control points in turn, so that their positions can be plotted on the table, and their distance apart measured and compared with the required distance.

A gear-box between the observing system and the plotting-table enables the ratio of plot to be controlled.

In a certain instance it was found that the correct distance between two points was 17,480 metres (17·48 cms. at a scale of 1/10,000) while the plotted distance was 15·48 cms. The length of base as read on the Autograph was 83·24 cms. and this had to be increased by $83{\cdot}24 \times \frac{17{\cdot}48}{15{\cdot}48}$. Having done this the two points were plotted again and found this time

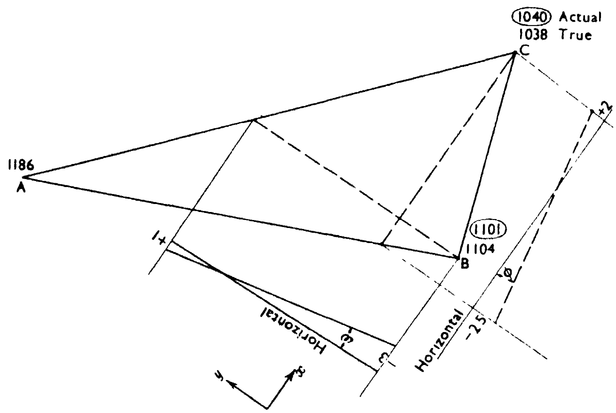



FIG. 131.

to be 17·47 cms. apart. The accuracy with which this scale adjustment can be made at the first attempt is a tribute to the accuracy of construction of the instrument.

The scale having been established the space model must be adjusted into the horizontal. In the diagram Fig. 131, the control points are A, B, and C, their heights being 1,186, 1,104, and 1,038 metres respectively. Taking A as the reference point, the height scale is set to 1,186 when the floating mark is on the point, and this is in turn set to B and C and the heights read off on the scale. In this case these were 1,101 and 1,040 metres respectively.

The next stage is to compute the longitudinal tilt of the model $\phi$ and the lateral tilt $\omega$. Through C is drawn a line in the $x$ direction cutting AB in D, the interpolated true level along this line being 1,046 metres.

Similarly a line drawn from B in the $y$ direction cuts AC in E, where the interpolated true level is 1,122 metres. By inspection of the observed levels from the instrument, the levels at D and E are respectively 1,043·5 and 1,122 metres. From these figures, the longitudinal and lateral tilts of the model may be computed. Sections drawn along CD and BE make this procedure clear. The values were found to be $\phi = 30{\cdot}5'$ and $\omega = 25{\cdot}7'$ (centesimal angles).

The correction for $\omega$ is made by giving each photograph a lateral tilt of 25·7′ in the correct direction, and although a common $\phi$ adjustment is provided on the instrument, it is found more convenient in practice to give each photograph an additional $\phi$ correction, leaving a small $bz$ correction to be made. This is said to be more convenient when a strip of photographs is being set.

The $bz$ correction required after making the independent $\phi$ correction $=$ base $\times$ $\sin \phi = 9390 \times \sin 30{\cdot}5' = 4{\cdot}4$ metres, and since the scale is 1/10,000, the vertical adjustment is 0·44 mm.

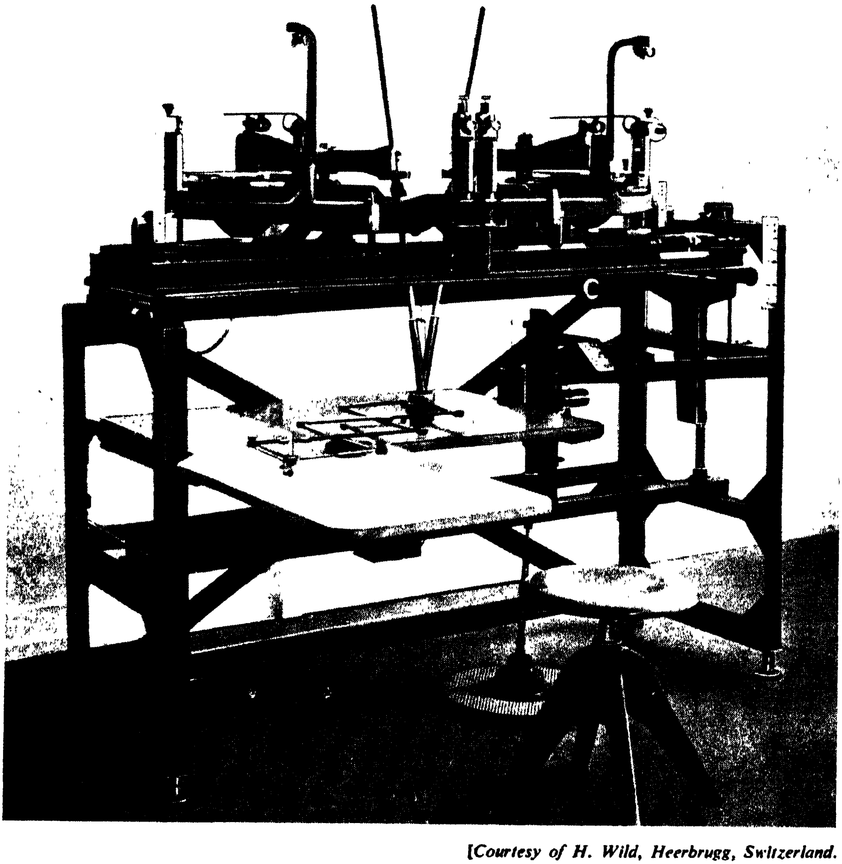
The floating mark is now set in turn to the control points and the height readings should correspond very closely to the required figures. The disagreement in such a case as this is not likely to be more than one metre.

The next stage is to insert a third photograph in place of the first one, and this time the adjustments must be made only to the photograph which has been added. Although theoretically, angular adjustments only should be made, in practice it is found that the setting is conveniently made by introducing some linear adjustment at this stage.

Finally when the whole of a strip has been through the instrument a control point is required on the last overlap. It is often found that the plan position agrees very well, but there is usually an appreciable closing error in level. This level error is a parabolic function and the best results are obtained when there is also a control point somewhere near the centre of the strip. Thus the corrected aerial co-ordinates of the points which correspond to a minor control point may be determined. In some cases the photographs are set in both directions along the strip for greater accuracy. In this way the instrument is used for control and the aerial co-ordinates of picture control points determined. The detail plotting is then carried out in an instrument of less precision, but which is capable of utilizing the values of swing, tilts, etc. obtained with the other instrument. It is found that for large surveys, instruments such as the A5 are best used for this control work only, or the progress is somewhat slow.

The spacing of control points will depend upon the scale and accuracy required. It is possible to carry forward a plot over a greater distance,

while obtaining accurate contours, than is possible with the radial-line method. In one case in an exploratory survey where a map was plotted on a very small scale, the distance between control points was forty miles




*[Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 132—WILD AUTOGRAPH MODEL A6.

for a contoured map plotted with the Autograph. For large-scale contoured plans, the control points must be closer together if contours at vertical intervals of a few feet are required. At these scales, the levels of points as determined from stereoscopic observations should be accurate to a foot or two, and it is unlikely that any stereoscopic method of observation will give greater accuracy.

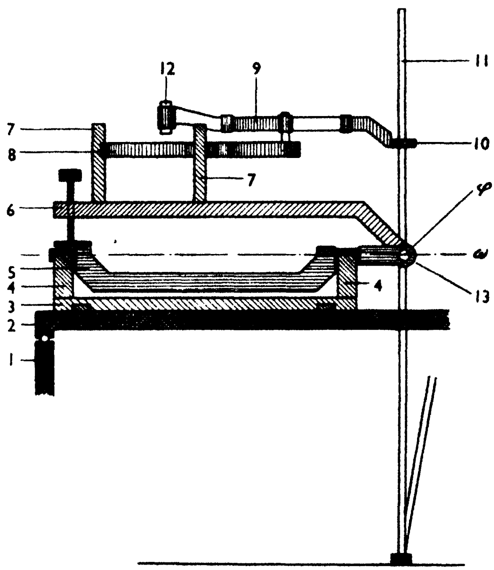
The Autograph A5 is being used in many parts of the world, and is capable of producing plans from air photographs with contours up to scales of 1 1,000, or somewhat larger. It can also be used for topographical maps.

## THE WILD AUTOGRAPH A6

This instrument has been recently designed by Wild and is intended for plotting on topographical scales from approximately vertical photographs. The maximum tilt which can be eliminated in the setting of the instrument is five degrees.

This plotter has been designed for plotting on topographical scales and for working in conjunction with the larger and more elaborate Autograph A5. Under such circumstances the latter instrument would be used primarily for aerial triangulation, the photographs being later transferred to the smaller instrument for detail plotting and contouring.

The instrument provides a purely mechanical projection of the epipolar plane of any point. The solution to the problem is strictly geometrical and A6 may be used as an independent machine.



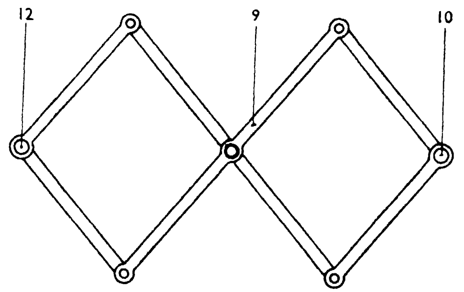

[*Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 133—WILD AUTOGRAPH A6. DIAGRAM OF LEFT CAMERA SYSTEM.

A photograph of the instrument is shown in Fig. 132, but the principle is shown in the diagram (Fig. 133). The support 1 may be levelled by foot-screws, and on this is carried a rigid frame 2 upon which the camera systems are mounted. The left-hand camera system shown is carried upon another frame 3. This frame is fixed, but that for the right-hand camera can be moved in the $x$ direction. A $Y$-bearing 4 is fixed at each side of the camera frame, and carries the axis-bolt of the cradle 5, enabling rotation to be made in the $\omega$ direction. Each cradle has at its end an axis $\phi$, for longitudinal tilt. The $\omega$ axes of both camera systems lie in the same straight line, which has one point of intersection with each

$\phi$-axis. The camera body 6 can therefore be rotated in the mutually perpendicular directions $\omega$ and $\phi$. The attached picture carrier 8 can be raised or lowered parallel to itself, while the photograph itself may be rotated in about a central axis to correct swing ($\kappa$).

It has been previously explained that one difficulty of providing a simple epipolar plane mechanical plotter is that of spacing the photographs so that they are free of the plotting arms. In this instrument, a double parallelogram on the "lazy tongs" principle has provided a simple and neat solution. This is illustrated in elevation in Fig. 133 and in plan in Fig. 134. The interior free end of each arm carries a cardan joint attached to a sleeve 10 for the reception of the guide rod 11. The exterior ends of the mechanism are hollow and contain the observing telescopes 12 with the measuring mark. It is this telescope which covers the area of photograph being observed, and observation of the stereoscopic pair is made through the eye-pieces seen at the front of the instrument. It will be noticed that the rod passes through the intersection of the $\omega$ and $\phi$ axes, where there is an additional cardan-mounted guide-rod sleeve 13. These guide-rod sleeves of the two cameras are quite close together, and their spacing is adjustable. This spacing is indicated on a scale and gives the length of the air-base.




[*Courtesy of H. Wild, Heerbrugg, Switzerland.*

FIG. 134– WILD AUTOGRAPH A.6. DOUBLE PARALLELOGRAM FOR SPACING PLOTTING ARMS.

The two guide-rods are connected together at the lower end. This lower end may be moved about over the horizontal table which can be raised or lowered by the foot-disc (Fig. 132).

The centres of the cardan-joints 13 represent the projection centres of the two photographs. The distance between them is the air-base, while the measuring planes are defined by the planes in which the cardan centres of the sleeves 10 move when the guide rod intersection on the table is displaced.

The vertical distance between the measuring planes and the cardans 13 corresponds to the focal length of the lens and is determined by the height of the picture carriers above the $\omega$-axis. This is adjustable between 98 and 215 mm.

The setting movements are similar to those employed on the large

Autograph A5, but are rather easier to manipulate while the accuracy of setting is not so great.

The maximum size of picture which can be used with the instrument is at present 18 $\times$ 18 cms., and negatives, diapositives, or paper prints may be used.

After placing the photographs in the holders, the usual relative orientation process is gone through on six points, by means of the two swing movements ($\kappa$), the two longitudinal tilts ($\phi$) and the relative lateral tilt ($\omega$). The scale is altered by changing the spacing of the joints as described above.

The absolute orientation of the space model is done by giving both camera holders a common adjustment about the $\omega$-axis for lateral tilt, and by adjusting the whole frame by a handle, the amount of movement being indicated by the vertical scale seen at the extreme right of Fig. 132. The height scale is observed considerably magnified in the window seen inclined at the right of the plotting-table and just above the stool.

When observing, after the photographs have been set, the point of intersection of the epipolar arms is moved over the table by hand, while the height disc is foot operated, so that the floating mark is kept in contact with the ground. The plotting-pencil is seen in the photograph attached to the intersection of the space-rods, the mechanism being that of a pantograph, with reduction ratios between 1:1 and 1:5.

If the photographs have been previously observed in the A5, the process of setting in this instrument consists of setting the various movements to those obtained in the other instrument. Such a process enables the more expensive instrument to be used on control work, while one or more of the less expensive instruments can be used for plotting.

The instrument is very simple to use and should not easily get out of adjustment. No results are yet available of work done with the instrument, but there seems to be little doubt that it has a considerable field of use in the future, not only in conjunction with the A5, but also as an independent instrument for topographical small-scale mapping.

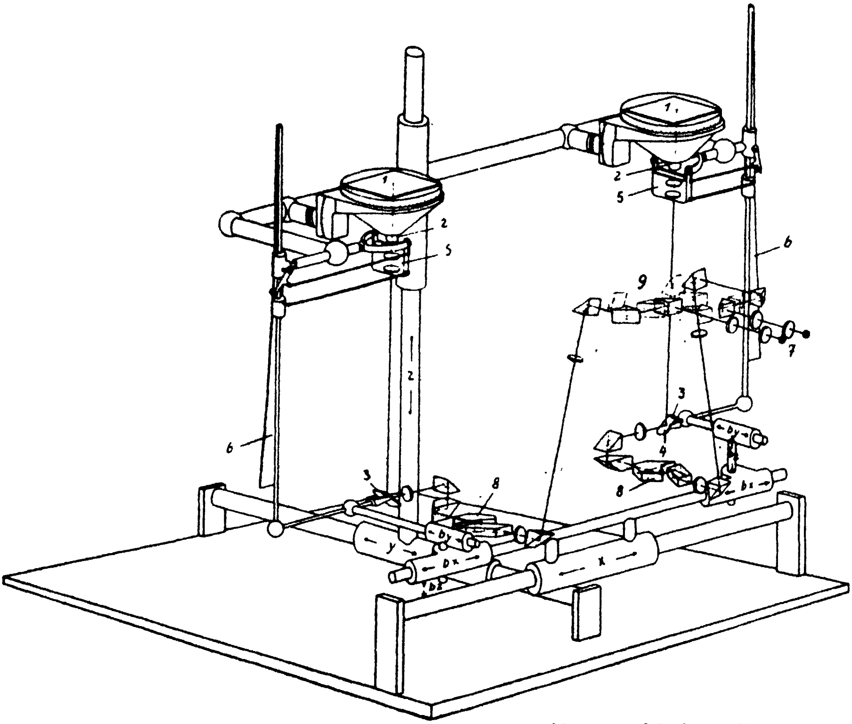
## ZEISS STEREOPLANIGRAPH

The Stereoplanigraph is a stereoscopic instrument fitted with a plotting apparatus, and is capable of providing a stereoscopic picture from pairs of photographs in all possible cases. It is a universal instrument which can deal with pairs of air photographs of any obliquity and also with terrestrial photographs as well as pairs of approximately vertical photographs.

An important feature of the instrument is that the projection is purely

optical, while the Porro-Koppe principle is rigidly employed. The schematic diagram in Fig. 135 serves to illustrate the general arrangement and conveys an idea of the relationships between the more important parts and of the various movements.

A photograph of the most recent model C5 is given in Fig. 136, while




[Courtesy of Carl Zeiss (London), Ltd.

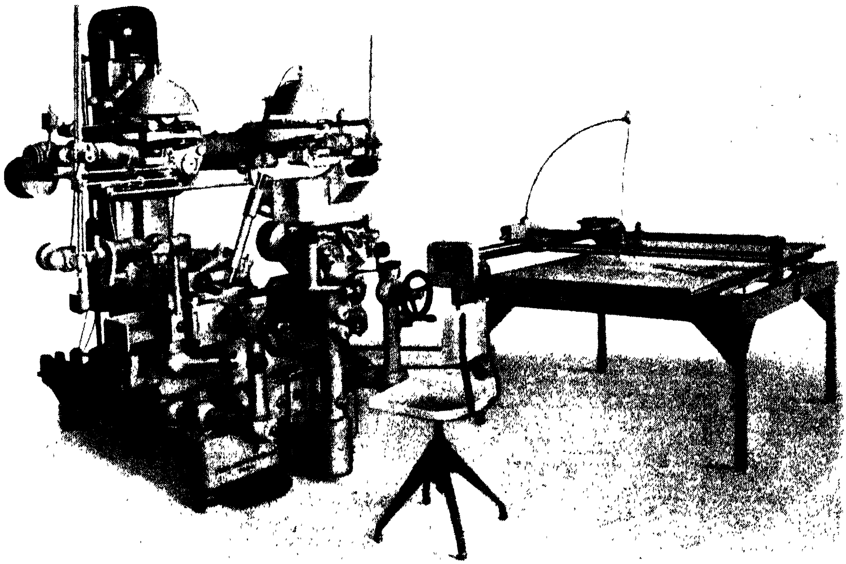
FIG. 135—DIAGRAM OF ARRANGEMENT OF ZEISS STEREOPLANIGRAPH.

the anaglyphic picture in Plate 1 will give some idea of the size of the instrument. [Coloured spectacles will be found in a pocket at the end of the book.]

When setting a pair of photographs in the instrument (either original film or plate negatives, or diapositives), each photograph is first set carefully in the picture carriers 1 (Fig. 135). To do this, the picture carriers are removed and mounted on a special table so that the collimating marks on the photographs may be set to correspond with those on the holder. By measurement of the distance between collimating marks, the amount of shrinkage of the film, for purposes of computation, may be allowed for by making a proportional adjustment to the focal length.

The picture-carriers are now replaced in the instrument and their images are projected downwards from a light source above through lenses 2, exactly similar to those used in the camera at exposure, on to mirrors 3, each having at its centre a measuring mark 4.

In their carriers, the photographs may be set to the same swing ($\kappa$),



[*Courtesy of Carl Zeiss (London), Ltd.*

FIG. 136—ZEISS STEREOPLANIGRAPH MODEL C.5. AND PLOTTING-TABLE.

longitudinal tilt ($\phi$), and lateral tilt ($\omega$) as those existing at the moment of exposure.

For each photograph, an auxiliary system 5, governed by a cam-rod 6, ensures that the image of that part of the photograph on the measuring-mark mirror is always sharp.

The projection cameras may be also simultaneously swung and tilted. Graduated circles are provided so that all angular movements can be measured.

The measuring marks 4 are engraved in the centre of the mirrors 3, from which rays are reflected into the observation stereoscope. In this manner the line of collimation between the image point and the measuring mark is represented by a ray of light. The images are then viewed simultaneously with both eyes (at 7) through a stereoscopic system composed

of several optical elements. When the photographs are correctly set the landscape appears in stereoscopic relief, while the fused measuring marks, now forming the floating mark, may be brought into contact with the ground model at a particular point by adjustment of the foot-height disc. By moving the floating mark over the model, the detail and contours can be plotted.

The mirrors 3 are each mounted on a cross slide, with motions by hand-wheel in three directions in space, $bx$, $by$, and $bz$, giving the space components with reference to the inclined plane of which the air-base defines the maximum slope. The setting in each case is read on a scale.

In order that the simultaneous spatial movements between the two measuring marks and the projecting cameras may be determined the measuring marks are moved by two hand-wheels in the plan directions, $x$ and $y$, of the instrument, while the projector cameras may be adjusted in the vertical direction $z$ by means of the foot disc. In the anaglyphic picture (Plate I), the observer is seen operating the $x$ and $y$ hand-wheels with his left and right hands respectively, and the height disc with his right foot. Scales are provided for the $x$ and $y$ movements and a counter for $z$ (height). The $x$ and $y$ movements are provided with reversing mechanism for convenience in working under certain conditions. Also the $x$ and $z$ movements can be interchanged for use when working with terrestrial photographs.

The height counter allows of any particular height to be set and this is used in absolute orientation of space models to ground control.

The relief model is seen through the eye-pieces 7, Fig. 135, at the same time as the floating mark. The eye-piece position remains fixed during observation, while the automatically governed erecting prisms 8 cause the image to appear stationary and upright at all times. Change-over prisms 9 permit of exchanging the paths of rays going to the eyes, so that when connecting up consecutive views, an orthoscopic model is always seen. This is useful when observing on a strip where the first photograph is removed and the third inserted. Also where diapositives are used, the eye-pieces 7 are changed for another standard pair having reversing prisms so that the orthoscopic image is again seen.

The $x$ and $y$ movements of the Stereoplanigraph are connected by coupling rods to a plotting-table, seen on the right in Fig. 136. When disconnected from the Stereoplanigraph, the plotting-table may be used for plotting co-ordinates mechanically, there being provided a geared transmission so that the scale may be suited to the plot. When the photographs in the Stereoplanigraph have been correctly set and the scale established, the plot is adjusted so that known points on the plot coincide with the

plotted positions of similar points on the photographs. The detail and contour plotting is then carried out by stereoscopic observation, while the plotting pencil on the table automatically records the positions where the floating mark is set to touch the ground model. In order that the plotting pencil shall be accurate a special accurate sharpening device is provided and it may be raised and lowered by a foot-switch operated by the observer.

Some of the special advantages claimed by Zeiss for the Stereoplanigraph are as follows.

Distortion errors of the camera objective are eliminated by using the Porro-Koppe principle. A projection lens similar to the camera lens is employed. It is considered that the distortions which arise from ultra wide-angle lenses are better dealt with optically than mechanically, by means of a guide-rod. The importance of this provision must be considered in relation to the amount of use of the ultra wide-angle lens. The Zeiss Orthometar Air Survey Lens has negligible distortion over its field of some 70°, and, for ordinary working, the necessity of the Porro-Koppe principle is not apparent. The designers of the Wild Autograph consider that for the accuracy required when plotting from ultra wide-angle photographs, it is sufficient to use a glass plate of variable thickness introducing a refraction equal and opposite to the lens distortion. It appears that the Porro-Koppe principle adhered to by Zeiss might give greater precision in this case, but against this must be considered the considerable economy of dispensing with photographic objectives in the plotting instrument.

A very real convenience of the Stereoplanigraph is the provision of optical means of interchanging the two images in front of the eyes, and applying all base components either "inside" or "outside" so that connection of consecutive views in aerial-triangulation can be easily and accurately effected.

Optical projection throughout is claimed to give maximum stereoscopic space impression and no adjustment of magnification is required over the area of the picture when the photographs are convergent.

Minor adjustments in focus of the objective may be carried out to allow for shrinkage of the film. This can be done over the small range required without spoiling the clarity of the image.

*Uses of the Stereoplanigraph.*

The range of uses quoted by Zeiss are: (*a*) Plotting from terrestrial photographs taken with a stereoscopic camera. (*b*) Plotting stereoscopic pairs taken with photo-theodolites, either without or with parallel axes.

(*c*) Plotting aerial views with the axes in any direction, in particular vertical views, low obliques, high obliques, and multi-lens camera views with the axes of the camera set to corresponding angles; single-picture pairs, and picture strips of any desired length, and in particular wide-angle views with complete utilization of the whole angular field. (*d*) Plotting single views of flat country by graphical rectification.

Views of unusual size or focal length can be plotted after enlargement or reduction. In the case of ultra wide-angle photographs, a special reducing camera is provided to reduce the 12 × 12 inch pictures to 6 × 6 inches, i.e. apparent focal length is reduced from 20 to 10 cms.

The results of measurement may be given graphically in the form of plan views, with contour lines and detail at any required scale, or in the form of aerial co-ordinates of required points. Both may be employed at the same time, although it is now becoming more usual to employ the Stereoplanigraph mainly for the determination of the accurate positions of control points.

The instrument may be used for plotting at any scale likely to be required in practice.

*Setting Photographs in the Instrument.*

The process of setting a pair of photographs in relative orientation follows a similar routine to that used with the Wild Autograph, and it is beyond the scope of this book to compare the technique. Also the process of absolute orientation to ground control is somewhat similar.

The complete orientation of a vertical pair of photographs takes a practised observer between twenty and thirty minutes.

After the first pair of photographs has been set in correspondence and the required observations made for aerial triangulation or detail plotting, the first photograph is removed and the third inserted in its place. It is here that great convenience is found in the $x$ and $y$ reversing mechanisms. Relative orientation is now made with reference to the third photograph only and if the first pair has been set to a horizontal reference plane, this absolute orientation may be continued along the strip. At the end of the strip adjustment will be necessary for heights.

It may be added that the instrument is a fine example of the instrument-maker's craft. Considering its weight, it is remarkable that the various movements are sensitive to a finger touch, and the accuracy of observation and plotting has to be seen and tested to be really appreciated.

Instruments of the latest type are in operation in many parts of the world.

Accurate plotting instruments are also produced by other makers; those described above do not represent the complete range of instruments available.

## Optical Epipolar Plane Plotters

The mechanical plotters previously described have all depended upon direct stereoscopic observation of the photographs; the plotting mechanism being connected in some manner to the movements of the floating mark.

The class of plotters which are termed "optical plotters," are those in which the common overlap of photographs is viewed when projected on to a screen. The essential condition is that the photographs should be set in their correct relationship as at exposure, so that by means of a source of light behind the negative or transparent positive (diapositive), the pictures, projected through a converging lens system, are superimposed on a screen. When the photographs are set in correspondence all pairs of corresponding points will be found in their epipolar plane, and if the table upon which the pictures are seen can be given a vertical movement, the two images of any point may be brought to coincidence, thus defining the point in space at the scale of the reconstruction.

Optical plotters operate on either the blink or anaglyphic process.

*Blink Projection.*

If two overlapping photographs are set in correspondence and projected as above, then if the height of the screen of projection is not the same as the height in space of the point of detail, the two separate images which are seen will be in the epipolar plane of the point, and the displacement between them will be parallel to the air-base. When the two pictures are projected rapidly and alternately, there will appear a flicker or "blink" between the two images of the point, parallel to the air-base, and this can be eliminated by moving the projector-table up or down until the two images coincide and the flicker is eliminated. The point of detail may then be plotted vertically below this point in plan and its height measured on a scale.

If an overlapping pair of photographs is set approximately in correspondence, and rapidly projected alternately, the blink will not now be necessarily parallel to the air-base, and the direction of this movement will give an indication of the "want of correspondence." Hence the photographs are easily set by adjusting them until the blink movement is parallel to the base-line.

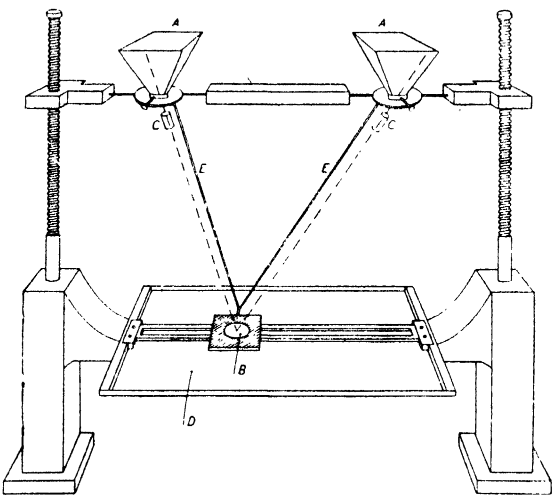
Two examples of plotting-machines employing this principle are the Nistri Photocartograph and the Gallus-Ferber Stereocartograph, both of

which have been used extensively on the Continent and elsewhere. Zeiss have also produced a Blink Comparator but this does not appear to have been used to any extent for plotting air surveys, though it has proved valuable in detection of bank-note forgeries, and also used in astronomical observations for measuring star displacements.

The monocular method of setting and plotting, although very simple, has the disadvantage that neither the stereoscopic nor the pseudoscopic impression can be seen during the plotting process, which may make interpretation rather difficult, although independent stereoscopic observation of the pairs of photographs is, of course, always possible.

*Gallus-Ferber Stereocartograph.*

A diagram of this instrument, an example of the group of instruments employing the "blink" principle, is shown in Fig. 137. The movements of which the instrument is capable correspond to those advocated by Fourcade. Each goniometer is free to move, while the horizontal bar, corresponding to the air-base, can be raised or lowered at either side by rotation of the screw-threaded uprights operated by an electric motor. The screen of projection B can be moved along a slider in the $x$ direction, while the slider itself can be moved in the $y$ direction. The bar is adjusted for a particular point in conjunction with the projection-table, until the "blink" of image of the point is eliminated.




[*Reproduced from the Report of the Air Survey Committee No. 2, 1935, by permission of the Controller of H.M. Stationery Office.*

FIG. 137—GALLUS-FERBER STEREOCARTOGRAPH.

It has been mentioned above that a very convenient feature of instruments working on this principle is that any "want of correspondence" can be detected when the "blink" of the images of a point is not parallel to the plan of the air-base.

Instruments employing the "blink" process are very similar in construction and operation to those employing anaglyphic projection.

*Anaglyphs.*

When a pair of eyes views a landscape, each point is automatically set in its epipolar plane, and the intersection of the rays in this plane from the left and right eyes fixes the point in the impression of space conceived by the brain. If a scale-model of the landscape is reconstructed, the eyes will receive the same impression of depth, the only difference being that the actual distance from the eyes is different.

The anaglyph provides a method whereby, by making use of the optical properties of complementary colours, this impression of the third dimension can be given to the human eye. If a picture is printed in one colour and viewed through spectacles which have lenses of the same colour, then no impression of the picture will reach the eye. For instance, if a picture is shown in a particular shade of red and viewed through a filter of the same shade of red the eye will receive no impression of the picture. Similarly no picture is seen when a blue picture is viewed through blue spectacles.

An anaglyph is formed when an overlapping pair of photographs set in correspondence are superimposed, with the two pictures in complementary colours. The result is as shown in Plates II, III and IV, where the left-hand picture is projected through a red filter and the right-hand one through a blue one. The separation of the red and blue images of a particular point is a measure of the difference of parallax.

To obtain the space impression in the form of the space-model of the overlap, the picture is observed through a pair of spectacles with a red lens before the left eye and a blue one before the right eye, the effect being that the view in the corresponding colour is cut out. The result of this is that left and right rays will appear to intersect at a definite point in space, and over the whole area of the overlap, a three-dimensional optical model is seen. By using the coloured spectacles provided at the back of the book the plates may be seen in stereoscopic relief.

The scale of Plate II is approximately 1/20,000, and the size of details which can be seen is easily appreciated. There are also three pairs of measuring marks which fuse in space to form floating marks and of these the point of one is above, one is just touching and the other is below the ground.

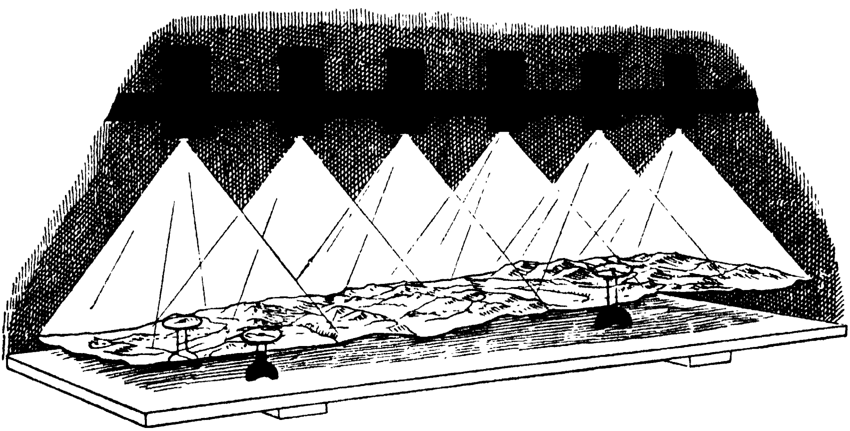
The scale of Plate III is approximately 1/2,200 and that of Plate IV approximately 1/4,400. In the three cases, it is possible to compare the amount and type of detail which may be appreciated and interpreted at different scales.

When these pictures are projected on to a screen from a light source,

the sighting must be done in darkness, and in order to fix the position of any point in space a special apparatus carrying a point spot of light is moved over the "model," the plotting-pencil being underneath, while the height can be measured by reference to a vertical scale. Such is the essential principle of the Zeiss Multiplex Aeroprojector.

*The Zeiss Multiplex Aeroprojector.**

This instrument employs the principle of anaglyphic projection, and is used for the plotting of medium and small-scale maps, when the more elaborate stereoplanigraph made by the same firm is unnecessarily precise. It may be used either as an independent instrument or for filling in detail



[*Courtesy of Carl Zeiss (London), Ltd.*

FIG. 138—DIAGRAM OF ZEISS MULTIPLEX AEROPROJECTOR.

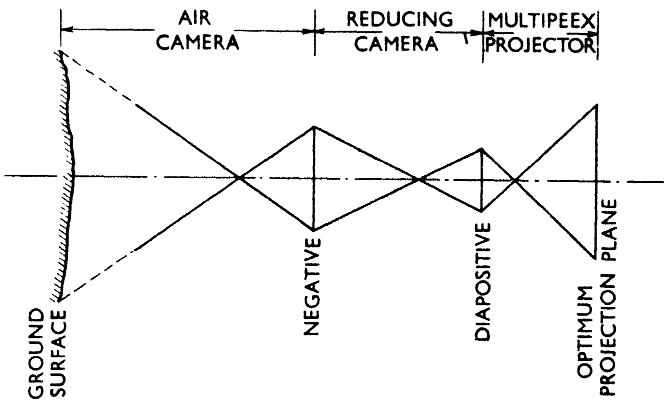
and for contouring only when the stereoplanigraph is used as a control instrument.

A diagram of the instrument is shown in Fig. 138. Several projectors are mounted along a horizontal bar and the pictures are projected from diapositives, alternately in red and blue, to form the space-model of the landscape as indicated, when the projection is viewed through the spectacles having one red and one blue lens.

One difficulty which has had to be overcome with this method of projection is that, in order to prevent the instrument from becoming very large, the photograph as taken must be reduced in size to form a diaposi-

*** Improved models of Williamson-Ross in U.K. and Bausch and Lomb in U.S.A. are now generally and extensively used.**

tive. There are, therefore, two stages in the work before the picture can be projected on to a screen, namely, the photography, and the reduction. If accurate results are to be obtained then accuracy must be maintained at all stages, and it must be possible to enlarge the diapositives considerably in projection. This has been made possible in recent years by improvements in the production of fine-grained photographic emulsions such as "Aerodiaplatte" made by the firm of Perutz. Fig. 139 shows how the plane of projection is related to the original ground surface. [54]



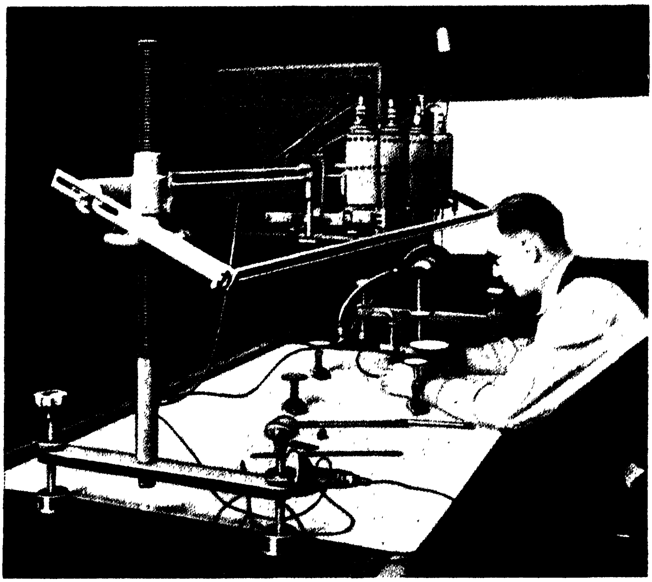

[*Courtesy of American Society of Civil Engineers.*

FIG. 139—INTER-RELATION OF VARIOUS MULTIPLEX MAPPING INSTRUMENTS.

The projectors are set in $x$, $y$ and $z$ directions as in other instruments with respect to a horizontal bar which represents the air-base. For mutual orientation, each projector is provided with the swing, cant and differential tilt motions which are provided for the mechanical plotters. In order to allow of absolute orientation without disturbing the relative setting of the components of the optical model, the main bar itself is provided with adjusting screws so that any inclination of the air-base can be set with respect to the drawing-tables. If, first of all, two photographs are set in mutual orientation, but to some unknown scale, others can be set in the same relationship by orienting a third photograph to the second without disturbing the second one; a fourth without disturbing the third, and so on up to a maximum of nine projectors. The result is then a space-model of a strip of nine photographs unknown in scale and orientation. An illustration of one of these instruments given by Higginson [54] with four projectors is shown on Fig. 140. The foot-screws for orientation of the space-model can be clearly seen.

*Preparation of Diapositives.* If the original size of photograph is used, the size of the instrument must be large, and in order to reduce this, diapositives are produced, having a picture size of 4 × 4 cms. (1½ × 1½ ins.). This is the size of photograph which would have been taken if the camera had a lens of the same focal length as that of the projector, 4·6 cms.

*Control Stands.* These are used to effect the absolute orientation of the space-model. Three of them can be seen on the drawing-paper in Fig. 140. They are small circular tables with a central dot mark set at the centre of a



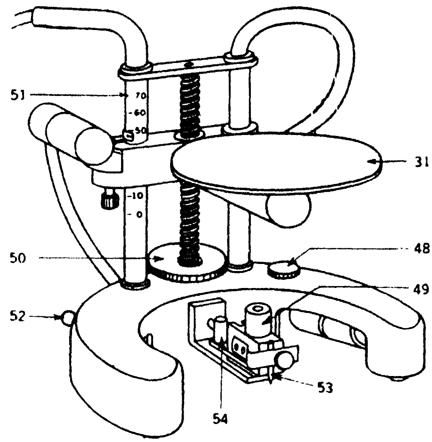
[*Courtesy of American Society of Civil Engineers.*

FIG. 140—MULTIPLEX AEROPROJECTOR* IN OPERATION.

small circle marked on the table to facilitate location of the central mark. The table may be adjusted vertically, and vertically below the dot is a needle which can be centred accurately over a control point plotted on the master plan. When the table is set to correspond with the known height of the control point which has been set in position on the plot, the process of orientation involves making the point of detail, as depicted by the photographs, agree with the control point in space.

*Tracing Table.* It is necessary that the plotting-table should be provided with vertical adjustment, so that the precise position of a point can be established in space. For this purpose a projector-table is provided. In

Fig. 140 the projector-table is seen just in front of the operator and between his hands. A sketch of one is shown in Fig. 141. The top of the table marked 31 is circular and is centred exactly over the point of a plotting pencil marked 53. The centre of the table provides the position for two alternative floating marks. One is a spot at the centre of the table which can be illuminated with different degrees of brightness by varying an electrical resistance. The other is a black arrow, with its point at the centre of the table which can be turned in order to be set in a favourable position relative to the ground detail. In order that the two corresponding projected images of the point may be set in a known position, the table is adjustable vertically, while the whole instrument can be moved over the plotting base by sliding the horseshoe base of the table on its three points of support. In this manner several complicated mechanisms such as those provided in the Stereoplanigraph to give similar motions are avoided. When the floating mark has been brought to the position of the point of detail in space, the plotting-pencil records its position in plan, while the height is read off on the vertical scale.




[*Courtesy of Carl Zeiss (London), Ltd.*

FIG. 141—TRACING-TABLE OF MULTIPLEX AEROPROJECTOR.

*Setting the Diapositives.* The setting process for the Multiplex is similar to that for other plotting-machines, and may also be divided into three stages. First the diapositives are set in the projectors with their principal points on the principal axis of the projector. The illumination is switched on and, without using the coloured filters, the two pictures are projected on to the table, and, by inspection, the pair of pictures is set approximately into correct mutual orientation. By reference to ground control the scale is made approximately that which is required. The coloured filters are now inserted and the space-model viewed through spectacles with glass of complementary colours. It will probably be possible to obtain the impression of stereoscopic relief at this stage, but a further adjustment of orientation may be necessary. By eliminating want of correspondence the two photographs are given their mutual orientation. When a third photograph

is added, only this one must be adjusted and thus it is possible to build up the space-model formed by nine or even more consecutive, overlapping photographs. The scale and orientation of this space-model is however as yet unknown.

It remains to relate the space-model to the ground control. The projector carrier can be adjusted for longitudinal and lateral inclination by the foot-screws which carry it. Three control points, two widely spaced laterally at one end of the strip, and the third at the other end, are plotted accurately and control tables centred accurately over them. This plot is placed under the projectors and moved about to obtain agreement with the projected images of the points. This is unlikely to be obtained at first, and the scales are made to agree by altering the length of air-base as represented by the spacing of two projectors. This does not interfere with the correspondence setting of the photographs.

No correction has yet been made for the slope of the air-base. The three control tables are now set to the known heights of the control points by making a vertical adjustment to the tables. The foot-screws carrying the main bar are adjusted, until the floating mark appears at ground level at the known height of each control point. The process of trial and error to effect this absolute orientation, necessitates adjustment of the air-base and its inclination, and the result should be a true-to-scale model of the overlap set on a horizontal plane. Before commencing plotting detail and contours it is advisable to check the correspondence setting and make any further final adjustments.

Contours are plotted by setting the tracing-table at a particular height, and moving it over the area so that the illuminated or black dot keeps always in contact with the ground. For detail plotting where height varies, the horizontal and vertical adjustments of the tracing-table are simple, and the pencil can be lowered into contact with the plan whenever required. When, as is frequently the case, the model is translated into the horizontal as soon as possible, it is usual to have three known points in the first overlap. For two of these points the aerial co-ordinates must be known, and for the third its height.

*Scope of the Multiplex Aeroprojector.* The instrument in its usual form is designed for plotting from photographs within 10° of the vertical, although projectors may be obtained for obliques. The Wide-angle Multiplex has been produced for plotting from photographs taken with the new ultra wide-angle Topogon lens. This has involved the provision of lenses covering the same angle in the projectors. The method of use is, however, the same as for the ordinary instrument.

There is a possibility that although the accuracy of point location is as great as 0·1 to 0·2 mm., the drawing-pencil which is moved by hand may be as much as 0·4 mm. out of position. As a result the technique of plotting with the multiplex has had to be somewhat modified to ensure that the final maximum error is within the allowed limit. It is customary to plot at a scale two to five times greater than that required, and to reduce the plot afterwards. This procedure must obviously considerably increase photographic costs if the mapping scale is the largest at which final mapping is likely. It may be assumed in almost all cases, however, that there is a good chance that, when there is development, a larger scale will be wanted and the extra cost of photography accordingly justified.

Table X.1 is given by Bricklacher[9] in connection with this instrument fitted with lenses of 70° field (the normal type).

TABLE X.1

| *Flying height ft.* *H.* | *Base length ft.* *B.* | *Scale of plotting.* | *Scale of final map.* |
|---|---|---|---|
| 4,900–8,200 | 1,640–2,790 | 1/5,000 | 1/20,000 |
| 8,200–16,400 | 2,790–5,570 | 1/10,000 | 1/25,000 |
| 16,400–21,400 | 5,570–11,150 | 1/20,000 | 1/50,000 |
| | | 1/25,000 | 1/100,000 |

When the special wide-angle instrument is employed it would appear that plotting can be carried out at 1/35,000 to 1/40,000, so that the reduction to a scale of 1/150,000 or 1/200,000 would not make the initial photographic costs too expensive.

The problems of, and suitable methods for, small-scale mapping are discussed in Chapter XI.

Bricklacher states that the instrument requires ground control of the order of secondary triangulation. He gives some results obtained over a length of some five miles between two control points for a scale of 1/7,500. The maximum errors of horizontal co-ordinates were of the order of 4 metres, while heights agreed to 1·5 metres.

Similar results were obtained from other experiments, and the instrument is being used extensively in many parts of the world, particularly on the Continent and in the United States. In the latter, for example, it is being used extensively by the various departments of the Federal Government. The United States Geological Survey has (1938) fifteen multiplex units engaged on mapping the Tennessee Valley and the Corps of Engineers United States Army has probably an equal number scattered over

the country. Here the scales required are particularly suitable for plotting with the Multiplex.

Mr. E. Haquinius, Chief of Surveys, Brazos River, Texas, (where the Multiplex is also used) remarks that the complete cost of mapping at 1/12,000 is $44 per square mile, which he estimates is between one-third and one-fifth the cost by normal ground methods, and he says: "It is safe to say that the mapping of property by this method on a scale of 1/12,000 is just as accurate as if the surveys were made by transit and tape and plotted to the same scale."

Mr. W. S. Higginson, of the United States Geological Survey, mentions that the first surveys plotted with the Multiplex were checked with "third-order transit traverse profiles" which were compared with corresponding profiles taken from Multiplex drawings. The results were favourable to the Multiplex method, as regards accuracy of both elevation and horizontal positions.

### Present Tendencies in the Employment of Plotting-Machines

Probably the greatest contribution towards the development of efficient practical methods of using plotting-machines has been made during the last few years by Professor Schermerhorn of the University of Delft in Holland. Professor Schermerhorn makes use of both Zeiss and Wild instruments.

In his technique, instruments of the Stereoplanigraph and Autograph types are used solely for control surveys, i.e., in determining the aerial co-ordinates of required points, and detail is subsequently plotted from the photographs in instruments such as the Multiplex or Autograph A6. These are quite adequate for plotting on scales of 1/5,000 or smaller, and for contouring, and since they are much cheaper than the larger ones, more of them may be kept in service, and delays do not occur at the large instrument, which will undoubtedly be the case if control is determined and detail plotted with the same instrument. The large instruments must, however, be used for plotting at large scales where levels are required.

Professor Schermerhorn employs the method of absolute orientation of the first pair of photographs to three ground control points and then works along the strip to a single point at the end, with occasionally an intermediate point as a check. The strip is generally set in the instrument both forward and backward so that the adjustments may be more reliable.

One interesting experiment made was by starting from three separate places where there were three ground control points, and flying three strips 100, 120 and 125 kilometres in length respectively, on to a point Z. After

applying corrections independently to the position of Z by the known theory of distortions, the maximum closing errors were ±30 metres in plan and ±6 metres in elevation.

Table X.2 resulting from another test also gives some idea of the accuracy which Professor Schermerhorn is obtaining. The details of three strips are shown as observed with a stereoplanigraph.

TABLE X.2

| Strip No. | 22 | 21 | 59 |
|---|---|---|---|
| No. of pictures | 845 | 500 | 840 |
| Error in length (before adjustment) parts per 1,000 | 3·2 | 12·1 | 2·8 |
| Error in length (after adjustment) | 1·7 | 1·2 | 0·7 |
| Error in azimuth (before adjustment) (centesimal minutes) | 17 | 34 | 20 |
| Error in azimuth (after adjustment) | 12 | 23 | 2 |
| Average length of strip (km.) | 40 | 135 | 95 |

It is found that about fifteen pictures may be bridged without control by manual flying without statoscope readings, but that this can be extended to some fifty pictures over a distance up to 120–150 kilometres when a statoscope is employed.

Professor Schermerhorn has carried out a large amount of work and in particular the survey of large areas for the Dutch-Shell Oil Company. He has also secured contracts for surveys in the British Empire.

While it is impossible here to go into the matter more fully, it does appear to be increasingly probable that the use of some such technique is desirable when mapping countries of the colonial type, since it is possible to obtain contoured maps with such a small amount of ground control. It seems likely in the future that stereoscopic plotting instruments will be increasingly used. In some cases doubts have been cast upon the accuracy of these instruments, but when one has once used instruments of the type suitable for control surveys, it becomes obvious that the extreme degree of exactitude obtainable could only be reached by precise construction of a very high order. In both the Stereoplanigraph and the Autograph, once the model is set, aerial co-ordinates of additional points may be read off on the appropriate scales, subject to certain small adjustments in some cases of the traverse type of correction. In this way a very considerable amount of computation is saved.

No doubt results of a similar standard will be obtained with the Thompson Plotter.

For surveys of less importance, instruments of the Multiplex and

Autograph A6 type may be used independently, and very satisfactory results can be obtained for topographical medium-scale mapping with suitable control.

Those who use these instruments do not appear to have made much use of the automatic pilot, although the statoscope is now a standard equipment for work when ground control points are few. When an automatic pilot is used with tilts reduced to $\frac{1}{4}°$ or so, the scope of the simple radial method is much enlarged, and the necessity for the more elaborate instruments not so apparent. Naturally the instrument makers are not concerned with the perfection of an automatic pilot which might reduce the scope of employment of their plotter.

No doubt both basic methods have a very considerable range of application and recently Continental and British methods have tended to approach more closely.

The relative development of the respective schools must be left to the future, but it is important that there should be some institute in Britain or in the British Empire where stereoscopic plotting instruments are set up so that the wider applications of plotting from photographs may be made. This would, for instance, apply to colonial surveys, where it will not always be possible to ensure accurate photography with an automatic pilot.

## CHAPTER XI

# SMALL-SCALE MAPPING FROM AIR PHOTOGRAPHS

## RANGE AND REQUIREMENTS OF SMALL-SCALE MAPPING

IT now remains to be seen how the methods that have been previously described can be adapted to small-scale mapping, and where other methods are applicable or desirable. For the purpose of this discussion the upper limit of small-scale mapping is taken as 1/30,000. The range to be considered is, therefore, from this scale to one at about 4 miles to 1 inch (1/253,440), the smallest scale upon which direct mapping is likely.

The technique of small-scale mapping primarily concerns the mapping surveyor, since the engineer or other economic user who wishes to take direct measurements will rarely concern himself with anything less than 1/25,000. There are, of course, instances, such as the work of the British Geological Survey, where the 1 inch to 1 mile map is used as a basis for the geological detail. It may, however, be assumed that in almost all cases of small-scale mapping, the economic user will take the maps prepared by the professional surveyor.

Air survey first became a practical and economic possibility for small-scale mapping, and although improvements in every direction have greatly widened its scope, it has been thought desirable to describe briefly the methods and technique applicable to small-scale mapping.

In many cases small-scale surveys involve the delineation of areas where there is little man-made detail, and one is primarily concerned with depicting the general configuration of the ground. In other cases complete direct small-scale mapping is unlikely to be required, because, if there is a likelihood of future development, maps on larger scales than, say, four miles to the inch will become necessary. Consequently, in order that one set of photographs will suffice, the surveyor will have to base his work upon the requirements of the largest scale likely to be plotted in the future.

For economy, each photograph should cover the largest possible area, consistent with fulfilment of photographic possibilities and requirements of accuracy. For the simple methods, such as the Arundel Method, many prefer to plot approximately at the scale of the map, so that when mapping at the military scale of 1/25,000, the plotting is done at a convenient figure

in close agreement with the mean photographic scale, but which may differ somewhat from 1/25,000. This facilitates the detail plotting, and the plot is adjusted for scale during photographic reproduction. Improvements during the last few years have, however, enabled the ratio of enlargement of photographs to be increased by the provision of very fine-grained emulsions, so that many operators prefer to photograph at a smaller scale than that of the map and enlarge by two diameters or more in the printing process. As long as such photographs can ensure the desired accuracy, then there is a definite economy in this procedure. On the other hand, when using the Zeiss Multiplex Aeroprojector, the final scale is usually smaller than the plotting scale in order to reduce possible errors in plotting which arise when this method is employed.

Until quite recently, the smallest practical scale of vertical photography was some 1/30,000, and therefore a map produced by the Arundel Method at this scale with a total size, say, ten feet square, would be about one foot square when reduced to 1/250,000. The labour of plotting and amount of ground control to be fixed under such conditions is excessive unless the preparation of a larger scale map is also contemplated. In this case suitable reduction and elimination of unnecessary detail for the smaller scale could be made with reference to the previous plotting.

Flying height is influenced by practical conditions. It is possible to photograph from heights of 20,000 feet or so; and although some photography has been done in the United States at over 30,000 feet, such heights are not suitable for practical mapping.

Working at a height of as much as 20,000 feet generally involves special design of aircraft and precautions in photography. Also the extreme cold and reduced atmospheric pressure make it very difficult for the photographer. Hence in this country a height of 15,000 feet is rarely exceeded, and heights of the order of 12,000 feet preferred.

If 15,000 feet is taken as the maximum flying height, and the smallest standard photograph 5 × 5 inches, taken with a lens of 5 inches focal length, then this scale $= \dfrac{f}{H} = \dfrac{5}{15{,}000 \times 12} = \dfrac{1}{36{,}000}$.

Vertical photographs on a smaller scale than this were not practicable until recently.

As a result of the great expense involved in mapping on small scales from vertical photographs taken on comparatively large scales, much work was done at first from oblique photographs. The inclination of the camera axis enables a much greater area to be covered with one exposure, and although the advantage of approximate uniform scale is sacrificed,

it is possible, by a comparatively simple perspective method, to produce reliable plans upon very small scales. Most of the pioneer work in this field was done by the Canadian authorities in the North, and some by the United States authorities in Alaska. Many thousands of square miles have been mapped in this way, with little ground control.

During the last few years, the oblique method has been improved so that it is applicable to ground of appreciable relief, and methods of levelling from obliques have been developed in India and elsewhere.

The rapid advance in the design of wide-angle and multi-lens cameras during the last year or two has tended to limit the scope of oblique photography, by making it possible to produce, either directly or after rectification, vertical photographs on very much smaller scales than was formerly possible. When it is suitable and economical, vertical photography at approximately the scale of mapping will generally be chosen in preference to oblique photography.

As a result of these developments, mapping from oblique photographs is now mainly applicable to very small scales. Obliques are, however, often of great value in conjunction with verticals for fixing distant control points, for plan and level.

Methods of photography for small-scale mapping can now be grouped as follows:

(*a*) Ultra wide-angle lens photographs—for the upper range.
(*b*) Multi-lens photographs—for the medium range,
(*c*) Oblique photographs—for the very small scales.

It should be remembered that where the small-scale map in hand is *not* the upper limit, it is often better to use the larger scale verticals or multi-lens photographs and reduce, instead of employing the rather more limited oblique method.

## Small-Scale Mapping from Vertical Photographs

Ultra wide-angle lenses, giving accuracy over an angular field of approximately 95° or so, have recently been introduced and are described in Chapter IV. Previously, the maximum field for an accurate wide-angle lens was some 70°. The advance is therefore considerable, and it becomes possible to take vertical photographs at much smaller scales than before, without exceeding convenient flying heights. The distortions are so small that no plottable inaccuracy is introduced. In addition, these lenses are remarkable for the uniformity of illumination which is produced over the whole of the field and the clarity of definition, even in the corners. Photographs taken at a scale of 1/40,000 with the new Ross Ultra Wide-angle

lens (see Chapter IV) have been used successfully for an experimental plot of a twenty-mile strip, by means of a Thompson Comparator, with only two ground control points.

Since the scale of a photograph is given by the ratio $f/H$, it follows that at a particular height the scale is inversely proportional to the focal length. At a flying height of 15,000 feet the scale of photography with a 7-inch lens is 1/25,700 while that with a 4-inch lens is 1/45,000. This reduction of scale is no advantage if the angle of the lens remains the same, because the area of photograph will be smaller for the 4-inch lens

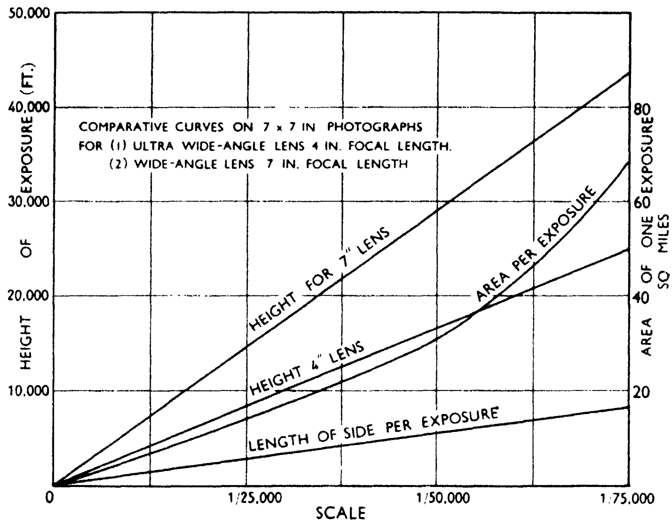



FIG. 142.

and there will be no economy of photography. For photographs 7 × 7 inches the above focal lengths represent respectively the old wide-angle lens of 70° field and the ultra wide-angle lens with 95° field.

It is therefore now possible to photograph directly on scales as small as 1/75,000, by using cameras fitted with these new lenses. At such scales, however, the photographic height, although quite easily attained, presents a problem owing to weather and temperature difficulties, so that about 1/60,000 is the convenient practical limit. In Fig. 142 may be seen the relationship between flying height and area photographed, for 4-inch and 7-inch lenses respectively on a 7 × 7 inch photograph.

When the area of each photograph is increased, the spacing of ground control for plotting by the Arundel Method can also be increased since the length of air-base is greater. A plan on a scale of 1 inch to 1 mile can

be produced, by the ordinary graphical Arundel Method, from photographs at approximately the correct scale, with ground control points spaced at a distance of some fifteen to twenty miles. Contours are usually required on these small scales and therefore spot heights must be determined at the rate of about one per square mile.

The method of working introduced by Thompson, for his Comparator, appears likely to make it possible to produce accurate maps at a scale of 1/40,000 from control points spaced at intervals of about twenty miles. When a comparator is used on photographs at these scales, it is hoped that the necessity for a tertiary system in the new Ordnance Survey triangulation will disappear. Additional ground control in the form of spot heights, related to detail identifiable in the photographs, must however be provided if accurate contours are required.

Obviously, wherever possible, air photography will be carried out with single-lens in preference to multi-lens cameras. Hence the field of the latter, which appeared wide a year or two ago, has been much reduced by the introduction of these ultra wide-angle lenses.

### SMALL-SCALE MAPPING FROM MULTI-LENS PHOTOGRAPHS

In the early days of surveying from air photographs obliques were the only economic way of mapping on small scales, because of the limitation of scale of vertical photography, and without increase of angular field no real improvement was possible. Cameras of various types have been developed, all with the idea of giving the same result as vertical photography with a single lens of very wide angular field, much wider, in fact, than can actually be obtained with a single lens. The basic idea is to have a central vertical of small size, taken with an ordinary lens of very short focal length; and, in addition, to take a group of obliques of adjoining areas with other lenses grouped round the central one. Each oblique lens axis has a fixed angular relationship to that of the central lens, and by a process of rectification, an equivalent vertical can be produced.

One of the first of these multi-lens cameras was the nine-lens camera designed by Aeschenbrenner in Munich, together with a rectifier which covered a field of 135°. This was considered in Britain to be too much on account of the difficulties of light transmission when the angle of obliquity of the rays is greater than 60°. Hence the seven-lens camera was designed on the basis of what was considered to be the maximum practicable angular field of 120°.

Several multiple cameras and multi-lens cameras have been developed in Britain, the United States and on the Continent, and it is proposed here to mention two only which cover the useful field of this type of camera:

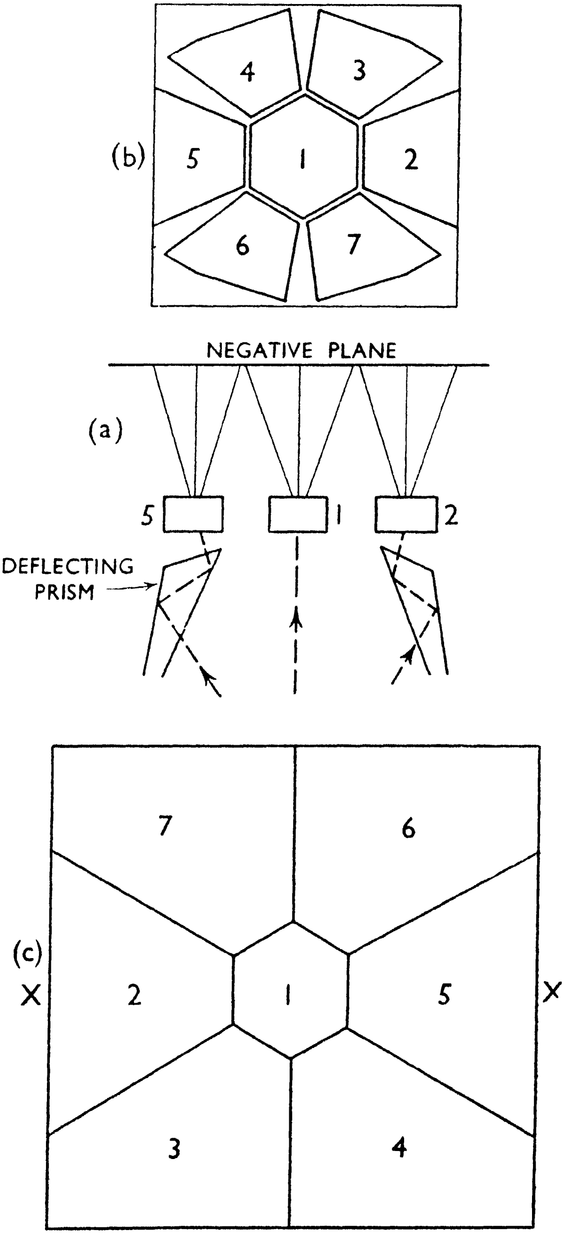


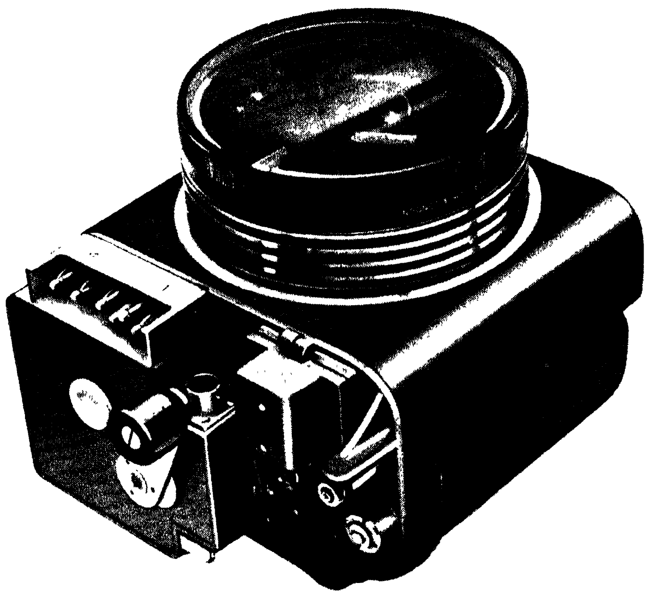

FIG. 143

(i) the British seven-lens camera of Barr and Stroud, and (ii) the nine-lens camera of the United States Coast and Geodetic Survey.

*Barr and Stroud Seven-Lens Camera.*

This camera and its application have been described by Thompson in the *Royal Engineers' Journal* and the *Empire Survey Review*. [85] It was designed as a composite unit because of the clumsiness and weight of multiple-lens cameras. Six lenses are mounted in the form of a regular hexagon and are grouped symmetrically round a central lens, the axis of which is approximately vertical at exposure. The axes of the outer lenses are all parallel to that of the central one, and below each is a deflecting prism, so that each side photograph becomes an oblique with boundaries of its area along the appropriate edge of the central photo-

graph and the adjacent obliques. It is held that the insertion of prisms resulting in deflection inwards across the central photograph makes the camera adjustment more stable. The general principle is shown in Fig. 143. The camera is arranged so that a rectangular area of ground (see Fig. 143c) is ultimately recorded as it would be by a lens of angular field of 120°. The area covered by each lens is seen in Fig. 143c. Although each lens is similar, the area covered by the outer lenses is much greater



[*Courtesy of the Institution of Royal Engineers.*

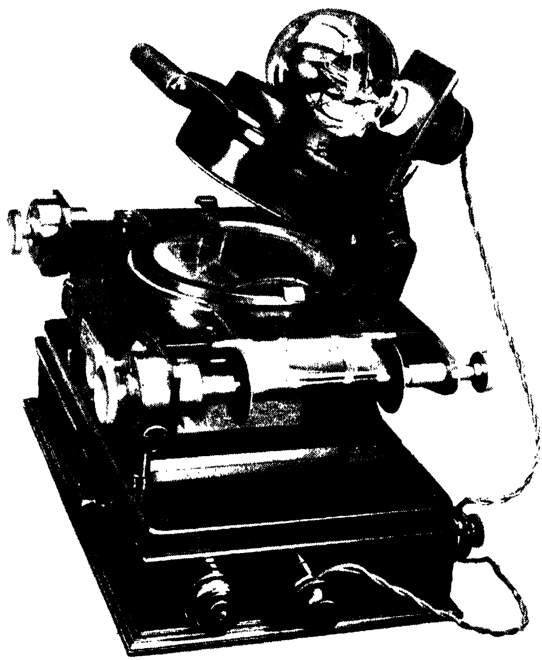
FIG. 144—BARR AND STROUD SEVEN-LENS CAMERA.

owing to the obliquity of the rays. Fig. 143a, representing a section along XX′, shows how the photographic images are formed of photographs 5, 1, 2.

All seven photographs are taken on one film, which is rolled round in the ordinary way, internal divisions being made in the camera body to keep each image distinct. The grouping of the photographs shown on Fig. 143b is so arranged that, by placing the negative in a rectifying printer, the result is an equivalent vertical. The rectifier has been very carefully designed so as to reproduce, as far as possible, the original conditions of Fig. 143c. Fig. 144 is a photograph of the camera. The camera

takes the photographs on an area of 5 inches square, the focal length of each lens being 1·7 inches. The shutter is of the focal-plane type.

The rectifier, Fig. 145, has been designed in accordance with the general principles laid down in Chapter VIII, the process being such that the vertical and six obliques are made into the equivalent vertical without distortion arising from the system of projection. The central vertical is



[*Courtesy of the Institution of Royal Engineers.*

FIG. 145—RECTIFYING PRINTER FOR BARR AND STROUD SEVEN-LENS CAMERA.

enlarged about one and a quarter times during rectification.

The problems which arise where there is rectification have also been mentioned in Chapter IX.

Thompson describes a test made with the camera from a height of about 12,000 feet over Salisbury Plain. The rectified prints gave a scale of approximately 1/60,000 and it was decided to prepare a map at a scale of 1 inch to 1 mile (1/63,360). The strip of five photographs had a length of twelve miles and was eight miles wide. It is interesting to note that sixty photographs would have been required when using a single 6-inch

lens from the same height. Ground control consisted of two triangulation stations and in order to check the plotting, some fifty points were chosen on the ground and their co-ordinates measured from the existing 1/25,000 sheet. These were compared with the co-ordinates of the points as determined from the plot by the Arundel Method. The average error in co-ordinates was 45 metres. On comparison with the existing 1-inch map the co-ordinates from the 1/25,000 map differed, on the average, by 23 metres, which is equivalent to a plan distance of 0·3 mm. Tests of a similar nature giving results of the same order were made of four other strips.

Contouring was also successfully carried out. When using the simple parallax method it is necessary to have six spot heights per overlap, so that in this case one point per five square miles is required instead of at least one per square mile by photography at the same height, with a lens of the Ross Xpres type (70° field). Here again the results were good. Heights for sixteen points of known height were measured on the photographs and the average error was 26 feet, which is not excessive on scales of the order of this one.

Thompson concluded that the seven-lens camera is suitable for mapping at scales of 1 inch to 1 mile, or smaller.

*The United States Nine-lens Camera.*

This camera was designed with rather a different idea from that which resulted in the design of the seven-lens camera. The United States Coast and Geodetic Survey required the greatest possible coverage at scales of 1/10,000 and 1/20,000. The camera was fitted with Ross Wide-angle Xpres lenses of focal length 8¼ inches (70° field), and was constructed by the Fairchild Aerial Camera Corporation. A special feature is that each lens has a separate shutter and these shutters are opened by solenoids in series which all operate simultaneously. The camera is much larger than the Barr and Stroud because of the much greater focal length of the lenses. A rectifier is also provided, and it is stated that "it is practicable with controlled temperature and humidity, modern low-shrinkage film and paper, and micrometer adjustments, to compensate for shrinkage and distortion observed to produce composite prints within 0·01 inch."

This camera is now being used by the United States Soil Conservation Service for rather different purposes from those for which it was anticipated the seven-lens camera would be used. Where the control is too widely spaced for single-lens photographs, it is expected that the camera will be valuable for providing a means of radial triangulation. Also, it is proposed to use the camera for reconnaissance surveys at a scale of 2 inches

to 1 mile, or smaller. In this latter case, the lenses will presumably be of shorter focal length.

The Coast and Geodetic Survey consider that nine-lens cameras fitted with a central lens of the new ultra wide-angle type will make it possible to reduce the area covered by obliques, thus retaining the maximum clarity of definition.

Since the introduction of the ultra wide-angle lenses, there is some disagreement among air surveyors as to the future and range of multi-lens cameras. Zeiss, among the first to produce multi-lens cameras, have now ceased their manufacture consequent upon the production of cameras fitted with the "Topogon" lens. This firm now puts forward a number of criticisms of multi-lens cameras, among them being the statement, that the camera is of necessarily complex design and consequently costly; that additional expense is involved in auxiliary instruments, i.e., rectifiers; and that there are many possible sources of error in reproducing the equivalent vertical from, possibly, ten component pictures.

It is now almost generally agreed that wherever the same results of scale can be achieved by a single lens, such a lens should be used in preference to a multi-lens camera. Thus the useful range of multi-lens photographs appears to be below the new limit of verticals, i.e., between about 1/60,000 or 1/70,000 and possibly 2 miles to 1 inch. The latter scale may need a certain amount of photographic reduction, if the aircraft is to be flown at a convenient height.

The multi-lens camera appears to have possibilities for surveys on small scales in countries which are not very fully developed, and this is particularly so where differences of ground height are not very great. This is because of the great exaggeration of stereoscopic relief impression, which makes it difficult to contour in very rugged areas, but facilitates measurements in flattish areas. One drawback is the hiding of detail by height distortions at the edges of the photographs.

## Small-Scale Mapping from Oblique Photographs

Mapping from high-oblique photographs (i.e., those in which the horizon appears) is effected by a process involving perspective transformation, so that the equivalent vertical picture of part of the area is reproduced. Owing to the direction of photography being considerably inclined to the vertical, a large area may be covered by each photograph, with corresponding economy in photography for scales of the order of

4 miles to 1 inch. A very considerable area has been mapped from obliques in Canada.

*The Original Canadian Method.*

The photographs were taken in sets of three; forward, right and left, the camera being mounted on a gun track round the cowling of an open-nosed flying-boat of the "pusher" type. The speed of the aircraft was between seventy and ninety miles per hour, and the camera operator set the camera right of the line by using a sight to ensure that the horizon showed, and then in succession swung the camera to straight ahead and left, and continued the process until the end of the strip. The altitude varied between 3,000 and 5,500 feet. Narraway [67] stated that by this method the strip was approximately eight miles wide, and that in three hours it was possible to photograph 1,680 square miles. He remarks that although methods have been developed for applying corrections due to height distortion, experience in Canada has shown that the labour involved does not make it worth while. Also, at that time (1929) with 125,000 square miles of Canada surveyed by this method, any point as scaled on the map should be within one-twentieth of an inch of its true geographical position. These maps are plotted on a scale of 4 miles to 1 inch, and the results are better than could be expected of a similar ground survey.

*The Improved Canadian Method.*

Aircraft now employed is of the cabin type, and three cameras coupled together have been substituted for the process of swinging one camera. Thus, once the positions of the photographs on the central strip have been ascertained, those of the side photographs are easily fixed. A special mounting is provided for the camera with one camera pointing directly back under the tail of the aircraft, the second pointing out of one window at, say, 45° to the other, and the third similarly in a position to the left. That is to say, all the pictures are taken simultaneously instead of in succession; they overlap at the edges and all three cameras point backwards. Sets of photographs are taken every 2½ to 3 miles, the ground coverage being as shown in Fig. 146. This procedure has resulted in several advantages over the old method. The speed of aircraft has been increased to between 140 and 150 miles per hour, thus giving a greater rate of climbing, economy of fuel and more chance of dodging the clouds. The endurance of the aircraft has been increased from about five to eight hours and this is important in a country where there are often considerable distances between places where fuel may be obtained. Photography is carried out at about 8,500 feet instead of 5,000 feet or less and this increased altitude

gives stability to the flying costs as well as reducing noticeably photographic costs by increasing the coverage.

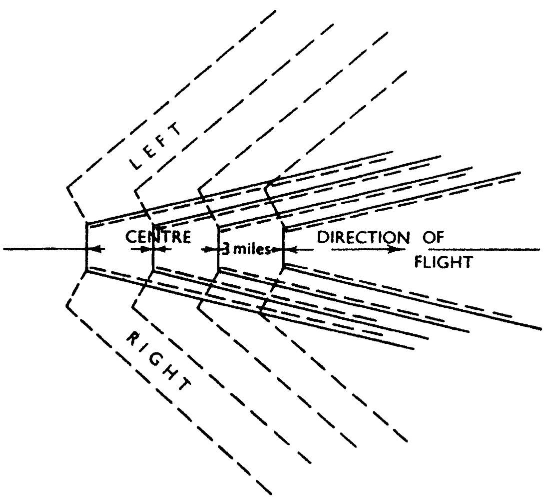



FIG. 146.

Also, by having the cameras in fixed relation to one another, it has been possible to reduce the overlap of the set, thus obtaining an angular field of about 160° against 130° by the method of successive photography. This has enabled the strips to be about 25 per cent farther apart. Burns [10] points out that plotting from a reduced scale of photography has been made possible by improved fine-grained photographic emulsions.

The cameras can be calibrated on the ground by photographing special targets. Although the horizon will not always be included on all three photographs, it is possible, from the known relationship of the three cameras, to establish the orientation of the other two when that for one has been determined.

*Characteristics of the Oblique Photograph.*

The essential perspective theorems have been given in Chapter V, and it now remains to show how they are applied in constructing a perspective grid upon a high oblique photograph. The method which follows has been used for hundreds of thousands of square miles of Canada, in those extensive areas of lake and forest country over which there is little variation of ground level, so that the masking of detail by that in front of it on the oblique is not considerable.

A full account of the method employed in Canada has been given by Narraway in *Applied Aerial Photography* by McKinley.[67] He mentions the difficulty of drawing a perspective grid accurately for each photograph taken, and the procedure has been to make a series of grids corresponding to altitudes at intervals of twenty-five feet over the required

range, and for varying distances, at intervals of 0·1 inch of the horizon from the upper margin of the photograph. This latter distance is a measure of the tilt of the photograph at exposure.

These grids are drawn accurately four times as large as required, i.e., 1 inch to 1 mile, and are then reduced photographically for reproduction on a transparent medium, such as glass or celluloid. When a grid is required for use on a photograph, the correct one is selected by trial and error with reference to control points. Narraway remarks: "In plotting, the photograph is treated as a perspective, the surface of the earth as a plane, and the position of the camera as the origin of another plane parallel with the assumed plane of the earth's surface. . . . The photograph, however, was taken from far above the ground and the resultant perspective is not in the ground plane but is in the camera plane, and the horizon line in which the parallels of the perspective merge is not the horizon line of the photographs but an imaginary horizon line in the plane of the camera."

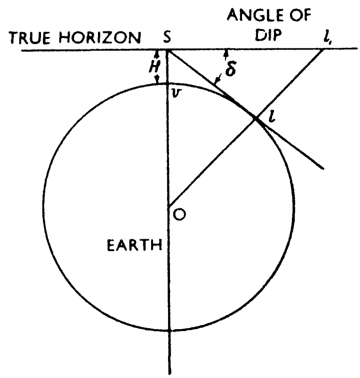



FIG. 147.

Thus the grid will show the true horizon, but the photograph will show the visible horizon. Hence corrections must be made for the difference between them, which is due to the curvature of the earth and to atmospheric refraction as a result of the greater density (and corresponding grèater refractive index) of air near the surface of the earth. This enables the depression of the camera axis to be determined.

In Fig. 147, $H$ is the height of the aircraft and $\delta$ is the dip of the visible horizon. Narraway has given an approximate formula for the dip $\delta$ in seconds—

$$\delta = 58{\cdot}82\sqrt{H} \qquad \text{(XI.1)}$$

This allows for curvature and assumes (without appreciable error) that the correction for atmospheric refraction remains constant.

*Construction of the Perspective Grid.*

Each grid is constructed for a specific case, and later, when required, the correct grid is selected for the photograph, in relation to the control and known data; if the height of aircraft be $H$ and the tilt from the vertical $\theta$ as before. In Fig. 148a, S is the camera station and SP the principal line, this view being drawn in the principal plane. The vertical line S$v$ is the

height $H$ of the aircraft, while the horizontal line through $v$ represents the true ground horizontal at the ground plumb point, the curved surface of the earth being indicated in the sketch. The horizontal line through S which cuts the principal line in $l_1$ is the true horizontal at the height of aircraft, and this point $l_1$, shown also in Fig. 148b, is the vanishing

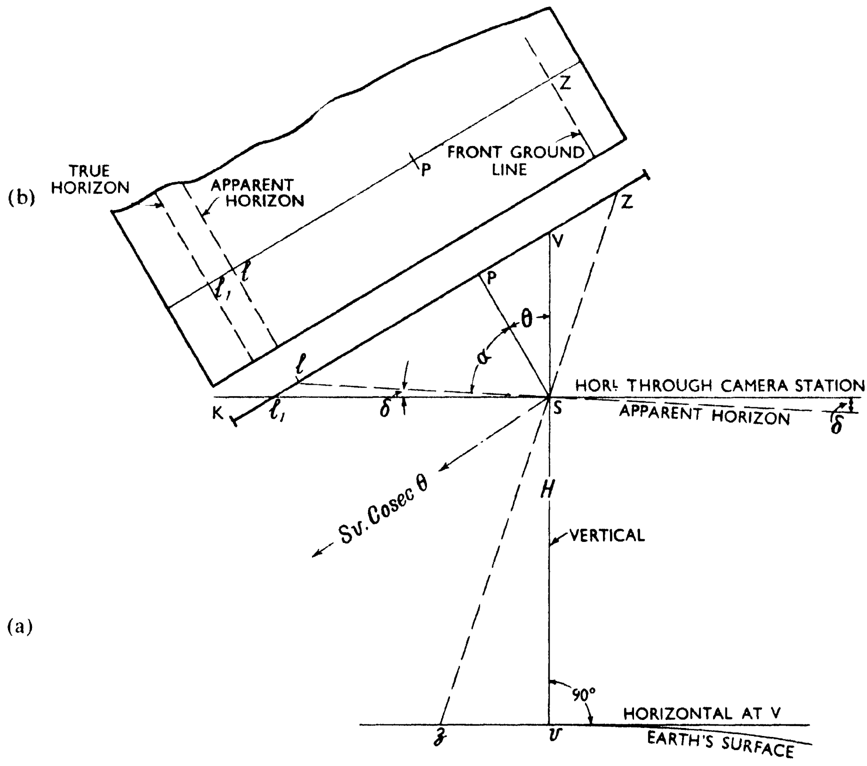



FIG. 148.

point for all lines on the ground parallel to the principal plane. This illustration is a view of the photograph looking in the direction of the principal line SP. A line drawn through $l_1$ in Fig. 148b perpendicular to the principal line through P is the true horizon. This true horizon does not appear on the photograph and the apparent horizon through $l$ is drawn parallel to the true horizon. These are both plotted so that the relationship between true and apparent horizon can be established for any case.

PS is the focal length $f$ of the camera lens, $\alpha$ the apparent inclination

of the photograph to the horizontal, $\theta$ the tilt of the camera axis from the vertical, and $\delta$ the curvature effect.

Then if $(90-\alpha)$ is the apparent tilt

$Pl_1 = f.\cot\delta - f.\tan(\alpha + \phi)$.

Now $\delta$ is known from Equation (1), and $\alpha$ may be determined, so that $Pl = f.\tan\alpha$ and therefore $ll_1 = Pl_1 - Pl$.

In this case the position of $l_1$ is assumed and $l$ found from the above. In practice, $l$P is measured to calculate $\alpha$, and thence $\theta$, by correcting for $\delta$. This enables the true horizon to be fixed on a photograph from the apparent horizon.

The first step in plotting the grid is to draw the meridians, i.e., the grid lines parallel to the principal line. It is found convenient to plot a "front ground line" along which the grid lines are one inch apart, the corresponding ground distances being ten Gunter's chains (660 feet).

The process assumes that the ground is a flat plane so that straight lines in this plane will reproduce as straight lines in the perspective view.

If the meridians are spaced one inch along a plate parallel through Z on the principal line, known as the "front ground line", the scale will be one inch to 660 feet.

From Fig. 148a scale at $Z = ZS/Sz = Zl_1/Sv.\text{cosec}\,\theta = 1/660$ (where distances on ground are measured in feet and those on the photograph are measured in inches).

$$\text{Hence } Zl_1 = \frac{H.\text{cosec}\,\theta}{660} \qquad \text{(XI.2)}$$

The plotting of the grid is shown in Fig. 149. First a horizontal line is drawn and $l'_1$, which corresponds to $l_1$ in Fig. 148, is marked and $l'_1Z'$ is set off perpendicular to this line and equal to $Zl_1$. Also $l'_1P'$ is set off equal to $l_1P$ to fix the position of the principal point. At right angles to the principal line and through Z′ are set off A′, C′, E′, F′, D′ and B′ at intervals of one inch, and these points are joined to $l_1'$, thus fixing the perspective meridians.

Next the plate-parallels must be drawn. It is clear from Chapter V that these lines will be parallel to the horizon and "front ground line" and it is necessary to place them at their correct distances apart, which will vary through the depth of the photograph (i.e. along ZP) owing to the constantly changing scale.

In Fig. 150 is shown a diagram of the grid which it is required to show on the photograph, $p$ being the ground principal point. The direction of the principal line is $px$ and $pd$ is the diagonal, making an angle $\alpha$ with this line. In Chapter V (Equation 6) it has been shown that when

angles are measured from the principal point, $\tan \alpha = \tan \beta . \cos \theta$, where $\beta$ is the corresponding angle on the photograph. Through P′ on the grid

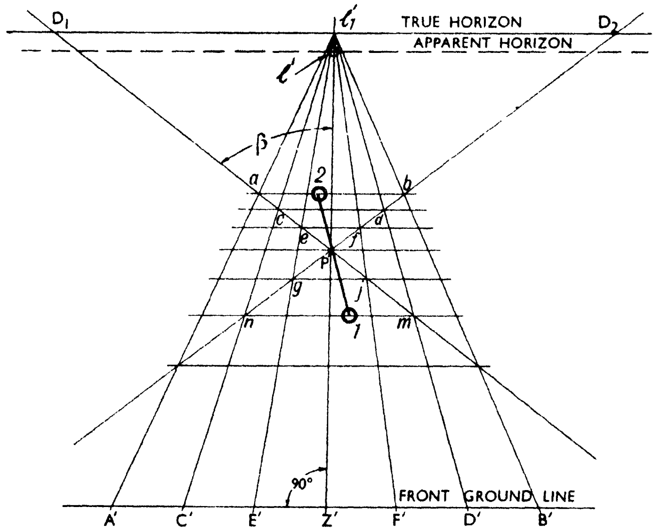



FIG. 149.

a line is drawn at an angle $\beta$ with the principal line, meeting the true horizon line in $D_1$. This is the vanishing point on the photograph for all lines parallel to *pd* on the ground.

From Fig. 149, $D_1 l_1' = P' l_1' . \tan \beta$.

Now $P' l_1'$ (equal to $Pl$ in Fig. 148) $= f \cot \theta$ and $\tan \alpha = \tan \beta . \cos \theta$.

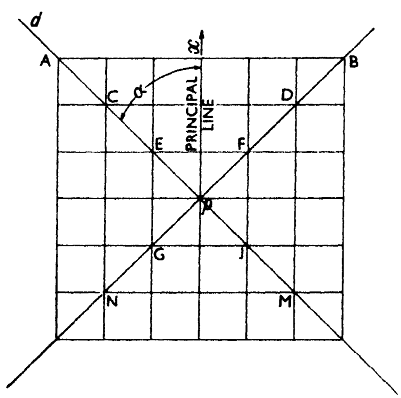



FIG. 150.

Therefore the distance of diagonal vanishing point from $D_1 l_1' = f . \operatorname{cosec} \theta . \tan \alpha$ . . . (XI.3)

If, as is usual, the grid to be reproduced is square, then $\alpha = 45°$ and $D_1 l_1' = f . \operatorname{cosec} \theta$ (XI.4)

The vanishing point for the other diagonal $D_2$ is similarly found.

The remainder of the grid may now be simply constructed. In Fig. 150 is a large square sub-divided into thirty-six equal small squares, which is to be

drawn in perspective. The corners of the squares, through which the diagonals pass, are indicated by capital letters. If it be assumed that the ground plane is level then all straight lines on the ground will also be straight lines on the grid, and the corners of the squares will be fixed on the perspective grid at the points of intersections of meridians and diagonals. Thus the points A and B on the ground appear respectively at *a* (intersection of $D_1P'$ and $l_1'A'$) and B at *b* (intersection of $D_2P'$ and $l_1'B$). The points *a* and *b* are symmetrical about the principal axis and *ab* is the perspective representation of the line AB. Similarly *cd* is the perspective representation of the line CD. Thus the grid may be completed.

*Selection of Appropriate Grid for a Photograph.*

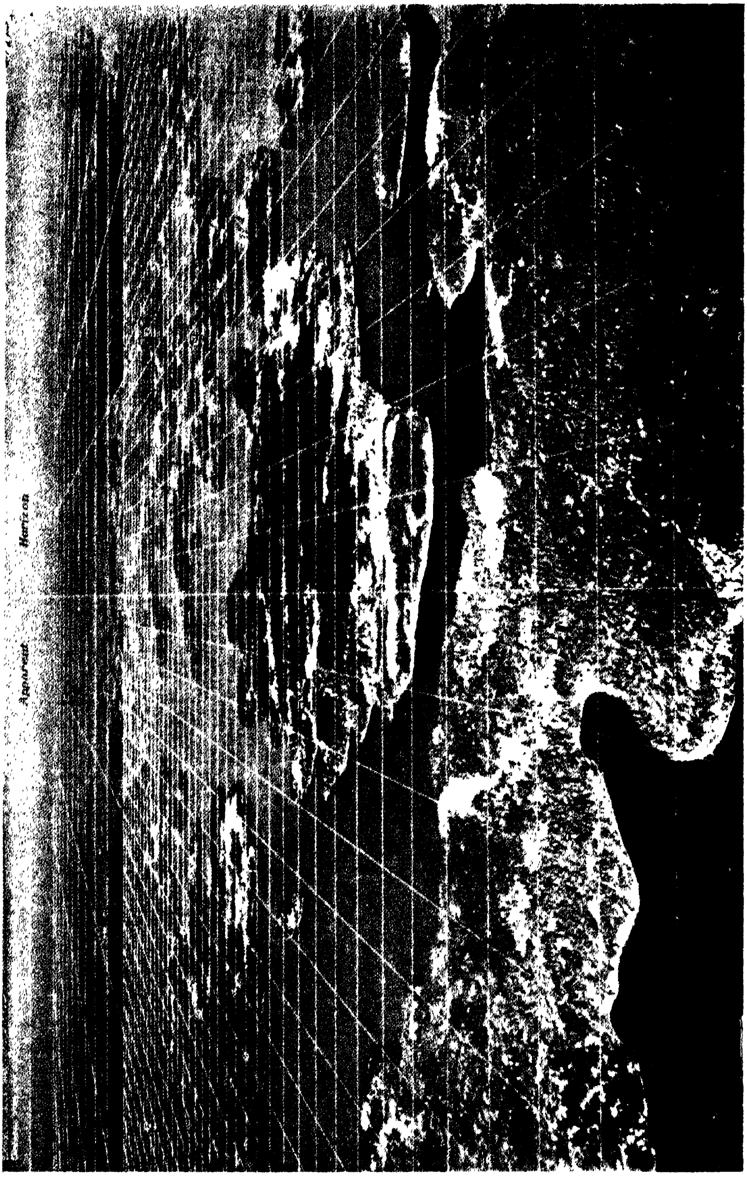
Grids are available for each 25 feet of height and for each 0·1 inch of marginal distance (K*l* in Fig. 148a). The principal point of the photograph is found at the intersection of the collimating marks, and the marginal distance is measured. Then, knowing the focal length, and having obtained the altitude as accurately as possible from the altimeter and statoscope readings with appropriate corrections, a possible grid may be selected. The correction for distances of true horizon from the apparent horizon is marked on each standard grid. When an accurate selection is required, this grid should be found from known ground points. In Fig. 149 points 1 and 2 are such points, their distance apart being known. From the assumed altitude and tilt, the distance between the two points may be found and compared with the correct distance. Thus suppose the distance is found to be 70·55 chains and it is actually 68·41 chains. It follows that the altitude, as taken, must be increased in the proportion of 70·55/68·41, and another grid selected. When this grid is tested it is unlikely that any further change will be necessary.

Much progress has recently been made in Canada with improved determination of height of aircraft and a number of surveys have been made with reference to occasional astronomically fixed points or with almost complete absence of ground control on small scales. Where ground control is employed it is necessary that the adjoining photographs should overlap so that plotting may be continued.

A specimen oblique photograph and rectified plot are shown in Figs. 151 and 152.

*Plotting the Map.*

As with other methods, the ground control points are plotted on a master grid at the required scale. The process involves plotting of the overlapping series of photographs in strips which also have a common overlap.

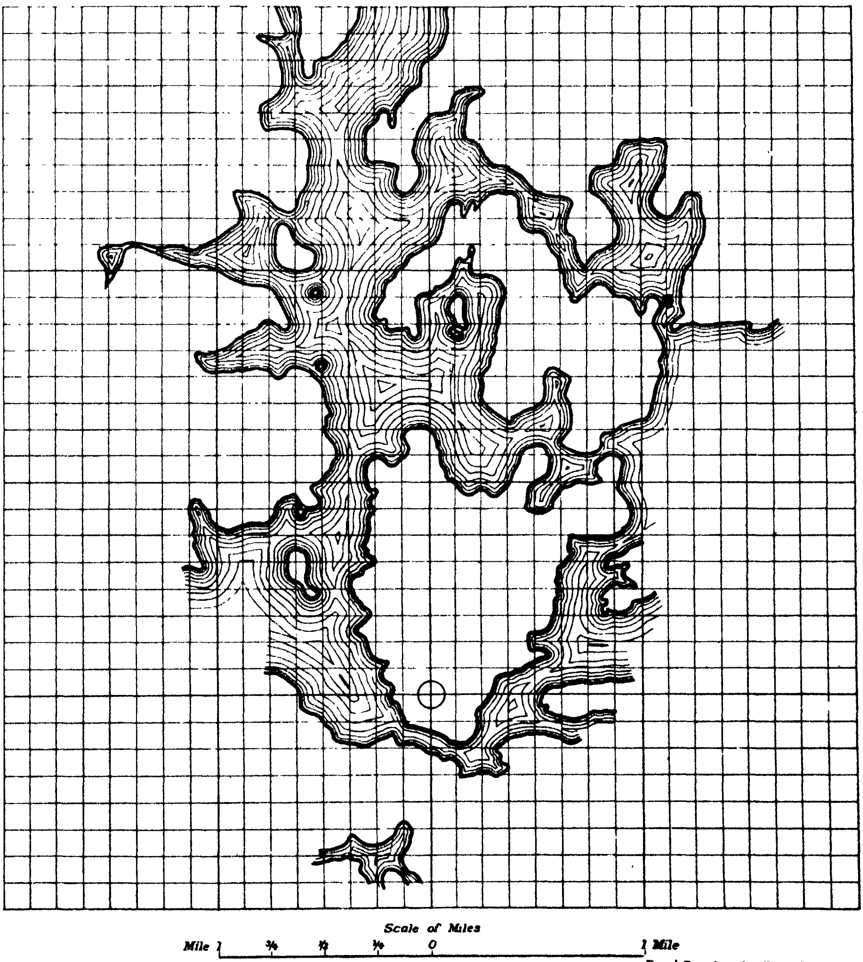



[*Royal Canadian Air Force Photograph.*

FIG. 151—TYPICAL CANADIAN HIGH OBLIQUE, WITH GRID SUPERIMPOSED.

Data:—Focal length 6·701 inches; distance from margin to apparent horizon 0·3 inch; elevation 3,000 feet; distance from grid centre to ground plumb point 118·34 chains.

An "azimuth line" is chosen on a photograph, near the centre of the strip, so that it passes through three or more points in the foreground and goes to a definite point, well in the background. These points are all chosen as nearly at water level as possible, to avoid height distortions. Owing to the tendency of the aircraft to drift, or otherwise deviate from the path when flying a strip, this line may move away from the principal line in successive photographs, and it may be necessary to choose another azimuth line, which can be continued along the whole strip.




Royal Canadian Air Force Photograph

RECTIFIED OUTLINE PLOT
RECTANGULAR GRID OF 10-CHAIN SQUARES.

FIG. 152.

First, a photograph with two points of ground control is taken and the correct grid is selected as previously described. The master grid having been previously produced on transparent paper and with reference to the ground control, the azimuth line may be transferred to it from the photograph. For the next photograph two points as far as possible from the azimuth line and which have been identified on the previous photograph are selected, and positions determined. One of these points will be in the background of the second photograph and about the middle of the first one, while the other will appear well in the foreground of the second photograph. In this manner the azimuth line is carried on to the end of the strip, the appropriate grid being used for each photograph, the plot giving a trial "traverse" along the line of the strip. Narraway points out that it is better to start from each end of the strip and work towards the middle.

By tracing off in succession from the azimuth line plot of each photograph, a trial plot is obtained which also shows the control points as they have been plotted in relation to the starting control. This plot is then adjusted to scale on the master plot in a manner similar to that adopted for the Arundel Method.

Owing to the obliquity of the photographs, a considerable area in the extreme background is not used for detail plotting because it will be shown nearer the foreground of another photograph. Hence, in plotting, the foreground of the first photograph is marked in and then as much of the next one as is required is similarly treated, and so on. The detail is plotted by reference to the corresponding squares which appear on the photograph and master grid.

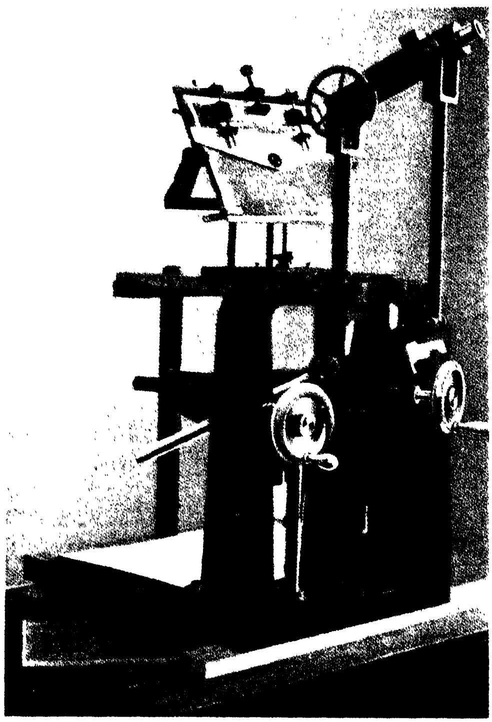
With regard to the right and left photographs (see Fig. 146) their relationship to the central one is known, so that the skewed grids can be related to the central strip, and the side detail thereby plotted.

*Canadian High Oblique Plotter.*

A plotter for use with the Canadian high obliques has been produced from designs of Colonel E. L. M. Burns of the Royal Canadian Engineers and Mr. R. H. Field, the Supervisor of the Physical Testing Laboratories of the Canadian National Research Laboratories. It has been described in a Canadian research publication.[11] The object of the design was to provide a simple mechanical solution of the perspective transformation effected by the grid method given above. By measuring the horizontal and vertical angles subtended by the camera station in the photographic field it is possible to resect the position of the camera station in space. A condition is that the apparent horizon must appear in the photographs.

The instrument embodies a telescope by means of which the direction of a perspective ray may be established in space by observing a correctly oriented photograph. A space rod is coupled to the telescope in such a way that, as the telescope is directed upon an image, the space rod moves a tracer which plots the corresponding ground plan of the homologous point.

"In this particular instrument the operation is simple, in that the telescope and space rod are connected by a parallelogram linkage so as to remain always parallel to one another, while both follow the direction of the perspective rays. The intersection of the axis of the space rod on a horizontal plane, will, therefore, be the ground plan of the point on whose image the telescope is directed."



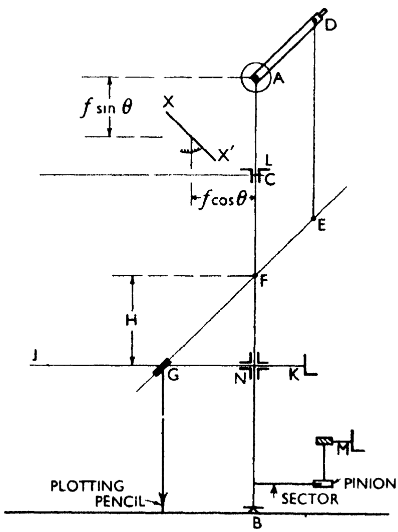
*[Courtesy of National Research Council of Canada.*

FIG. 153—CANADIAN HIGH OBLIQUE PLOTTER.

The plotter is illustrated in Fig. 153 and by a diagram in the principal plane of the picture given in Fig. 154. The tilt $\theta$ and height $H$ are determined as before. The photographic principal plane is represented by XX′ which is set to the angle of tilt while the height of slide N is set so that the screw KJ is a distance $H$ to scale vertically below the pivot F. When the telescope DA is directed to any point of detail on the photograph, the parallelogram DAFE transmits this direction to the rod FG. The position of G fixes the point of detail in plan according to the scale of setting, and a pencil vertically below G plots the position of the point on the plan. In operation, positions are set by turning the two hand-wheels K and M. Horizontal and vertical angles are measured on the graduated circles at L and A. The jig shown in position on the photograph of the instrument (Fig. 153) was used to set the principal point and to bring the apparent

horizon into the horizontal position. It was found in operation that this could be done as well, and more quickly, with the telescope.

The machine was designed for the small-scale Canadian Surveys with application to areas of small relief and consequently there are many possible sources of inaccuracy which could not pass disregarded in the design of a more elaborate instrument. These errors are not great at very small scales.




[*Courtesy of National Research Council of Canada.*

FIG. 154—DIAGRAM OF CANADIAN HIGH OBLIQUE PLOTTER.

The plotter was found suitable for planimetry where ground heights are not very variable, and experiments were made with it to test its possibilities for contouring. The photographs were set in the plotter and horizontal and vertical angles measured to the control points, their directions being plotted by the machine on tracing paper, which was later used for the resection of the plumb point. Subsequent computation of height of aircraft gave small errors of tilt and of orientation of the photograph, and these were adjusted in position on the photograph table, so that finally exposure heights, as determined, showed agreement.

After measurement of vertical and horizontal angles to all height control points, intersections of the rays to the points on different photographs give the plan position, and the heights of the points are then computed from the photographs. By obtaining three values, accuracy can be checked. From these points, contouring can be carried out by stereoscopic observation, but the impression is flatter for obliques than for verticals and the method is not likely to be used in competition with ultra wide-angle or multi-lens photography.

Burns and Field consider that the chief value of this apparatus for levelling is where the country is inaccessible, and where the distant control points upon which Crone's method is dependent are not available. They say, "Maps might be required of areas where distant instrumental control* would not be possible, as when a ridge interposed between accessible

* i.e., intersected instrumental rays.

country and the ground it was desired to map." Here the method is feasible.

Another experiment (covering 325 square miles at one end of Great Slave Lake) was made with the three-coupled cameras. It was found possible to combine a process of intersection and resection "to give a good aerial triangulation, in a manner similar to the radial-line method with verticals." Unlike the simple method of plotting from obliques, control can be fixed despite variations of ground height. In Canada much of the small-scale plotting is over areas where these variations of ground height may be neglected, so that the additional complication of all these intersections is not needed.

From these experiments it was found that the plotting of a single photograph having an area of about six square miles, took between half and three-quarters of an hour, including setting the photograph, and the instrument seems likely to be very useful for plotting.

*Other Methods of Plotting from Obliques.*

Other methods than those employed in Canada for plotting from oblique photographs have been evolved and used from time to time. There is, however, little likelihood that oblique methods will be developed extensively in the future. Where there are no appreciable changes of level obliques may be used for planimetry. Usually, however, small-scale mapping must also include contours, and here the oblique method tends to become inaccurate, without special precautions. By using methods for determining height of aircraft from control points known in position and height (such as those of Crone of the Survey of India as described in Chapter IX, or of Miller of the American Geographical Society's School of Surveying), and thence fixing auxiliary points, it is possible to allow for height distortions and, by stereoscopic observation on the obliques, to obtain approximate contours.

Miller [96] has also devised a plotting instrument with a single eye-piece for obliques. Above the drawing-board used for plotting there is an arm which is pivoted round the perspective centre. This arm carries the plotting-pencil and above it there is an illuminated mark. The photographic plate is set in a camera which is set at the correct orientation tilt and height (to scale). Thus the perspective conditions are reconstructed. The illuminated mark is projected through the camera lens on to the plane of the plate, and appears to the observer to be superimposed on the picture. As the spot of light is moved over the photograph, the pencil point moves in such a manner that the plan position is plotted on the drawing-board. By plotting also features of a lateral oblique, in which similar

points are established from a different angle, it is possible to eliminate height distortions.

Obviously, in plotting, when the correct position of a point has been established by intersections from two or more photographs, a measure of the height of the point can be made by finding the vertical movement of the photograph in order to bring the illuminated mark back on to the point of detail in a particular photograph.

### General Conclusions as to the Choice of Method for Small-Scale Maps

The method will be dependent upon the final scale required. In almost every area of country it is probable that a map to the scale of 1 inch to 1 mile will eventually be made, and photographs taken with ultra wide-angle lenses, or those with multi-lens cameras, will probably fulfil the requirements of almost every case. Not only will it be possible to plot the survey by the radial-line method, but stereoscopic observation, either by a simple stereoscope in relation to ample ground control, or by a more elaborate instrument with minimum of ground control, will enable adequate contours to be plotted. There appears to be an advantage in fixing some control points for height in the cases instanced by Crone.

Reduction to a scale of 4 miles to 1 inch can be achieved without much difficulty, where this scale is required, from multi-lens or ultra wide-angle photographs.

It appears that in the future the obliques will be limited to those areas unlikely to be developed to any great extent and where it is expected that a map scale larger than 4 miles to 1 inch is unlikely to be required.

It must be remembered, however, that oblique photographs are very valuable for interpretation where non-technical persons may have occasion to study them. Oblique photography will often be required in addition to the verticals directly for mapping.

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| A.S.C. | Air Survey Committee of the War Office, London. |
| Am. Soc. C.E. | American Society of Civil Engineers. |
| Am. Soc. Ph. | American Society of Photogrammetry. |
| *C.E.* (Am.) | *Civil Engineering* (American). |
| *E. S. R.* | *Empire Survey Review.* |
| H.M.S.O. | His Majesty's Stationery Office, London. |
| I.C.E. | Institution of Civil Engineers. |
| Con. E.S.O. | Conference of Empire Survey Officers, London, 1935. |
| *P. E.* | *Photogrammetric Engineering*, Washington. |


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Authors whose work has been consulted, but who have not been specifically mentioned in the text, are referenced in the bibliography.

# INDEX

PLATE I

About $\frac{1}{20}$ original Size

Zeiss Stereoplanigraph

Instrument in position for plotting from oblique photographs

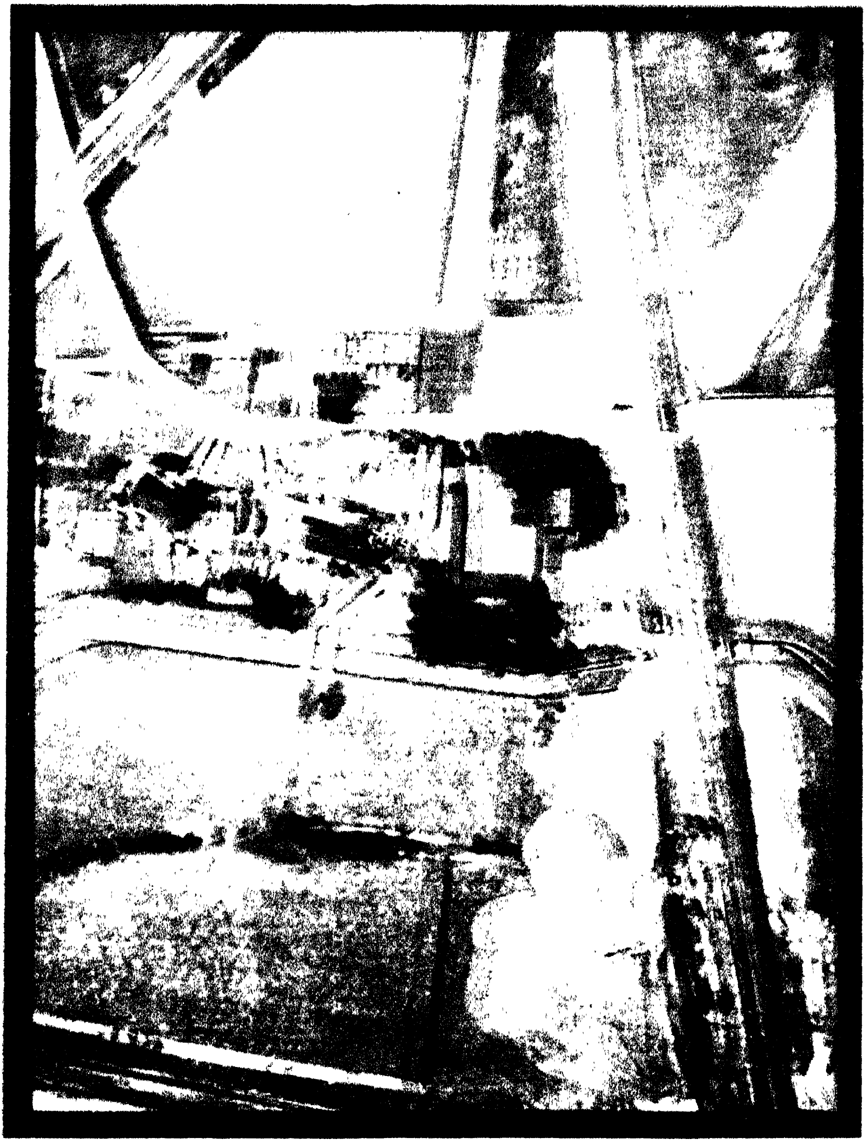
PLATE II

Landscape in Ecuador

Photograph taken with Zeiss Air Survey Camera RMK P 10

fitted with Topogon lens

PLATE III



German Landscape: Farm Houses and Communications

Taken with Zeiss Air Survey Camera RMK. 21

f = $8\frac{1}{4}$ in.   Flying Height 1,500 ft.

# PLATE IV

German Town in Central Mountain Region

Taken with Zeiss Air Survey Camera RMK. 21

$f = 8\frac{1}{4}$ in. Flying Height 3000 ft.

Approximate Scale 1 : 4400

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